流体动力学基础

§4.2　实际流体的能量方程

- 理想流体恒定元流的能量方程

动能定理：运动物体在某一时段内动能的增量等于各外力对物体所作的功之和

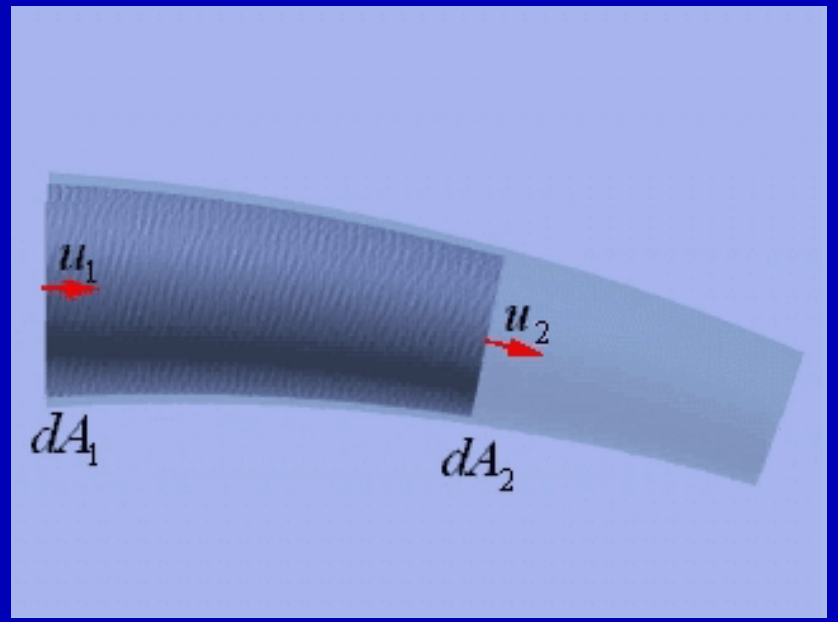
$$\sum W = \frac{1}{2}mV^2 - \frac{1}{2}mV_o^2$$

