

# 各章基本题详解

## 习题一

### 一、选择题

1. (A)  $A \subset B \longrightarrow A \cup B = B$ ;

(B)  $\bar{B} \subset \bar{A} \longrightarrow A \subset B \longrightarrow A \cup B = B$ ;

(C)  $A\bar{B} = \phi \longrightarrow A \subset B \longrightarrow A \cup B = B$ ;

(D)  $\bar{A}B = \phi \longrightarrow B \subset A$  不一定能推出  $A \cup B = B$  (除非  $A = B$ )

所以 选 (D)

2.  $P(A - B) = P(A\bar{B}) = P(A\bar{B}) - P(A) - P(\bar{B}) + P(A) + P(\bar{B})$

$$= P(A) + P(\bar{B}) - P(A \cup \bar{B})$$

所以 选 (C)

3.  $A \subset B \longrightarrow P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)} \longrightarrow \frac{P(A)}{P(B)} \geq P(A)$

所以 选 (B)

4.  $P(A) = P(AB) = P(A)P(B) \longrightarrow P(A) = 0 \text{ 或 } P(B) = 1$

所以 选 (B)

5. (A) 若  $A = \bar{B}$ , 则  $AB = \phi$ , 且  $\bar{A}\bar{B} = \bar{A}A = \phi$ , 即  $\bar{A}, \bar{B}$  不相容

(B) 若  $A \supset B \neq \phi$ , 且  $A \neq \Omega$ , 则  $AB \neq \phi$ , 且  $\bar{A}\bar{B} = \bar{A} \neq \phi$ , 即  $\bar{A}, \bar{B}$  相容

(C) 若  $A = \phi, B \neq \phi$ , 则  $AB = \phi$ , 且  $\bar{A}\bar{B} = B \neq \phi$ , 即  $\bar{A}, B$  相容

(D) 若  $AB \neq \phi$ , 不一定能推出  $\bar{A}B = \phi$

所以 选 (D)

6. (A) 若  $AB \neq \phi$ , 不一定能推出  $P(AB) = P(A)P(B)$

(B) 若  $P(A) = 1$ , 且  $A \supset B \neq \phi$ , 则  $P(AB) = P(B) = P(A)P(B)$ , 即 A,B 独立

(C) 若  $AB = \phi$ ,  $0 < P(A) < 1, 0 < P(B) < 1$ , 则  $P(AB) \neq P(A)P(B)$

(D) 若  $P(A)=1$ , 则 A 与任何事件都相互独立

所以 选 (B)

7. 射击  $n$  次才命中  $k$  次, 即前  $n-1$  次射击恰好命中  $k-1$  次, 且第  $n$  次射击时命中目标, 所以 选 (C)

## 二、填空题

$$\begin{aligned} 8. (A \cap \bar{C})(\bar{A} \cap \bar{C}) \cap \overline{A \cap C} \cap \overline{\bar{A} \cap C} &= (A \cap \bar{A}) \cap \bar{C} \cap \overline{A \cap C} \cap \overline{\bar{A} \cap C} \\ &= \bar{C} \cap (\bar{A} \cap \bar{C}) \cap (A \cap \bar{C}) = \bar{C} \cap ((\bar{A} \cap A) \cap \bar{C}) = \bar{C} \cap \bar{C} = \bar{C} \end{aligned}$$

所以  $C = \bar{B}$

9. 共有  $4 \times 4$  种基本事件, 向后两个邮筒投信有  $2 \times 2$  种基本事件, 故所求概率为  $\frac{2 \times 2}{4 \times 4} = \frac{1}{4}$

10. 设事件 A 表示两数之和大于  $\frac{1}{2}$ , 则

样本空间  $\Omega = \{(x, y) \mid 0 < x < 1, 0 < y < 1\}$ ,

$$A = \{(x, y) \mid x + y > \frac{1}{2}, 0 < x < 1, 0 < y < 1\}$$

$$P = \frac{S_A}{S_\Omega} = 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{7}{8}$$

11. 由  $P(A) = 0.8, P(A - B) = 0.1$ , 得  $P(AB) = 0.7$ , 故  $P(\overline{AB}) = 0.3$

12. 由  $P(A) = 0.2, P(B) = 0.3, P(A \cap B) = 0.4$ , 得  $P(AB) = 0.1$ ,

$$\text{故 } P(B\bar{A}) = P(B) - P(AB) = 0.2$$

13.  $P(AB) = P(A)P(B|A) = 0.2$ , 故  $P(B) = \frac{P(AB)}{P(A|B)} = 0.8$

$$\begin{aligned} 14. P(A \cap B \cap C) &= P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC) \\ &= P(A) + P(B) + P(C) - P(A)P(B) - P(B)P(C) - P(C)P(A) + P(A)P(B)P(C) \\ &= \frac{19}{27} \end{aligned}$$

15. 由于 A,B 相互独立, 可得  $P(\overline{A}\overline{B}) = P(\overline{A})P(\overline{B}) = \frac{1}{9}$ ,  $P(A\overline{B}) = P(\overline{A}B)$ , 于是

$$P(\overline{A}) = P(\overline{B}) = \frac{1}{3}, \text{ 故 } P(B) = \frac{2}{3}$$

