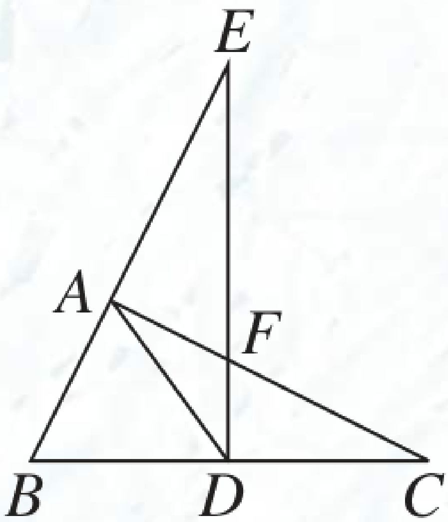
阶段拔尖专训12 证明等积式或比例式的技巧

技巧1 三点定型法

1.如图所示，在$\triangle ABC$中，$AB \perp AC$，D为BC的中点.$DE \perp BC$交AC于点F，交BA的延长线于点E.求证：$AD^2 = DE \cdot DF$.



【证明】$\because AB \perp AC$, D为BC的中点，

$\therefore AD = \frac{1}{2}BC = DC. \therefore \angle DAC = \angle C.$

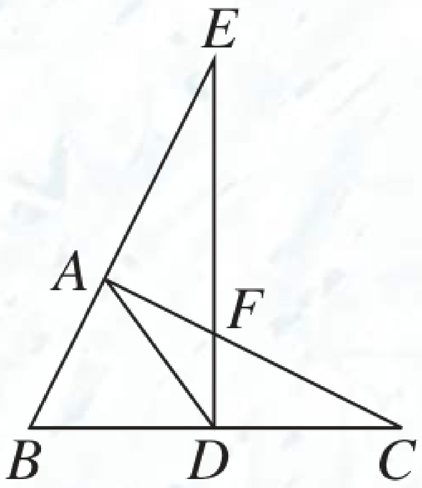
$\because AB \perp AC$，$DE \perp BC$，

$\therefore \angle C + \angle B = 90°$，$\angle E + \angle B = 90°$.

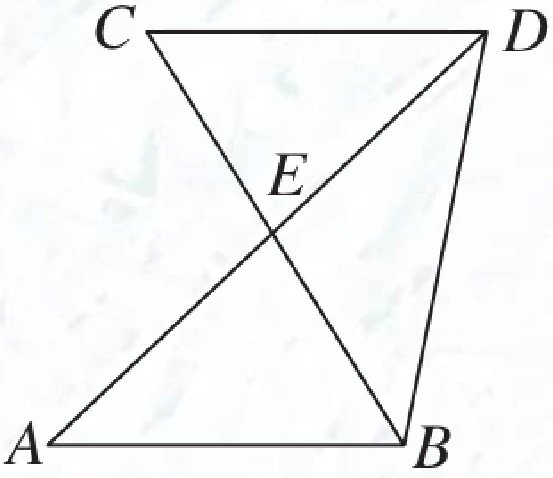
$\therefore \angle C = \angle E. \therefore \angle DAC = \angle E.$

又$\because \angle ADE = \angle FDA$, $\therefore \triangle DAE \backsim \triangle DFA.$

$\therefore \frac{DE}{AD} = \frac{AD}{DF}. \therefore AD^2 = DE \cdot DF.$



2.[2024·合肥包河区期中] 如图，$AB//CD$，AD与BC相交于点E，$\angle A=\angle CBD$.



（1）求证：$CD^2=BC\cdot CE$;

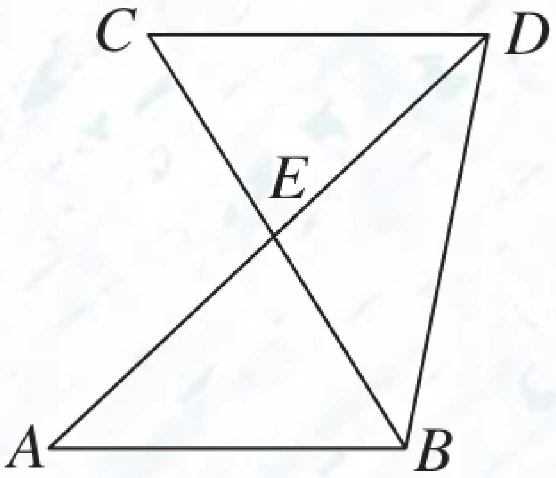
【证明】$\because AB//CD$，$\therefore \angle A=\angle ADC$.

