

教材习题答案

第一章 空间向量与立体几何

1.1 空间向量及其运算

1.1.1 空间向量及其线性运算

练习

1. 解析 答案不唯一. 例如

三棱锥 $V-ABC$ 中, $\overrightarrow{VA}, \overrightarrow{VB}, \overrightarrow{VC}$ 表示三个不同在一个平面内的向量.

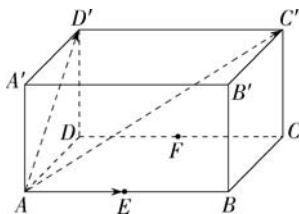
2. 解析 (1) $\overrightarrow{AA'} - \overrightarrow{CB} = \overrightarrow{AA'} - \overrightarrow{DA} = \overrightarrow{AA'} + \overrightarrow{AD} = \overrightarrow{AD'}$;

(2) $\overrightarrow{AA'} + \overrightarrow{AB} + \overrightarrow{B'C'} = \overrightarrow{AB'} + \overrightarrow{B'C'} = \overrightarrow{AC'}$;

(3) $\overrightarrow{AB} - \overrightarrow{AD} + \overrightarrow{B'D'} = \overrightarrow{DB} + \overrightarrow{BD} = \mathbf{0}$;

(4) $\overrightarrow{AB} + \overrightarrow{CF} = \overrightarrow{AB} + \overrightarrow{BE} = \overrightarrow{AE}$.

向量 $\overrightarrow{AD'}$, $\overrightarrow{AC'}$, $\overrightarrow{AE}$ 如图所示.



3. 解析 $\overrightarrow{A'C} = \overrightarrow{A'A} + \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{AA'} + \overrightarrow{AB} + \overrightarrow{AD}$;

$\overrightarrow{BD'} = \overrightarrow{BA} + \overrightarrow{AA'} + \overrightarrow{A'D'} = -\overrightarrow{AB} + \overrightarrow{AA'} + \overrightarrow{AD}$;

$\overrightarrow{DB'} = \overrightarrow{DA} + \overrightarrow{AB} + \overrightarrow{BB'} = -\overrightarrow{AD} + \overrightarrow{AB} + \overrightarrow{AA'}$.

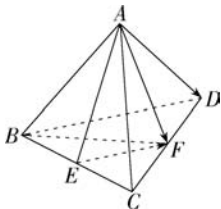
4. 解析 如图, 连接 BF .

(1) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$;

(2) $\overrightarrow{AB} + \frac{1}{2}(\overrightarrow{BD} + \overrightarrow{BC}) = \overrightarrow{AB} + \overrightarrow{BF} = \overrightarrow{AF}$;

(3) $\overrightarrow{AF} - \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC}) = \overrightarrow{AF} - \overrightarrow{AE} = \overrightarrow{EF}$.

向量 $\overrightarrow{AD}$, $\overrightarrow{AF}$, $\overrightarrow{EF}$ 如图所示.



5. 解析 (1) $\because \overrightarrow{AC'} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CC'}$,

$\therefore x=1$.

(2) $\because \overrightarrow{AE} = \overrightarrow{AA'} + \overrightarrow{A'E} = \overrightarrow{AA'} + \frac{1}{2}\overrightarrow{A'C'} = \overrightarrow{AA'} + \frac{1}{2}\overrightarrow{AC} = \overrightarrow{AA'} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD}$,

$\therefore x = \frac{1}{2}, y = \frac{1}{2}$.