### 三、计算题

16. (1)  $\Omega = \{(H, H, H), (H, H, T), (H, T, H), (H, T, T),$   
 $(T, H, H), (T, H, T), (T, T, H), (T, T, T)\}$ ;

(2)  $\Omega = \{0, 1, 2, 3\}$ ;

(3)  $\Omega = \{(x, y) | x^2 + y^2 \leq 1\}$ ;

(4)  $\Omega = \{5:0, 5:1, 5:2, 5:3, 5:4, 4:5, 3:5, 2:5, 1:5, 0:5\}$

17. (1)  $A \sqcup B \sqcup C$ ;

(2)  $\overline{A}(B \sqcup C)$ ;

(3)  $\overline{A}\overline{B}\overline{C} \sqcup \overline{A}B\overline{C} \sqcup \overline{A}BC$ ;

(4)  $AB \sqcup BC \sqcup AC$ ;

(5)  $\overline{A}\overline{B}\overline{C}$ ;

(6)  $\overline{A} \sqcup \overline{B} \sqcup \overline{C}$ ;

(7)  $\overline{ABC}$

18. 法一, 由古典概率可知, 所求概率为:  $\frac{C_{20}^4 \cdot 9^{16}}{10^{20}}$ ;

法二, 由伯努利定理可知, 所求概率为:  $C_{20}^4 \cdot 0.1^4 \cdot 0.9^{16}$

19. 只有唯一的一个六位数号码能打开锁。因此由古典概率可知, 试开一次就能打开锁的

概率是  $\frac{1}{10^6}$ ; 如果要求这 6 个数字全不相同, 试开一次就能打开锁的概率是  $\frac{1}{A_{10}^6}$

20. 一枚均匀硬币抛掷三次, 共有  $2^3 = 8$  种不同情况

(1)至少连续两次出现正面有: (正, 正, 正), (正, 正, 反), (反, 正, 正)3 种情况, 故其概率为  $\frac{3}{8}$

(2)恰好出现两次正面有: (正, 正, 反), (正, 反, 正), (反, 正, 正)3 种情况, 故其概率为  $\frac{3}{8}$

(3) 正面与反面都出现的概率为  $1 - P(\text{仅出现反面或仅出现正面}) = \frac{3}{4}$

$$21. (1) \frac{1}{7^6}; \quad (2) \left(\frac{6}{7}\right)^6; \quad (3) 1 - \frac{1}{7^6}$$

22. 设事件 A 表示两数之积小于  $\frac{2}{9}$ , 则

$$\text{样本空间 } \Omega = \{(x, y) \mid 0 < x < 1, 0 < y < 1\},$$

$$A = \{(x, y) \mid xy < \frac{2}{9}, 0 < x < 1, 0 < y < 1\}$$

$$P = \frac{S_A}{S_\Omega} = 1 - \int_{\frac{2}{9}}^1 dx \int_{\frac{2}{9x}}^1 dy = \frac{2}{9} + \frac{2}{9} \ln \frac{9}{2}$$

23. 设事件 A 表示方程有实根, 则

$$\text{样本空间 } \Omega = \{(u, v) \mid 0 < u < 1, 0 < v < 1\},$$

$$A = \{(u, v) \mid u \leq v^2, 0 < u < 1, 0 < v < 1\}$$

$$P = \frac{S_A}{S_\Omega} = \int_0^1 dv \int_0^{v^2} du = \frac{1}{3}$$

$$24. P(A - B) = P(A) - P(AB) = P(A) - 1 + P(\overline{AB}) = P(A) - 1 + P(\overline{A} \cap \overline{B}) = 0.7$$

$$25. (1) P(\overline{A} \cap \overline{B}) = P(\overline{AB}) = 1 - P(AB) = 1 - c$$

$$(2) P(\overline{AB}) = P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - a - b + c$$

$$(3) P(\overline{A}B) = P(B - AB) = P(B) - P(AB) = b - c$$

$$(4) P(\overline{A} \cap B) = P(\overline{A}) + P(B) - P(\overline{A}B) = 1 - a + b - b + c = 1 - a + c$$

$$26. 1 - \frac{P(\overline{B})}{P(A)} = \frac{P(A) + P(B) - 1}{P(A)} \leq \frac{P(A) + P(B) - P(A \cap B)}{P(A)} = \frac{P(AB)}{P(A)} = P(A|B)$$

27. 设  $A_i$ : 第  $i$  次调试能调试好,  $i=1,2,3$ ,  $B$ : 调 3 次能调试好

$$\text{则 } P(A_1) = \frac{1}{3}, P(A_2 | \overline{A}_1) = \frac{3}{8}, P(A_3 | \overline{A}_1 \overline{A}_2) = \frac{9}{10}, B = A_1 \cap \overline{A}_1 A_2 \cap \overline{A}_1 \overline{A}_2 A_3$$

$$\begin{aligned}
P(B) &= P(A_1) + P(\bar{A}_1 A_2) + P(\bar{A}_1 \bar{A}_2 A_3) \\
&= P(A_1) + P(\bar{A}_1)P(A_2 | \bar{A}_1) + P(\bar{A}_1)P(\bar{A}_2 | \bar{A}_1)P(A_3 | \bar{A}_1 \bar{A}_2) \\
&= \frac{1}{3} + \frac{2}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{5}{8} \times \frac{9}{10} = \frac{23}{24} = 0.958
\end{aligned}$$

