

Single-Source Shortest Paths

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Topics

1. Bellman-Ford algorithm
2. Single-source shortest paths in directed acyclic graphs
3. Dijkstra's algorithm
4. Difference constraints and shortest paths
5. Conclusion

Shortest-paths Problem



Shortest-paths Problem



Shortest-paths Problem



Shortest-paths Problem

Problem 1 Given a weighted, directed graph $G = (V, E)$. The weight of a path $p = \cdot v_0, v_1, \dots, v_k$ is

$$w(p) = \sum_{i=1}^k w(v_{i-1}, v_i) \quad (1)$$

The **shortest-path weight** from u to v is

$$\delta(u, v) = \begin{cases} \min\{w(p) \mid u \xrightarrow{p} v\} & \text{if there is a path from } u \text{ to } v \\ \infty & \text{otherwise} \end{cases} \quad (2)$$

A **shortest path** from u to v is the defined as any path p with weight $\delta(u, v)$.

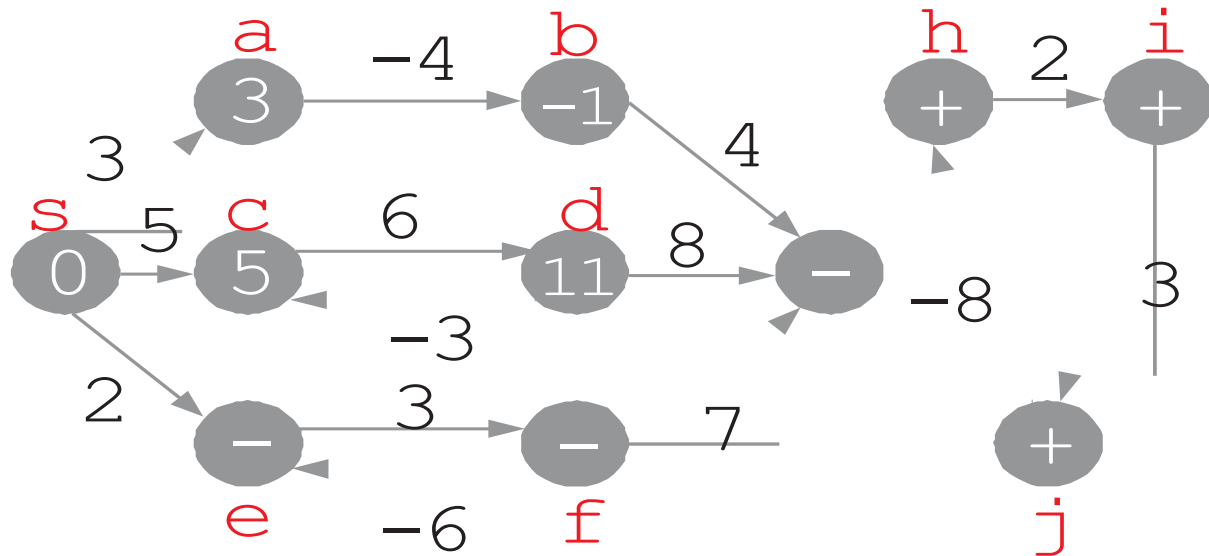
In this course, we just focus on the problem of single-source shortest-paths.

Optimal Substructure of Shortest Path

Homework 1 Given a weighted, directed graph $G = (V, E)$ with weight function $w : E \rightarrow \mathbb{R}$, let $p = \cdot v_1, v_2, \cdot \cdot \cdot, v_k$ be a shortest path from vertex v_1 to vertex v_k and, for any i and j such that $1 \leq i \leq j \leq k$, let $p_{ij} = \cdot v_i, v_{i+1}, \cdot \cdot \cdot, v_j$ be the subpath of p from v_i to v_j . Then, p_{ij} is a shortest path from v_i to v_j .