(3) $\because \overrightarrow{AF} = \overrightarrow{AD} + \overrightarrow{DF} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} = \overrightarrow{AD} + \frac{1}{2}(\overrightarrow{DC} + \overrightarrow{CC'}) = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AA'}$,

$$\frac{1}{2}(\overrightarrow{DC} + \overrightarrow{CC'}) = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AA'},$$

$$\therefore x = \frac{1}{2}, y = \frac{1}{2}.$$

1.1.2 空间向量的数量积运算

练习

1. B 设 $BB_1 = 1$, 则 $AB = \sqrt{2}$, $\overrightarrow{AB_1} = \overrightarrow{BB_1} - \overrightarrow{BA}$, $\overrightarrow{BC_1} = \overrightarrow{BB_1} + \overrightarrow{B_1C_1} = \overrightarrow{BB_1} + \overrightarrow{BC}$,

$$\therefore \overrightarrow{AB_1} \cdot \overrightarrow{BC_1} = (\overrightarrow{BB_1} - \overrightarrow{BA}) \cdot (\overrightarrow{BB_1} + \overrightarrow{BC})$$

$$= \overrightarrow{BB_1}^2 + \overrightarrow{BB_1} \cdot \overrightarrow{BC} - \overrightarrow{BA} \cdot \overrightarrow{BB_1} - \overrightarrow{BA} \cdot \overrightarrow{BC}$$

$$= 1 - \sqrt{2} \times \sqrt{2} \times \cos 60^\circ = 0,$$

$$\therefore \overrightarrow{AB_1} \perp \overrightarrow{BC_1}, \therefore AB_1 \perp BC_1.$$

$\therefore AB_1$ 与 BC_1 所成角的大小为 90° , 故选 B.

2. 解析 (1) $a \cdot (b+c) = a \cdot b + a \cdot c = 0$.

$$(2) a \cdot (a+b+c) = a \cdot a + a \cdot b + a \cdot c = 1.$$

$$(3) (a+b) \cdot (b+c) = a \cdot b + a \cdot c + b \cdot b + b \cdot c = 1.$$

3. 解析 (1) $\overrightarrow{AA'} \cdot \overrightarrow{AB} = |\overrightarrow{AA'}| |\overrightarrow{AB}| \cos \angle BAA'$

$$= 5 \times 4 \times \cos 60^\circ = 20 \times \frac{1}{2} = 10.$$

$$(2) \because \overrightarrow{AB'} = \overrightarrow{AA'} + \overrightarrow{AB},$$

$$\therefore |\overrightarrow{AB'}|^2 = (\overrightarrow{AA'} + \overrightarrow{AB})^2 = |\overrightarrow{AA'}|^2 + 2 \overrightarrow{AA'} \cdot \overrightarrow{AB} + |\overrightarrow{AB}|^2 = 5^2 + 2 \times 10 + 4^2 = 61.$$

$$\therefore |\overrightarrow{AB'}| = \sqrt{61}, \text{ 即 } AB' \text{ 的长为 } \sqrt{61}.$$

$$(3) \because \overrightarrow{AC'} = \overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{AA'},$$

$$\therefore |\overrightarrow{AC'}|^2 = |\overrightarrow{AB}|^2 + |\overrightarrow{AD}|^2 + |\overrightarrow{AA'}|^2 + 2 \overrightarrow{AB} \cdot \overrightarrow{AD} + 2 \overrightarrow{AB} \cdot \overrightarrow{AA'} + 2 \overrightarrow{AD} \cdot \overrightarrow{AA'}$$

$$= 16 + 9 + 25 + 2 \times 4 \times 5 \times \frac{1}{2} + 2 \times 3 \times 5 \times \frac{1}{2} = 85,$$

$$\therefore |\overrightarrow{AC'}| = \sqrt{85}, \text{ 即 } AC' \text{ 的长为 } \sqrt{85}.$$

4. 解析 $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AB} + \overrightarrow{BD}$,

$$\therefore |\overrightarrow{CD}|^2 = |\overrightarrow{CA}|^2 + |\overrightarrow{AB}|^2 + |\overrightarrow{BD}|^2 + 2 \overrightarrow{CA} \cdot \overrightarrow{AB} + 2 \overrightarrow{CA} \cdot \overrightarrow{BD} + 2 \overrightarrow{AB} \cdot \overrightarrow{BD} = a^2 + b^2 + c^2 + 2 \overrightarrow{CA} \cdot \overrightarrow{BD} + 2 \overrightarrow{AB} \cdot \overrightarrow{BD}.$$

$$\therefore |\overrightarrow{CD}| = \sqrt{a^2 + b^2 + c^2},$$

$$\text{即 } C, D \text{ 两点间的距离为 } \sqrt{a^2 + b^2 + c^2}.$$

◆ 习题 1.1

复习巩固

1. 解析 (1) 与向量 $\overrightarrow{BC}$ 相等的向量有: $\overrightarrow{AD}, \overrightarrow{A'D'}, \overrightarrow{B'C'}$.