动能的增量

$$\Delta E_K = E_{K1'-2'} - E_{K1-2}$$

$$= (E_{K1'-2} + E_{K2-2'}) - (E_{K1-1'} + E_{K1'-2})$$

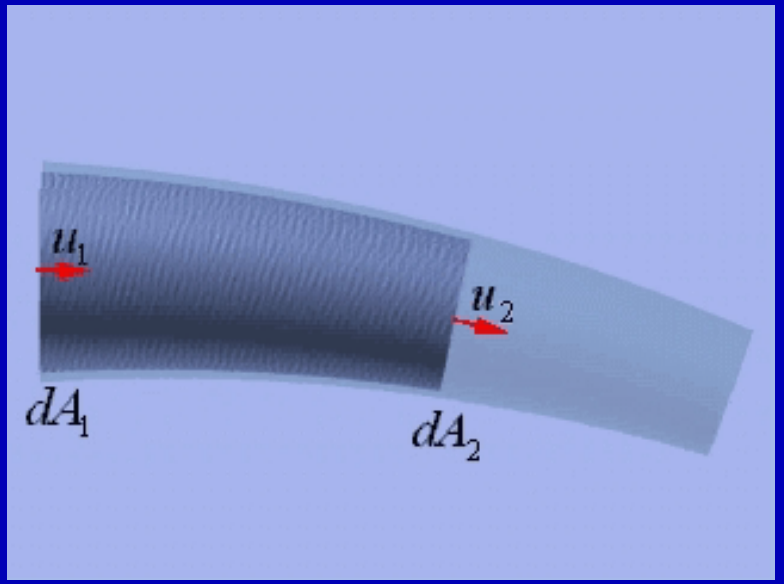
$$= E_{K2-2'} - E_{K1-1'}$$



$$\Delta E_K = \frac{1}{2} dm U_2^2 - \frac{1}{2} dm U_1^2$$

$$= \frac{1}{2} \rho dQ dt (U_2^2 - U_1^2)$$

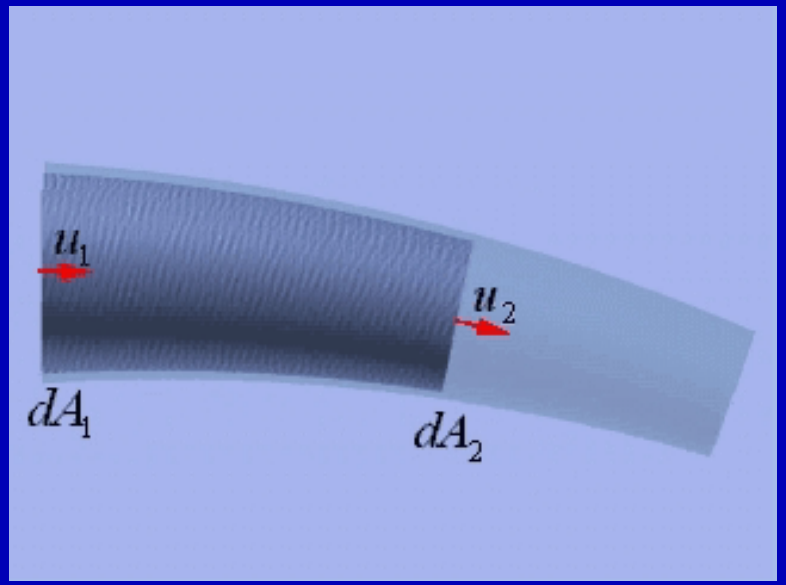
$$= \rho g dQ dt (\frac{U_2^2}{2g} - \frac{U_1^2}{2g})$$



- 重力作功：

$$W_G = dmg(z_1 - z_2)$$

$$= \rho g dQ dt(z_1 - z_2)$$

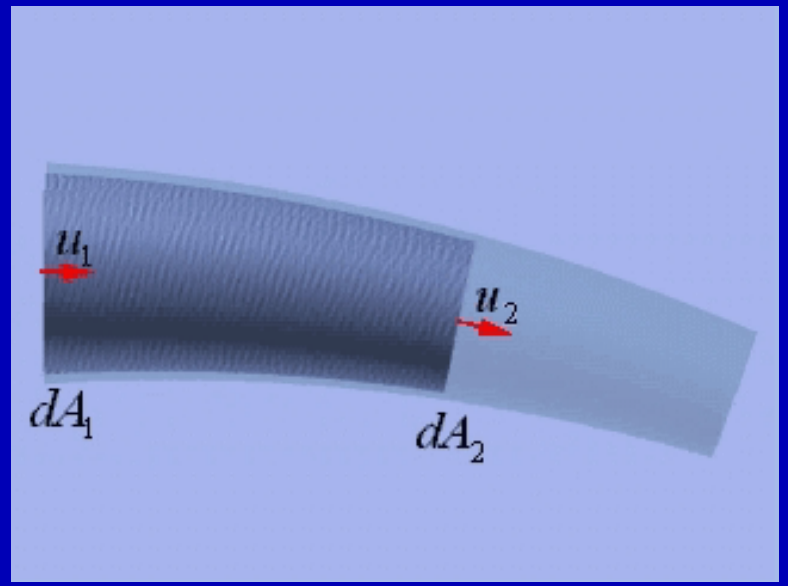


- 压力作功：

$$W_P = p_1 dA_1 dS_1 - p_2 dA_2 dS_2$$

$$= p_1 dA_1 U_1 dt - p_2 dA_2 U_2 dt$$

$$= dQdt(p_1 - p_2)$$



$$W_G + W_P = \Delta E_K$$

$$\rho g dQdt(z_1 - z_2) + dQdt(p_1 - p_2) = \rho g dQdt(\frac{U_2^2}{2g} - \frac{U_1^2}{2g})$$

$$z_1 + \frac{p_1}{\rho g} + \frac{U_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{U_2^2}{2g}$$