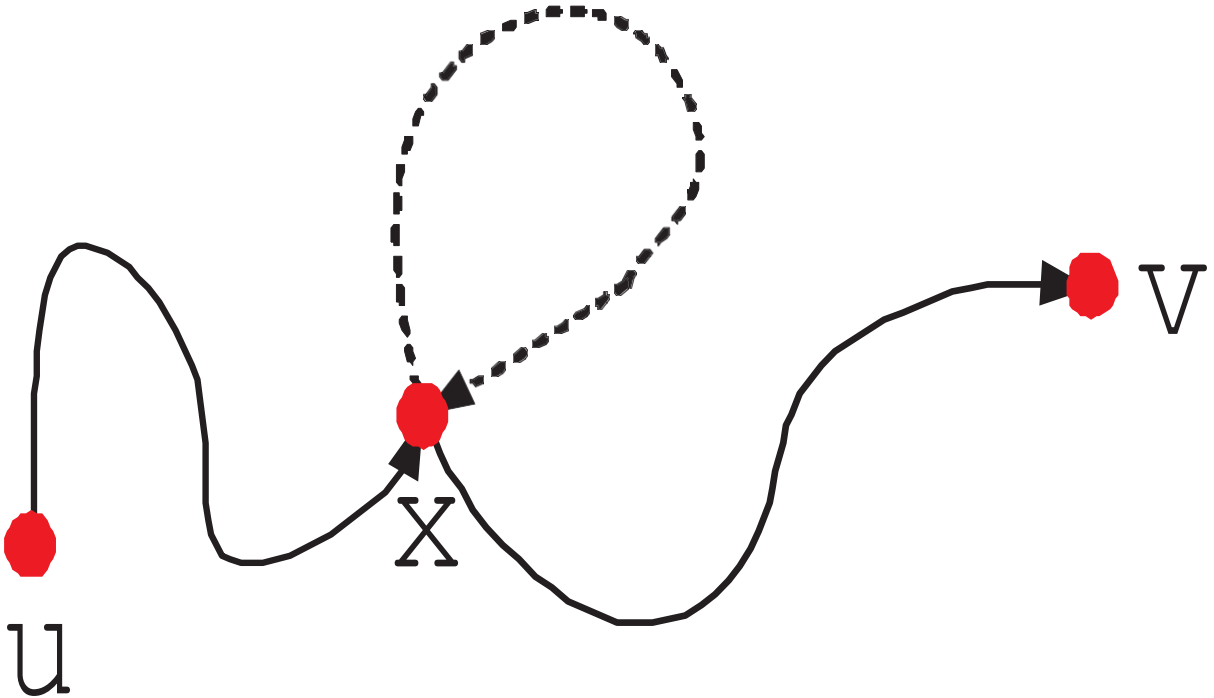
The property leads us to **Dynamic Programming** and **Greedy Method**.

Negative-weighted Edges



- Dijkstra's algorithm assumes that all edge weights are nonnegative.
- Bellman-Ford algorithm allow negative-weight edges, but no negative-weight cycles are reachable from the source.

No Cycles

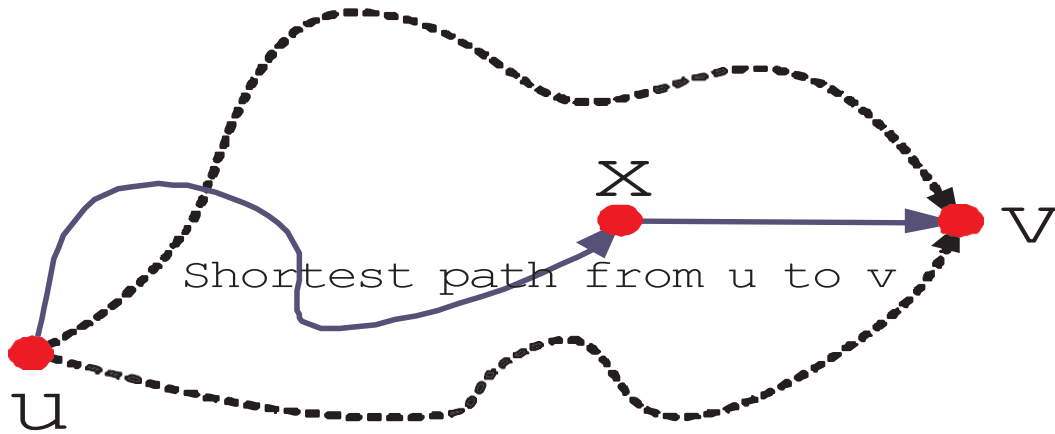


- We assume that the shortest paths have no cycles.
- A shortest path has at most $|V| - 1$ edges.

Predecessor

Definition 1 The predecessor of each vertex v , say $\pi[v]$, is another vertex or NIL.