(2) 与向量 $\overrightarrow{BC}$ 相反的向量有: $\overrightarrow{CB}, \overrightarrow{DA}, \overrightarrow{D'A'}, \overrightarrow{C'B'}$.

(3) 与向量 $\overrightarrow{EF}$ 平行的向量有: $\overrightarrow{FE}, \overrightarrow{A'B}, \overrightarrow{BA'}, \overrightarrow{D'C}, \overrightarrow{CD}$.

2. 解析 (1) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

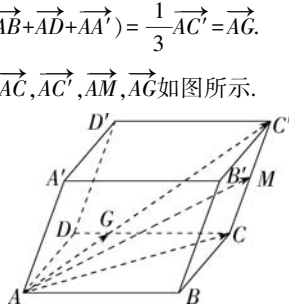
$$(2) \overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CC'} = \overrightarrow{AC} + \overrightarrow{CC'} = \overrightarrow{AC'}.$$

(3) 设点 M 是线段 CC' 的中点, 则 $\overrightarrow{AB} + \overrightarrow{AD} + \frac{1}{2}\overrightarrow{CC'} = \overrightarrow{AC} + \overrightarrow{CM} = \overrightarrow{AM}$.

(4) 设点 G 是线段 AC' 上靠近点 A 的三等分点, 则

$$\frac{1}{3}(\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{AA'}) = \frac{1}{3}\overrightarrow{AC'} = \overrightarrow{AG}.$$

向量 $\overrightarrow{AC}, \overrightarrow{AC'}, \overrightarrow{AM}, \overrightarrow{AG}$ 如图所示.



3. 证明 因为向量 a, b 共线, 所以根据向量共线的充要条件, 假设存在实数 λ , 使得 $b = \lambda a$,

则 $2a + b = (2 + \lambda)a$,

所以向量 $2a + b$ 与 a 共线.

4. 解析 (1) $\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{AB}| |\overrightarrow{AC}| \cos 60^\circ$

$$= \frac{1}{2}a^2.$$

$$(2) \overrightarrow{AD} \cdot \overrightarrow{DB} = |\overrightarrow{AD}| |\overrightarrow{DB}| \cos 120^\circ$$

$$= -\frac{1}{2}a^2.$$

$$(3) \overrightarrow{GF} \cdot \overrightarrow{AC} = |\overrightarrow{GF}| |\overrightarrow{AC}| \cos 180^\circ$$

$$= -\frac{1}{2}a^2 \left(|\overrightarrow{GF}| = \frac{1}{2} |\overrightarrow{AC}| = \frac{1}{2}a \right).$$

$$(4) \overrightarrow{EF} \cdot \overrightarrow{BC} = |\overrightarrow{EF}| |\overrightarrow{BC}| \cos 60^\circ$$

$$= \frac{1}{4}a^2 \left(|\overrightarrow{EF}| = \frac{1}{2} |\overrightarrow{BD}| = \frac{1}{2}a \right).$$

$$(5) \overrightarrow{FG} \cdot \overrightarrow{BA} = |\overrightarrow{FG}| |\overrightarrow{BA}| \cos 120^\circ$$

$$= -\frac{1}{4}a^2 \left(|\overrightarrow{FG}| = \frac{1}{2} |\overrightarrow{AC}| = \frac{1}{2}a \right).$$

$$(6) \overrightarrow{GE} \cdot \overrightarrow{GF} = \left(\overrightarrow{GC} + \overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA} \right) \cdot \frac{1}{2}\overrightarrow{CA}$$