28. 设  $A_i$ : 10 个球中有  $i$  个红球,  $i=0,1,\dots,10$ ,  $B$ : 任取一球是红球

$$\text{则 } P(A_i) = \frac{1}{11}, P(B | A_i) = \frac{i+1}{11}, i = 0, 1, \dots, 10,$$

$$P(B) = \sum_{i=0}^{10} P(A_i)P(B | A_i) = \frac{6}{11}$$

29. 设  $A$ : 邻居记得浇水,  $B$ : 树已死去

$$\text{则 } P(A) = 0.9, P(B | A) = 0.15, P(\bar{A}) = 0.1, P(B | \bar{A}) = 0.8$$

$$P(\bar{B}) = 1 - P(B) = 1 - [P(A)P(B | A) + P(\bar{A})P(B | \bar{A})] = 1 - 0.215 = 0.785$$

$$P(\bar{A} | B) = \frac{P(\bar{A}B)}{P(B)} = \frac{P(\bar{A})P(B | \bar{A})}{1 - P(\bar{B})} = 0.372$$

30. 设  $A$ : 某人患肺癌,  $B$ : 某人吸烟

$$\text{则 } P(A) = 0.001, P(B | A) = 0.9, P(\bar{A}) = 0.999, P(B | \bar{A}) = 0.2$$

$$P(A | B) = \frac{P(AB)}{P(B)} = \frac{P(A)P(B | A)}{P(A)P(B | A) + P(\bar{A})P(B | \bar{A})} = \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.2} = 0.00448$$

$$P(A | \bar{B}) = \frac{P(A\bar{B})}{P(\bar{B})} = \frac{P(A)P(\bar{B} | A)}{1 - P(B)} = \frac{0.001 \times 0.1}{1 - 0.001 \times 0.9 - 0.999 \times 0.2} = 0.000125$$

31. 设每次击中目标的概率为  $p$ , 则由  $1 - (1 - p)^3 = \frac{7}{8}$ , 可得  $p = \frac{1}{2}$

32. 设应选出  $n$  件产品, 则由  $1 - (1 - 0.01)^n \geq 0.95$ , 可得  $n \geq 299$

33. (1)  $p$ ; (2)  $(1 - p)^{k-1} p$ ;

(3) 即前  $k - 1$  次射击恰好命中  $r - 1$  次, 且第  $k$  次射击时命中目标, 故所求概率为

$$C_{k-1}^{r-1} p^{r-1} (1 - p)^{k-r};$$

(4) 即共射击  $k + r$  次, 前  $k + r - 1$  次射击恰好命中  $k - 1$ , 且最后一次射击时命中目标,

故所求概率为  $C_{k+r-1}^r p^k (1 - p)^r$

## 习题二

### 一. 选择题

1. 因为  $P(X \geq 1) = 1 - P(X = 0) = 1 - (1 - p)^3 = 19/27$ , 故  $p = 1/3$ , 选择 (A)
2. 由正态分布密度关于  $x = \mu$  对称知,  $P(X < 0) = [1 - P(0 < X < 4)]/2 = 0.35$ , 选择 (C)
3. 因为密度为偶函数, 所以  $F(-a) = 1/2 - \int_{-a}^0 f(x)dx = 1/2 - \int_0^a f(x)dx$ , 选择 (B)
4. 因为  $P(|X - \mu| < \sigma) = P(|\frac{X - \mu}{\sigma}| < 1) = 2\Phi(1) - 1$ , 所以选择 (D)
5. 由严格单调条件下的定理 2.4.1 可知  $f_Y(y) = f_X(\frac{1-y}{3})|-\frac{1}{3}|$ , 选择 (A)

### 二. 填空题

6. 代入泊松分布律可解得  $\lambda$ , 填: 2
7.  $P(a/2 < X < b) = F(b-0) - F(a/2) = 0.4$ , 填: 0.4
8.  $P(X > 2) = \frac{a-2}{a+2} = 0.6$  或者  $P(X \leq 2) = \frac{4}{a+2} = 0.4$ , 填: 8
9. 由  $\int_0^1 (kx+2)dx = 1$  得出  $k = -2$  的值, 然后计算  $P(X < 0.5) = \int_0^{0.5} (2-2x)dx = 3/4$ , 填: 3/4
10.  $P(120 < X \leq 200) = P(|\frac{X-160}{\sigma}| < \frac{40}{\sigma}) = 2\Phi(\frac{40}{\sigma}) = 0.8$ , 查表, 填: 31.25
11.  $P(Y=1) = P(X=1) + P(X=2)$ , 填: 0.72

### 三. 计算题

12. 1 颗骰子掷 2 次总共有 36 种可能, 将这些可能排除二维表格形式,

|   | 1 | 2 | 3 | 4  | 5  | 6  |
|---|---|---|---|----|----|----|
| 1 | 2 | 3 | 4 | 5  | 6  | 7  |
| 2 | 3 | 4 | 5 | 6  | 7  | 8  |
| 3 | 4 | 5 | 6 | 7  | 8  | 9  |
| 4 | 5 | 6 | 7 | 8  | 9  | 10 |
| 5 | 6 | 7 | 8 | 9  | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

