计算机问题求解 – 论题1-10

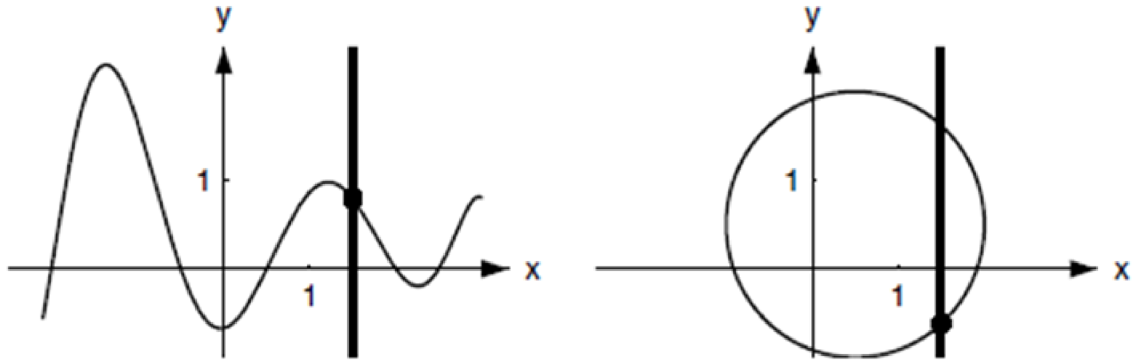
函数

问题1:

"函数"与"关系"有什么异同?

问题2：

这里的function与你中学时熟悉的函数有什么异同？



You probably learned that a function $f : \mathbb{R} \to \mathbb{R}$ can be represented by a graph, and that there is a vertical line test to determine whether or not f is a function (See Figure above) . Which condition in the definition corresponds to the vertical line test? Why?

问题3：

你是否能解释一下？

问题4：

关于函数自变量的集合只有一个 (domain)，关于函数值的集合却有两个 (codomain和range)，为什么？

When you define a new mathematical concept, it's always a good idea to think about it and pose questions. Of course, it's also a good idea to answer those questions, if you can. We now turn to some questions that we find interesting. See if you can think of some questions on your own.

问题5:

书中提出了什么问题? 你想出了什么"自己"的问题吗?

几种特殊的函数

- **满射**
  - f:A→B是满射的：ran$f$=B, iff. ∀y∈B, ∃x∈A, 使得f(x)=y
- **单射（一对一的）**
  - f:A→B是单射的：∀y∈ ran$f$, ∃!x∈A, 使得f(x)=y iff. ∀x_1,x_2∈A, 若$x_1 \neq x_2$，则$f(x_1) \neq f(x_2)$ iff. ∀x_1,x_2∈A, 若$f(x_1) = f(x_2)$，则$x_1=x_2$。
- **双射（一一对应的）**
  - 满射+单射

几种特殊的函数：例子

- f:$R \to R$, $f(x) = -x^2+2x-1$
- f:$Z^+ \to R$, $f(x) = \ln x$, 单射
- f:$R \to Z$, $f(x) = \lfloor x \rfloor$, 满射
- f:$R \to R$, $f(x) = 2x-1$，双射
- f:$R^+ \to R^+$, $f(x) = (x^2+1)/x$
  - 注意：$f(x) \geq 2$, 而对任意正实数x，$f(x)=f(1/x)$
- f:$R \times R \to R \times R$, $f(<x,y>) = <x+y, x-y>$, 双射。
- f:$N \times N \to N$, $f(<x,y>) = | x^2-y^2|$

问题6：为什么？

有限集合上一一对应的函数的例子

- S={1,2,3}, 可以在S上定义6个不同的一一对应的函数 (每一个称为一个“置换”):

$$e=\begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix}\qquad \alpha=\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}\qquad \beta=\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}$$

$$\gamma=\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix}\qquad \delta=\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}\qquad \varepsilon=\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix}$$

Given functions $f : A \to B$ and $g : C \to D$ with $\text{ran}(f) \subseteq C$, we can define a third function called the **composite function** from A to D. (We will usually call this the **composition**, rather than the composite function.) This composition is the function $g \circ f : A \to D$ defined by $(g \circ f)(x) = g(f(x))$.

Let R be a relation from A to B and S be a relation from B to C. Then we can define a relation, the composition of R and S written as SoR. The relations SoR is a relation from the set A to the set C and is defined as follows:

If $a \in A$, and $c \in A$, then $(a, c) \in S o R$ if and only if for some $b \in B$, we have $(a, b) \in R$ and $(b, c) \in S$.

问题7: 这两个定义有什么关联?