又$\because \angle A=\angle CBD$，$\therefore \angle ADC=\angle CBD$.

又$\because \angle C=\angle C$，$\therefore \triangle CDE\backsim\triangle CBD$.

$\therefore \frac{CD}{BC}=\frac{CE}{CD}.\therefore CD^2=BC\cdot CE$.

（2）若$CD=1$，$BD=2$，$AB=3$，求DE的长.



【解】$\because AB // CD$,

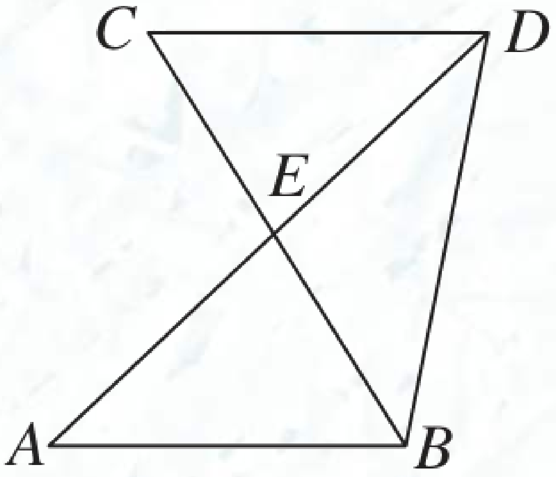
$\therefore \angle EAB = \angle EDC$, $\angle EBA = \angle ECD$.

$\therefore \triangle CDE \backsim \triangle BAE$. $\therefore \frac{CD}{AB} = \frac{CE}{BE}$.

$\because CD = 1$, $AB = 3$, $\therefore \frac{CE}{BE} = \frac{1}{3}$.

设$CE = x$，则$BE = 3x$, $\therefore BC = 4x$.

$\because CD^2 = BC \cdot CE$,

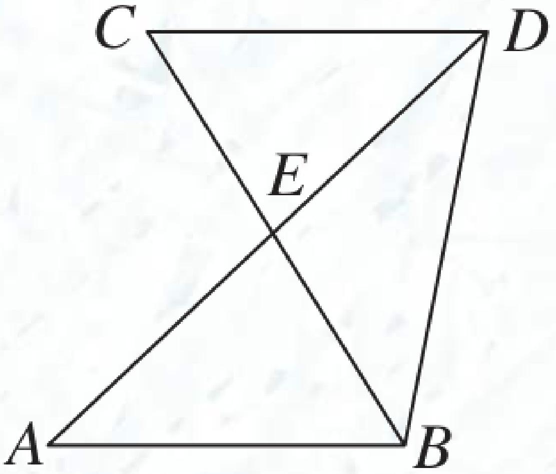


$\therefore 1 = 4x \cdot x$，解得$x = \frac{1}{2}$（负值已舍去）．

$\therefore CE = \frac{1}{2}$.

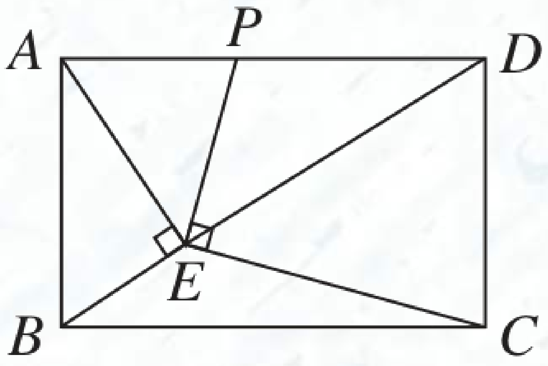
$\because \triangle CDE \backsim \triangle CBD$，$\therefore \frac{DE}{BD} = \frac{CE}{CD}$.

$\therefore \frac{DE}{2} = \frac{\frac{1}{2}}{1}$，解得$DE = 1$.