$$= \left(\frac{1}{2}\overrightarrow{DC} + \overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA} \right) \cdot \frac{1}{2}\overrightarrow{CA} = \frac{1}{4}\overrightarrow{DC} \cdot \overrightarrow{CA} + \overrightarrow{CB} \cdot \frac{1}{2}\overrightarrow{CA} + \frac{1}{4}\overrightarrow{BA} \cdot \overrightarrow{CA} = \frac{1}{4}|\overrightarrow{DC}|$$

$$\cdot |\overrightarrow{CA}| \cos 120^\circ + \frac{1}{2}|\overrightarrow{CB}| |\overrightarrow{CA}| \cos 60^\circ + \frac{1}{4}|\overrightarrow{BA}| |\overrightarrow{CA}| \cos 60^\circ = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$\therefore \overrightarrow{GE} \cdot \overrightarrow{GF} = \frac{1}{4}a^2.$$

$$= -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} + \mathbf{c}.$$

6. 证明 $\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC}, \overrightarrow{HG} = \overrightarrow{HD} + \overrightarrow{DG} = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{DC}) = \frac{1}{2}\overrightarrow{AC},$

$$\text{所以 } \overrightarrow{EF} = \overrightarrow{HG},$$

$$\text{所以 } EF \parallel HG, EF = HG,$$

$$\text{所以四边形 } EFGH \text{ 是平行四边形,}$$

$$\text{所以 } E, F, G, H \text{ 四点共面.}$$

7. 解析 (1) $\overrightarrow{A'B} = \overrightarrow{A'A} + \overrightarrow{AB}, \overrightarrow{B'C} = \overrightarrow{B'B} + \overrightarrow{BC} = \overrightarrow{A'A} + \overrightarrow{AD},$

$$\therefore \overrightarrow{A'B} \cdot \overrightarrow{B'C} = (\overrightarrow{A'A} + \overrightarrow{AB}) \cdot (\overrightarrow{A'A} + \overrightarrow{AD}) = \overrightarrow{A'A} \cdot \overrightarrow{A'A} + \overrightarrow{A'A} \cdot \overrightarrow{AD} + \overrightarrow{AB} \cdot \overrightarrow{A'A} + \overrightarrow{AB} \cdot \overrightarrow{AD} = a^2.$$

$$\text{又 } |\overrightarrow{A'B}| = \sqrt{2}a, |\overrightarrow{B'C}| = \sqrt{2}a,$$

$$\therefore \cos \langle \overrightarrow{A'B}, \overrightarrow{B'C} \rangle = \frac{\overrightarrow{A'B} \cdot \overrightarrow{B'C}}{|\overrightarrow{A'B}| \cdot |\overrightarrow{B'C}|} = \frac{a^2}{2a^2} = \frac{1}{2},$$

$$\therefore \overrightarrow{A'B} \text{ 和 } \overrightarrow{B'C} \text{ 的夹角为 } 60^\circ.$$

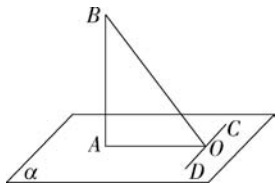
$$(2) \text{ 证明: } \overrightarrow{A'B} = \overrightarrow{A'A} + \overrightarrow{AB} = \overrightarrow{AB} - \overrightarrow{AA'}, \overrightarrow{AC'} = \overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{AA'},$$

$$\therefore \overrightarrow{A'B} \cdot \overrightarrow{AC'} = (\overrightarrow{AB} - \overrightarrow{AA'}) \cdot (\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{AA'}) = \overrightarrow{AB} \cdot \overrightarrow{AB} + \overrightarrow{AB} \cdot \overrightarrow{AD} + \overrightarrow{AB} \cdot \overrightarrow{AA'} - \overrightarrow{AA'} \cdot \overrightarrow{AB} - \overrightarrow{AA'} \cdot \overrightarrow{AD} - \overrightarrow{AA'} \cdot \overrightarrow{AA'} = a^2 - a^2 = 0,$$

$$\therefore \overrightarrow{A'B} \perp \overrightarrow{AC'},$$

$$\therefore \overrightarrow{A'B} \perp \overrightarrow{AC'}.$$

8. 证明 如图, BO 是平面 α 的斜线, O 为斜足, $BA \perp \alpha$, A 为垂足, $CD \subset$ 平面 α , 且 $CD \perp AO$.