易得分布律

| $X$ | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   |
|-----|------|------|------|------|------|------|------|------|------|------|------|
| $P$ | 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 |

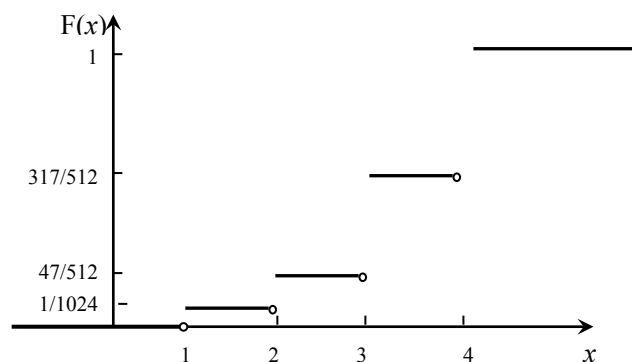
$$13. P(X=1) = \frac{C_4^1}{4^6} = \frac{1}{4^5} = \frac{1}{1024}, \quad P(X=2) = \frac{C_4^2(2^6-2)}{4^6} = \frac{93}{4^5} = \frac{93}{1024}$$

$$P(X=3) = \frac{C_4^3(3^6 - C_3^2(2^6-2) - 3)}{4^6} = \frac{540}{4^5} = \frac{540}{1024}, \quad P(X=4) = 1 - \sum_{k=1}^3 P(X=k) = \frac{390}{1024}$$

所以  $X$  的分布列为

| $X$ | 1                | 2                 | 3                  | 4                  |
|-----|------------------|-------------------|--------------------|--------------------|
| $P$ | $\frac{1}{1024}$ | $\frac{93}{1024}$ | $\frac{540}{1024}$ | $\frac{390}{1024}$ |

$$X \text{ 的分布函数 } F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{1024}, & 1 \leq x < 2 \\ \frac{47}{512}, & 2 \leq x < 3 \\ \frac{317}{512}, & 3 \leq x < 4 \\ 1, & x \geq 4 \end{cases}$$



$$14. \text{ 一次打开 } P(X=1) = \frac{1}{n}, P(X=2) = \frac{n-1}{n} \frac{1}{n-1} = \frac{1}{n}, \text{ 以此类推得}$$

$$P(X=k) = \frac{1}{n}, k=1, 2, \dots, n$$

15. 因为  $F(\pi/2) < F(0)$  不满足单调不减性, 或者  $F(-k\pi) = 1, k \in \mathbb{Z}^+, \rightarrow +\infty$ , 所以不是某个变量的分布函数

$$16. (1) \sum_{k=1}^N \frac{k}{C_1} = \frac{N(N+1)}{2C_1} = 1 \quad \text{得} \quad C_1 = \frac{N(N+1)}{2}$$

$$(2) C_2 \left(\frac{2}{3}\right)^1 + C_2 \left(\frac{2}{3}\right)^2 + C_2 \left(\frac{2}{3}\right)^3 = C_2 \frac{38}{27} = 1, \quad \text{得} \quad C_2 = \frac{27}{38}$$

$$(3) \sum_{k=1}^{+\infty} C_3 \frac{\lambda^k}{k!} = C_3 \left( \sum_{k=0}^{+\infty} \frac{\lambda^k}{k!} - 1 \right) = C_3 (e^\lambda - 1) = 1, \quad \text{得} \quad C_3 = \frac{1}{e^\lambda - 1}$$

17. 易知  $F(x)$  的间断点为 -1, 1, 3, 再由离散分布函数的性质可得

$$P(X = -1) = F(-1) - F(-1-0) = 0.4, \quad P(X = 1) = F(1) - F(1-0) = 0.8 - 0.4 = 0.4$$

$$P(X = 3) = F(3) - F(3-0) = 1 - 0.8 = 0.2$$

|     |     |     |     |
|-----|-----|-----|-----|
| $X$ | -1  | 1   | 3   |
| $P$ | 0.4 | 0.4 | 0.2 |

18. 已知  $X \sim P(4)$ , 则 (1)  $P(X = 6) = e^{-4} \frac{4^6}{6!} = F(6) - F(5) = 0.1042$  查表得

$$(2) \quad P(X > 8) = 1 - P(X \leq 8) = 1 - F(8) = 0.0214 \quad \text{查表得}$$

19. 设患者中病的人数为  $X$ , 则由题意得  $X \sim B(5000, 0.001)$ , 由于  $n$  较大,  $p$  较小,

$$\text{所以取 } \lambda = np = 5$$

$$\text{所求概率为 } P(X \leq 6) \approx \sum_{k=0}^6 e^{-5} \frac{5^k}{k!} = 0.7622 \quad (\text{泊松近似, 查表得})$$