伯努利方程

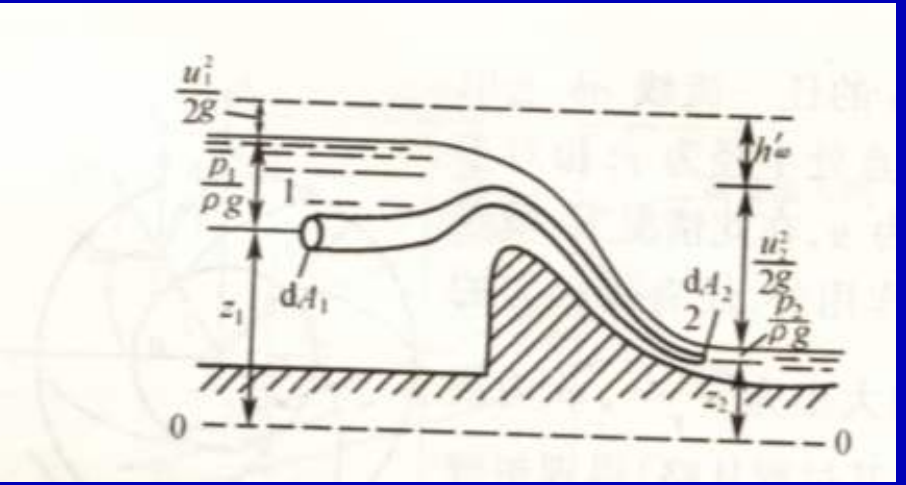
- 实际流体恒定元流的能量方程

$$z_1+\frac{p_1}{\rho g}+\frac{U_1^2}{2g}=z_2+\frac{p_2}{\rho g}+\frac{U_2^2}{2g}+h_w'$$

- 实际流体恒定总流的能量方程

$$z_1+\frac{p_1}{\rho g}+\frac{U_1^2}{2g}=z_2+\frac{p_2}{\rho g}+\frac{U_2^2}{2g}+h_w'$$

$$(z_1+\frac{p_1}{\rho g}+\frac{U_1^2}{2g})\rho g U_1 dA_1 dt$$



$$=(z_2+\frac{p_2}{\rho g}+\frac{U_2^2}{2g})\rho g U_2 dA_2 dt+h_w'\rho g dQ dt$$

$$\int_{A_1}(z_1+\frac{p_1}{\rho g}+\frac{U_1^2}{2g})\rho g U_1 dA_1 dt$$

$$=\int_{A_2}(z_2+\frac{p_2}{\rho g}+\frac{U_2^2}{2g})\rho g U_2 dA_2 dt+\int_Q h_w^{'}\rho g dQ dt$$

$$\frac{1}{Q}\int_{A_1}(z_1+\frac{p_1}{\rho g})U_1 dA_1+\frac{1}{Q}\int_{A_1}\frac{U_1^2}{2g}U_1 dA_1$$

$$=\frac{1}{Q}\int_{A_2}(z_2+\frac{p_2}{\rho g})U_2 dA_2+\frac{1}{Q}\int_{A_2}\frac{U_2^2}{2g}U_2 dA_2+\frac{1}{Q}\int_Q h_w^{'}dQ$$

- 势能积分

$$\frac{1}{Q}\int_A (z+\frac{p}{\rho g})UdA = \frac{1}{Q}(z+\frac{p}{\rho g})\int_A UdA$$

$$= (z+\frac{p}{\rho g})\frac{1}{Q}\int_A dQ = z+\frac{p}{\rho g}$$

- 动能积分

$$\frac{1}{Q}\int_A \frac{U^2}{2g} U dA = \alpha \frac{V^2}{2g}$$

$$a = \frac{\frac{1}{Q}\int_A \frac{U^3}{2g} dA}{\frac{V^2}{2g}} = \frac{\int_A U^3 dA}{V^2 \cdot VA} = \frac{\int_A U^3 dA}{V^3 A}$$

- 水头损失积分

$$\frac{1}{Q}\int_{Q} h_w' dQ = h_w$$

代入整理后得到恒定总流能量方程

$$z_1 + \frac{p_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + h_w$$