$$\text{因为 } BA \perp \alpha, CD \subset \alpha, \text{ 所以 } BA \perp CD, \text{ 所以 } \overrightarrow{BA} \perp \overrightarrow{CD}, \text{ 所以 } \overrightarrow{BA} \cdot \overrightarrow{CD} = 0,$$

$$\text{因为 } CD \perp AO, \text{ 所以 } \overrightarrow{CD} \perp \overrightarrow{AO},$$

$$\text{所以 } \overrightarrow{CD} \cdot \overrightarrow{AO} = 0,$$

$$\text{又因为 } \overrightarrow{BO} = \overrightarrow{BA} + \overrightarrow{AO},$$

$$\text{所以 } \overrightarrow{CD} \cdot \overrightarrow{BO} = \overrightarrow{CD} \cdot (\overrightarrow{BA} + \overrightarrow{AO}) = \overrightarrow{CD} \cdot \overrightarrow{BA} + \overrightarrow{CD} \cdot \overrightarrow{AO} = 0,$$

$$\text{所以 } \overrightarrow{CD} \perp \overrightarrow{BO},$$

$$\text{所以 } CD \perp BO.$$

拓展探索

9. 证明 因为 $OA \perp BC$, 所以 $\overrightarrow{OA} \cdot \overrightarrow{BC} = \overrightarrow{OA} \cdot (\overrightarrow{OC} - \overrightarrow{OB}) = \overrightarrow{OA} \cdot \overrightarrow{OC} - \overrightarrow{OA} \cdot \overrightarrow{OB} = 0,$

$$\text{所以 } \overrightarrow{OA} \cdot \overrightarrow{OC} = \overrightarrow{OA} \cdot \overrightarrow{OB}.$$

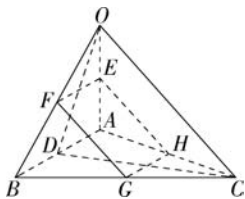
$$\text{因为 } OB \perp AC, \text{ 所以 } \overrightarrow{OB} \cdot \overrightarrow{AC} = \overrightarrow{OB} \cdot (\overrightarrow{OC} - \overrightarrow{OA}) = \overrightarrow{OB} \cdot \overrightarrow{OC} - \overrightarrow{OB} \cdot \overrightarrow{OA} = 0,$$

$$\text{所以 } \overrightarrow{OB} \cdot \overrightarrow{OC} = \overrightarrow{OB} \cdot \overrightarrow{OA}.$$

$$\text{所以 } \overrightarrow{OC} \cdot \overrightarrow{AB} = \overrightarrow{OC} \cdot (\overrightarrow{OB} - \overrightarrow{OA}) = \overrightarrow{OC} \cdot \overrightarrow{OB} - \overrightarrow{OC} \cdot \overrightarrow{OA} = \overrightarrow{OA} \cdot \overrightarrow{OB} - \overrightarrow{OA} \cdot \overrightarrow{OB} = 0,$$

$$\text{所以 } \overrightarrow{OC} \perp \overrightarrow{AB}, \text{ 所以 } OC \perp AB.$$

10. 证明 如图, 取 AB 的中点 D , 连接 CD, OD , 则有 $OD \perp AB, CD \perp AB$.