20. 设发生故障的设备数为  $X$ , 维修人员数为  $m$ , 则由题意可构建概率方程:

$$P(X > m) < 0.01$$

$$\text{因为 } X \sim B(300, 0.01), \quad P(X > m) = 1 - P(X \leq m) \approx 1 - \sum_{k=0}^m e^{-3} \frac{3^k}{k!} < 0.01$$

查表可得  $m \geq 8$ , 即至少配备 8 个维修人员才能满足题目要求。

$$21. (1) \quad \int_{-1}^1 \frac{C}{\sqrt{1-x^2}} dx = C \arcsin(x) \Big|_{-1}^1 = \pi C = 1 \Rightarrow C = \frac{1}{\pi}$$

$$(2) \quad F(x) = \begin{cases} 0, & x \leq -1 \\ \int_{-1}^x \frac{1}{\pi \sqrt{1-x^2}} dx = \frac{1}{2} + \frac{1}{\pi} \arcsin(x), & -1 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

$$22. \quad F(x) = \begin{cases} 0, & x < 0 \\ \int_0^x x dx = \frac{x^2}{2}, & 0 \leq x < 1 \\ \int_0^1 x dx + \int_1^x (2-x) dx = 2x - \frac{x^2}{2} - 1, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

$$23. (1) \int_0^2 (Cx^2 + x)dx = \frac{Cx^3}{3} + \frac{x^2}{2} \Big|_0^2 = \frac{8C}{3} + 2 = 1 \Rightarrow C = -\frac{3}{8}$$

$$(2) P(X \geq 1) = 1 - \int_0^1 (x - \frac{3}{8}x^2)dx = \frac{5}{8}$$

$$24. (1) F(+\infty) = \frac{A}{1} = 1 \Rightarrow A = 1$$

$$(2) f(x) = F'(x) = \frac{e^{-x}}{(1+e^{-x})^2}, -\infty < x < +\infty$$

$$(3) P(X \leq 0) = F(0) = 1/2$$

25. 已知  $X \sim E(0.2)$ , 指数分布具有无记忆特性, 所以

$$(1) P(X > 15 | X > 5) = P(X > 10) = e^{-0.2 \times 10} = e^{-2}$$

$$(2) \text{ 因为 } P(X > 10) = e^{-0.2 \times 10} = e^{-2}$$

假设  $Y$  表示三次等待不到服务而离开窗口的次数, 由题意得  $Y \sim B(3, e^{-2})$

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1 - e^{-2})^3$$

26. 已知  $X \sim E(1/1000)$ , 3 个元件的使用相互独立, 故所求概率为

$$[P(X > 1000)]^3 = [e^{-1000 \times 1/1000}]^3 = e^{-3}$$

27. 已知  $X \sim E(1/2)$ , 则 (1)  $P(X > 2) = e^{-2 \times 1/2} = e^{-1}$

(2) 由指数分布的无记忆特性得,  $P(X > 10 | X > 9) = P(X > 1) = e^{-1/2}$

$$28. (1) P(2 < X \leq 5) = \Phi(\frac{5-3}{2}) - \Phi(\frac{2-3}{2}) = \Phi(1) - \Phi(-\frac{1}{2}) = \Phi(1) + 1 - \Phi(\frac{1}{2}) = 0.5328$$

$$(2) P(-4 < X \leq 10) = \Phi(\frac{10-3}{2}) - \Phi(\frac{-4-3}{2}) = 2\Phi(3.5) - 1 = 0.9996$$

$$(3) P\{|X| > 2\} = 1 - P(-2 \leq X \leq 2)$$

$$= 1 - \Phi(\frac{2-3}{2}) - \Phi(\frac{-2-3}{2}) = \Phi(\frac{1}{2}) + 1 - \Phi(2.5) = 0.6977$$

$$(4) P\{|X| < 3\} = \Phi(0) - \Phi(-3) = \Phi(3) - 0.5 = 0.4987$$

(5) 有正态分布的特性, 知  $C$  等于  $\mu$  的时候正好满足题目要求, 所以  $C = 3$

29. 由题意得 次品率为  $1 - P(|X - 10.05| \leq 0.12) = 2 - 2\Phi(2) = 0.0456$

30. 由  $P(X > 96) = 1 - \Phi(\frac{96 - 72}{\sigma}) = 0.023$  得  $\sigma = 12$

$$\text{则 } P(60 \leq X \leq 84) = 2\Phi(\frac{12}{12}) - 1 = 0.6826$$

31. (1) 因为  $y = e^x$  为严格单调函数, 且  $y > 0$ , 所以

$$f_Y(y) = \begin{cases} f_X(\ln y) \frac{1}{y} = \frac{1}{\sqrt{2\pi}} e^{-\frac{\ln^2 y}{2}}, y > 0 \\ 0, \text{其他} \end{cases}$$

$$(2) F_Y(y) = P(|X| \leq y) = \begin{cases} P(-y \leq X \leq y) = F_X(y) - F_X(-y), y > 0 \\ 0, \text{其他} \end{cases}$$

$$f_Y(y) = \begin{cases} f_X(y) + f_X(-y) = \frac{2}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}, y > 0 \\ 0, \text{其他} \end{cases}$$