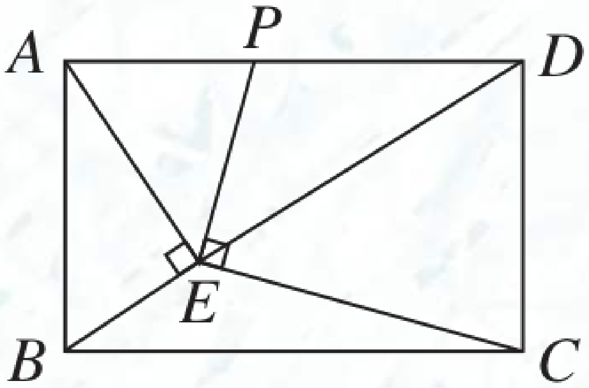


技巧2 等线段代换法

3.[2024·济南历下区一模] 如图，在矩形$ABCD$中，$AE \perp BD$于点E，点P是边AD上一点.若$PE \perp EC$，求证：$AE \cdot AB = DE \cdot AP$.



【证明】∵ 四边形$ABCD$是矩形,



$\therefore \angle ADC = 90°$ ，$AB = CD.$

$\because AE \perp BD, PE \perp EC,$

$\therefore \angle AED = \angle PEC = 90°$.

$\therefore \angle EAD + \angle ADE = 90°$ ，

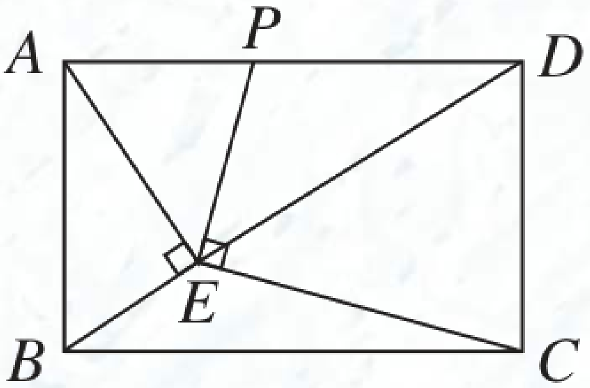
$\angle AEP = \angle DEC.$

$\because \angle ADE + \angle CDE = 90°$ ，

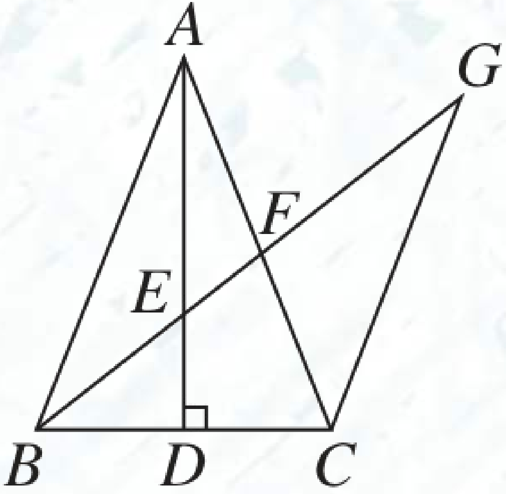
$\therefore \angle EAP = \angle EDC. \therefore \triangle AEP \backsim \triangle DEC.$

$\therefore \frac{AE}{DE} = \frac{AP}{DC}. \therefore AE \cdot DC = DE \cdot AP.$

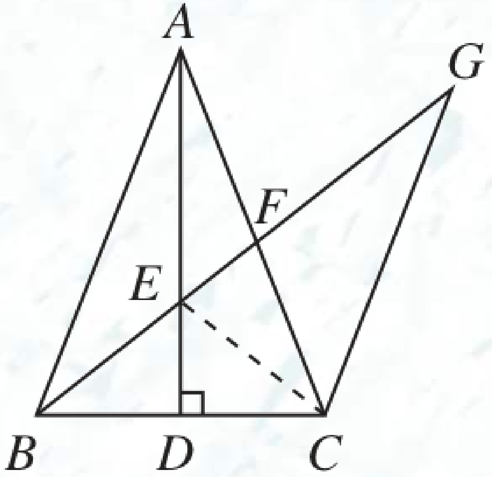
又$\because AB = CD, \therefore AE \cdot AB = DE \cdot AP.$



4.如图，在等腰三角形ABC中，$AB=AC$，$AD\perp BC$于D，$CG//AB$，BG分别交AD，AC于E，F.求证：$BE^2=EF\cdot EG$.



【证明】连接CE,如图所示.



$\because AB = AC, AD \perp BC,$