The shortest-path algorithms in this course set the π attribute:



Predecessor Subgraph

Definition 2 Induced by π values, $G_\pi = (V_\pi, E_\pi)$ is a subgraph of G :

1. $V_\pi = \{v \in V \mid \pi[v] = \text{NIL}\} \cup \{s\}$
2. $E_\pi = \{(\pi[v], v) \in E \mid v \in V_\pi - \{s\}\}$

Note

Remind the predecessor subgraph in breadth-first search.

Shortest-path Tree

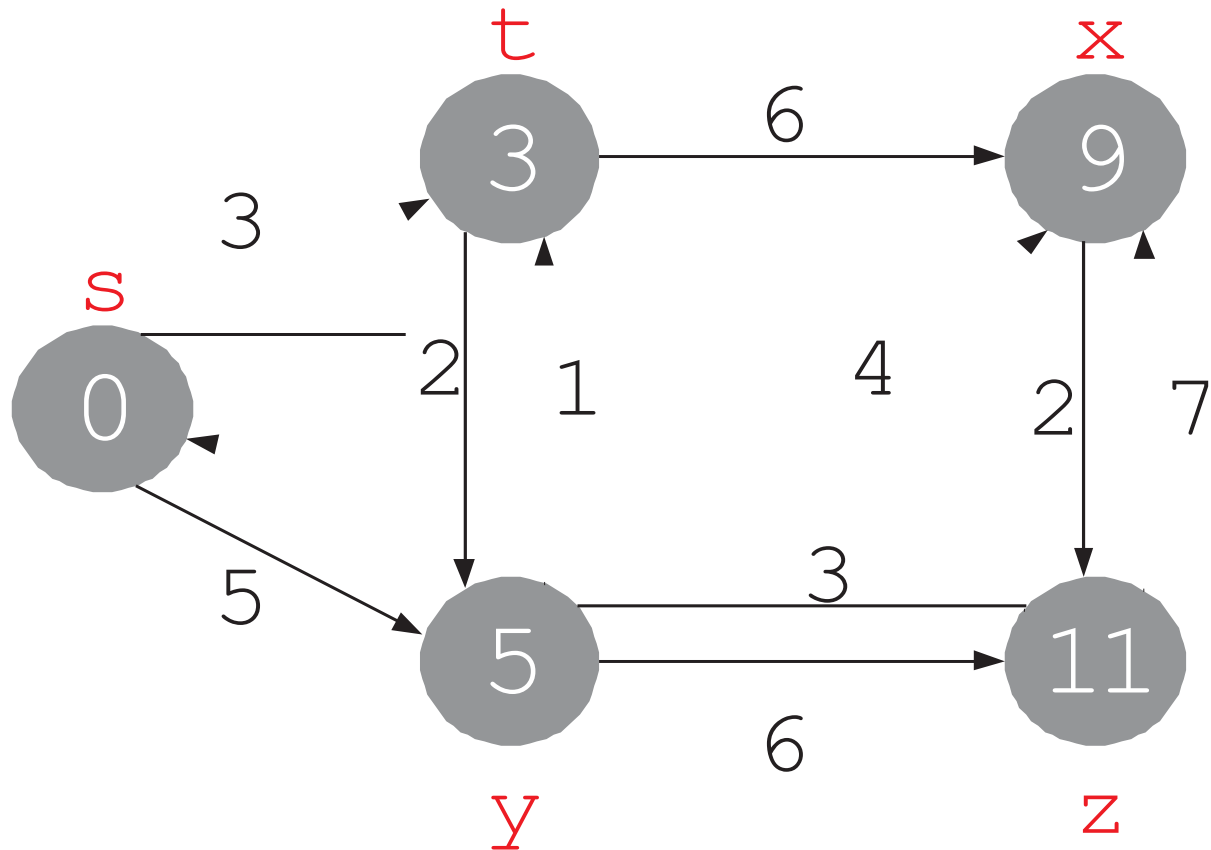
Definition 3 Given a weighted, directed graph $G = (V, E)$, a **shortest-path tree** of G is $G = (V, E)$ where

1. V is the set of vertices reachable from s in G ,
2. G forms a rooted tree with root s , and
3. for all $v \in V$, the unique simple path from s to v in G is a shortest path from s to v in G .

We will prove that

Theorem 1 G_π is a shortest-path tree.

Example of Shortest-path Tree



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