32. (1)  $y = 1 - 2x$  严格单调, 所以  $f_Y(y) = \begin{cases} f_X(\frac{1-y}{2}) \frac{1}{2} = \frac{1}{8}, -5 \leq y \leq 2 \\ 0, \text{其他} \end{cases}$

$$(2) F_Y(y) = P(X^2 \leq y) = \begin{cases} P(-\sqrt{y} \leq X \leq \sqrt{y}) = F_X(\sqrt{y}) - F_X(-\sqrt{y}), y > 0 \\ 0, \text{其他} \end{cases}$$

$$f_Y(y) = \begin{cases} f_X(\sqrt{y}) \frac{1}{2\sqrt{y}} + f_X(-\sqrt{y}) \frac{1}{2\sqrt{y}} = \frac{1}{4\sqrt{y}}, 0 < y \leq 1 \\ f_X(\sqrt{y}) \frac{1}{2\sqrt{y}} = \frac{1}{8\sqrt{y}}, 1 < y \leq 9 \\ 0, \text{其他} \end{cases}$$

33. 圆片面积  $Y = \pi(\frac{X}{2})^2 = \frac{\pi X^2}{4}$ , 函数  $y = \frac{\pi x^2}{4}$  在区间  $[5, 6]$  上严格单调, 所以

$$f_Y(y) = \begin{cases} f_X(\frac{2\sqrt{y}}{\sqrt{\pi}}) \frac{2}{2\sqrt{\pi y}} = \frac{1}{\sqrt{\pi y}}, y \in [25\pi/4, 9\pi] \\ 0, \text{其他} \end{cases}$$

## 习题三

### 一、选择题

1.  $P(X=Y) = P(X=-1, Y=-1) + P(X=1, Y=1) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$ , 故选 (A)

2.  $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y) dx dy = \int_0^2 \int_0^2 c x dx dy = 4c = 1$ , 则  $c=1/4$  故选 (A)

3. 选 (D)

4. 由题意可得  $X+Y \sim N(1,2)$ , 所以  $P(X+Y \leq 1) = \frac{1}{2}$  故选 (B)

5.  $(X,Y)$  服从区域  $G = \{(x,y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}$  上的均匀分布,

$$P(X+Y > 1) = \int_{0.5}^1 dx \int_{-x+1.5}^1 dy = 0.125, \quad \text{故选 (A)}$$

### 二、填空题

6.  $P(X=1) = \frac{1}{6} + \frac{1}{9} + \frac{1}{18} = \frac{1}{3}$ , 所以  $P(X=2) = \frac{1}{3} + \alpha + \beta = \frac{2}{3}$ ,

且由  $X$  与  $Y$  独立可得  $P(X=1, Y=2) = P(X=1)P(Y=2)$ , 因此  $P(Y=2) = \frac{1}{3}$

所以  $\alpha = P(X=2, Y=2) = P(X=2) \cdot P(Y=2) = \frac{2}{9}$ ,  $\beta = \frac{2}{3} - \frac{1}{3} - \frac{2}{9} = \frac{1}{9}$

7

$$P(\max(X,Y) \geq 0) = P(\{X \geq 0\} \cup \{Y \geq 0\}) = P(X \geq 0) + P(Y \geq 0) - P(X \geq 0, Y \geq 0)$$

$$= \frac{4}{7} + \frac{4}{7} - \frac{3}{7} = \frac{5}{7}$$

$$8. F_X(x) = F(x, +\infty) = \begin{cases} 1 - e^{-3x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

故当  $x > 0$  时,  $(X,Y)$  关于  $X$  的边缘概率密度  $f_X(x) = F'_X(x) = 3e^{-3x}$

$$9. f_X(x) = \int_{-\infty}^{+\infty} f(x,y) dy = \begin{cases} \int_{-1}^{+1} \frac{1}{4} dy = \frac{1}{2}, & |x| < 1 \\ 0, & \text{其他} \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_{-1}^{+1} \frac{1}{4} dx = \frac{1}{2}, & |y| < 1 \\ 0, & \text{其他} \end{cases}$$

$$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)} = \begin{cases} \frac{1}{2}, & |x| < 1 \\ 0, & \text{其他} \end{cases}$$

10. 由正态分布的性质可得  $X-Y \sim N(0, 2)$ , 所以  $Z = X - Y$  的概率分布  $f(z) = \frac{e^{-\frac{z^2}{4}}}{2\sqrt{\pi}}$

### 三、计算题

11.

| $X \backslash Y$ | 1              | 2             | 3              |
|------------------|----------------|---------------|----------------|
| 1                | 0              | $\frac{1}{6}$ | $\frac{1}{12}$ |
| 2                | $\frac{1}{6}$  | $\frac{1}{6}$ | $\frac{1}{6}$  |
| 3                | $\frac{1}{12}$ | $\frac{1}{6}$ | 0              |

12.

$$P(X=i, Y=j) = C_5^i C_{5-i}^j 0.5^i 0.3^j 0.2^{5-i-j} = \frac{5!}{i!j!(5-i-j)!} 0.5^i 0.3^j 0.2^{5-i-j}$$

$$(i, j = 0, 1, \dots, 5; i+j \leq 5)$$

13.

| $X \backslash Y$ | 1               | 2              | 3             | $p_{i \cdot}$ |
|------------------|-----------------|----------------|---------------|---------------|
| 1                | $\frac{1}{3}$   | 0              | 0             | $\frac{1}{3}$ |
| 2                | $\frac{1}{6}$   | $\frac{1}{6}$  | 0             | $\frac{1}{3}$ |
| 3                | $\frac{1}{9}$   | $\frac{1}{9}$  | $\frac{1}{9}$ | $\frac{1}{3}$ |
| $p_{\cdot j}$    | $\frac{11}{18}$ | $\frac{5}{18}$ | $\frac{1}{9}$ |               |

14. (1)  $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = \int_0^{+\infty} dx \int_0^1 A y e^{-x} dy = \frac{A}{2} = 1$ , 故得  $A=2$ .

$$(2) P(X < 1) = \int_0^1 dx \int_0^1 2 y e^{-x} dy = 1 - e^{-1}$$

$$(3) F(x, y) = \int_{-\infty}^x ds \int_{-\infty}^y f(s, t) dt = \begin{cases} \int_0^x ds \int_0^y 2te^{-s} dt = y^2(1 - e^{-x}), & x > 0, 0 < y < 1 \\ \int_0^x ds \int_0^1 2te^{-s} dt = 1 - e^{-x}, & x > 0, y \geq 1 \\ 0, & \text{其它} \end{cases}$$

$$15. (1) \text{ 区域 } G \text{ 的面积} = \int_0^1 x - x^2 dx = \frac{1}{6}, \quad \text{故 } f(x, y) = \begin{cases} 6, & (x, y) \in G \\ 0, & \text{其它} \end{cases}$$

$$(2) f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_{x^2}^x 6 dy = 6(x - x^2), & 0 \leq x \leq 1 \\ 0, & \text{其它} \end{cases},$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_y^{\sqrt{y}} 6 dx = 6(\sqrt{y} - y), & 0 \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$$

$$16. (1) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = \int_0^1 dx \int_0^1 Cxy^3 dy = \frac{C}{8} = 1, \text{故得 } C=8.$$

$$(2) P(Y > X) = \int_0^1 dx \int_0^x 8xy^3 dy = \frac{2}{3}$$

$$(3) f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_0^1 8xy^3 dy = 2x, & 0 \leq x \leq 1 \\ 0, & \text{其它} \end{cases},$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_0^1 8xy^3 dx = 4y^3, & 0 \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$$

因为  $f(x, y) = f_X(x)f_Y(y)$ , 故 X 与 Y 相互独立。

$$17. (1) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = \int_0^1 dx \int_0^x Cy(1-x) dy = \frac{C}{24} = 1, \text{故得 } C=24.$$

$$(2) f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_0^x 24y(1-x) dy = 12x^2(1-x), & 0 \leq x \leq 1 \\ 0, & \text{其它} \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_y^1 24y(1-x) dx = 12y(1-y)^2, & 0 \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$$

故 X 与 Y 不独立。

$$18. (1) \text{ 区域 } G \text{ 的面积} = 2\sqrt{2}, \quad \text{故 } f(x, y) = \begin{cases} \frac{\sqrt{2}}{4}, & (x, y) \in G \\ 0, & \text{其它} \end{cases}$$

$$(2) P(X+Y < 1) = \frac{\sqrt{2} + \frac{1}{2}}{2\sqrt{2}} = \frac{1}{2} + \frac{\sqrt{2}}{8}$$

$$(3) f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_{-\sqrt{2}}^{\sqrt{2}} \frac{\sqrt{2}}{4} dy = 1, & 0 \leq x \leq 1, \\ 0, & \text{其它} \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_0^1 \frac{\sqrt{2}}{4} dx = \frac{\sqrt{2}}{4}, & 0 \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$$

因为  $f(x, y) = f_X(x)f_Y(y)$ ，故  $X$  与  $Y$  相互独立。

$$19. P(X=m) = \sum_{n=m+1}^{+\infty} p^2 q^{n-2} = pq^{m-1}, m=1, 2, \dots$$

$$P(Y=n) = \sum_{m=1}^{n-1} p^2 q^{n-2} = (n-1)p^2 q^{n-2}, n=2, 3, \dots$$

20. (1) 因为  $X$  与  $Y$  相互独立，则  $(X, Y)$  的联合分布律为

| $X \backslash Y$ | -1   | 1    | 2    | $p_{i.}$ |
|------------------|------|------|------|----------|
| -1               | 0.06 | 0.09 | 0.15 | 0.3      |
| 0                | 0.08 | 0.12 | 0.2  | 0.4      |
| 1                | 0.06 | 0.09 | 0.15 | 0.3      |
| $p_{.j}$         | 0.2  | 0.3  | 0.5  | 1        |

(2)

| $Z_1$ | -2   | -1   | 0    | 1    | 2    | 3    |
|-------|------|------|------|------|------|------|
| $P_z$ | 0.06 | 0.08 | 0.15 | 0.27 | 0.29 | 0.15 |

| $Z_2$ | -2   | -1   | 0   | 1    | 2    |
|-------|------|------|-----|------|------|
| $P_z$ | 0.15 | 0.15 | 0.4 | 0.15 | 0.15 |

21.(1)由题意

$$P(U=0, V=0) = P(\max\{X, Y\} = 0, \min\{X, Y\} = 0) = P(X=0, Y=0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(U=1, V=1) = P(\max\{X, Y\} = 1, \min\{X, Y\} = 1) = P(X=1, Y=1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(U=0, V=1) = P(\max\{X, Y\} = 0, \min\{X, Y\} = 1) = 0$$

故  $P(U=1, V=0) = \frac{1}{2}$ , 则  $(U, V)$  的联合分布律为

| $U \backslash V$ | 0             | 1             | $p_{i\cdot}$  |
|------------------|---------------|---------------|---------------|
| 0                | $\frac{1}{4}$ | 0             | $\frac{1}{4}$ |
| 1                | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ |
| $p_{\cdot j}$    | $\frac{3}{4}$ | $\frac{1}{4}$ | 1             |

(2) 由  $P(U=0, V=1) = 0 \neq P(U=0) \cdot P(V=1)$ ,

所以  $U$  与  $V$  不独立。

22. (1)  $Z=X+Y$ , 由连续型卷积公式可得

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x)f_Y(z-x)dx = \begin{cases} \int_0^z 1dx = z, & 0 < z \leq 1 \\ \int_{z-1}^1 1dx = 2-z, & 1 < z \leq 2 \\ 0, & \text{其它} \end{cases}$$

(2)  $Z = \min\{X, Y\}$  的分布函数为

$$\begin{aligned} F_Z(z) &= P(Z \leq z) = 1 - P(Z > z) = 1 - P(\min\{X, Y\} > z) \\ &= 1 - P(X > z, Y > z) = 1 - P(X > z)P(Y > z) \\ &= 1 - (1 - F_X(z))(1 - F_Y(z)) \\ &= \begin{cases} 0, & z \leq 0 \\ 1 - (1 - z)^2, & 0 < z < 1 \\ 1, & z \geq 1 \end{cases} \end{aligned}$$

所以  $Z = \min\{X, Y\}$  的概率密度函数为  $f_Z(z) = \begin{cases} 2(1-z), & 0 < z < 1 \\ 0, & \text{其他} \end{cases}$

23.  $(X, Y)$  的联合密度函数为  $f(x, y) = \begin{cases} \frac{1}{\pi R^2}, & x^2 + y^2 < R^2 \\ 0, & \text{其他} \end{cases}$

$Z$  的分布函数

$$F_Z(z) = P(Z \leq z) = P(\sqrt{X^2 + Y^2} \leq z) = \begin{cases} \iint_{\sqrt{x^2+y^2} \leq z} \frac{1}{\pi R^2} dx dy = \frac{z^2}{R^2}, & 0 \leq z \leq R \\ 0, & z < 0 \\ 1, & z > R \end{cases}$$

$$\text{所以 } Z \text{ 的概率密度函数 } f_Z(z) = \begin{cases} 2z/R^2, & 0 \leq z \leq R \\ 0, & \text{其它} \end{cases}$$

24. 当  $z < 0$  时  $F(z) = P\{Z \leq z\} = P\{X - Y \leq z\} = 0$

当  $0 \leq z \leq 1$  时

$$F(z) = P\{Z \leq z\} = P\{X - Y \leq z\} = \int_0^z dx \int_0^x 3xdy + \int_z^1 dx \int_{x-z}^x 3xdy = \frac{3}{2}z - \frac{1}{2}z^3$$

当  $z > 1$  时  $F(z) = 1$

$$\text{故 } f(z) = \begin{cases} \frac{3}{2}z - \frac{1}{2}z^3, & 0 \leq z \leq 1 \\ 0, & \text{其它} \end{cases}$$

25.

$$\begin{aligned} f_X(x) &= \int_{-\infty}^{+\infty} f(x, y) dy \\ &= \begin{cases} \int_0^{+\infty} x e^{-x(y+1)} dy = e^{-x}, & x > 0 \\ 0, & \text{其它} \end{cases} \end{aligned}$$

$$\begin{aligned} f_Y(y) &= \int_{-\infty}^{+\infty} f(x, y) dx \\ &= \begin{cases} \int_0^{+\infty} x e^{-x(y+1)} dx = \frac{1}{(1+y)^2}, & y > 0 \\ 0, & \text{其它} \end{cases} \end{aligned}$$

$$\text{在 } Y=y (y>0) \text{ 条件下, } f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)} = \begin{cases} \frac{x e^{-x(y+1)}}{(1+y)^2}, & x > 0 \\ 0, & \text{其他} \end{cases}$$

$$\text{在 } X=x (x>0) \text{ 条件下, } f_{Y|X}(y|x) = \frac{f(x, y)}{f_X(x)} = \begin{cases} x e^{-xy}, & y > 0 \\ 0, & \text{其他} \end{cases}$$

$X$  与  $Y$  不独立。

26. 已知随机变量  $X$  与  $Y$  独立同分布, 且  $P(X = k) = p(1-p)^{k-1}, k = 1, 2, \dots$

$$\text{证明 } P(X = k | X + Y = n) = \frac{1}{n-1}, k = 1, \dots, n-1.$$

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