$$\therefore \overrightarrow{OD} \cdot \overrightarrow{AB} = 0, \overrightarrow{DC} \cdot \overrightarrow{AB} = 0,$$

$$\therefore \overrightarrow{OD} \cdot \overrightarrow{AB} + \overrightarrow{DC} \cdot \overrightarrow{AB} = \overrightarrow{AB} \cdot (\overrightarrow{OD} + \overrightarrow{DC}) = \overrightarrow{AB} \cdot \overrightarrow{OC} = 0.$$

$$\therefore \overrightarrow{AB} \perp \overrightarrow{OC}, \therefore AB \perp OC.$$

$$\therefore \text{点 } E, F, G, H \text{ 分别是 } OA, OB, BC, CA \text{ 的中点,}$$

$$\therefore \overrightarrow{EF} = \overrightarrow{HG} = \frac{1}{2}\overrightarrow{AB}, \overrightarrow{EH} = \overrightarrow{FG} = \frac{1}{2}\overrightarrow{OC}.$$

$$\therefore \text{四边形 } EFGH \text{ 为平行四边形.}$$

$$\text{又 } \because AB \perp OC, \therefore EF \perp EH,$$

$$\therefore \text{四边形 } EFGH \text{ 为矩形.}$$

1.2 空间向量基本定理

练习

1. 解析 $\because p = a + b, q = a - b$ 与 a, b 共面, 与 c 不共面, $\therefore$ 向量 c 一定可以与 p, q 一起构成空间的另一个基底.

2. 解析 $\because \overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ 不构成空间的一个基底, $\therefore \overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ 共面, $\therefore O, A, B, C$ 四点共面.

3. 解析 (1) $\because OA, OC, OO'$ 是平行六面体中有公共点的三条棱, 由平行六面体的结构特征, 知 $\overrightarrow{OA}, \overrightarrow{OC}, \overrightarrow{OO'}$ 不共面, $\therefore \{a, b, c\}$ 能构成空间的一个基底.

$$(2) \overrightarrow{OB'} = a + b + c, \overrightarrow{BA'} = c - b, \overrightarrow{CA'} = a - b + c,$$

$$\overrightarrow{OG} = \overrightarrow{OC} + \overrightarrow{CG} = b + \frac{1}{2}(\overrightarrow{CB} + \overrightarrow{BB'}) = \frac{1}{2}a + b + \frac{1}{2}c.$$

练习

1. 证明 因为 $OB = OC$, 所以 $|\overrightarrow{OB}| = |\overrightarrow{OC}|,$

$$\text{所以 } \cos \langle \overrightarrow{OA}, \overrightarrow{BC} \rangle = \frac{\overrightarrow{OA} \cdot \overrightarrow{BC}}{|\overrightarrow{OA}| |\overrightarrow{BC}|} = \frac{\overrightarrow{OA} \cdot (\overrightarrow{OC} - \overrightarrow{OB})}{|\overrightarrow{OA}| |\overrightarrow{BC}|} = \frac{\overrightarrow{OA} \cdot \overrightarrow{OC} - \overrightarrow{OA} \cdot \overrightarrow{OB}}{|\overrightarrow{OA}| |\overrightarrow{BC}|} = \frac{|\overrightarrow{OA}| |\overrightarrow{OC}| \cos \theta - |\overrightarrow{OA}| |\overrightarrow{OB}| \cos \theta}{|\overrightarrow{OA}| |\overrightarrow{BC}|} = 0,$$

$$\text{所以 } \overrightarrow{OA} \perp \overrightarrow{BC}, \text{ 所以 } OA \perp BC.$$

2. 解析 设 $\overrightarrow{AB} = a, \overrightarrow{AD} = b, \overrightarrow{AA'} = c$, 因为这三个向量不共面, 所以 $\{a, b, c\}$ 构成空间的一个基底.

$$\text{则 } \overrightarrow{BC'} = \overrightarrow{BC} + \overrightarrow{CC'} = \overrightarrow{AD} + \overrightarrow{AA'} = b + c,$$

$$\overrightarrow{CA'} = \overrightarrow{AA'} - \overrightarrow{AC} = \overrightarrow{AA'} - \overrightarrow{AB} - \overrightarrow{AD} = -a - b + c,$$

$$\text{所以 } \overrightarrow{BC'} \cdot \overrightarrow{CA'} = (b + c) \cdot (-a - b + c) = -a \cdot b - b \cdot b + b \cdot c - a \cdot c - b \cdot c + c \cdot c = -2 \times 2 \times \frac{1}{2} - 2^2 + 2 \times 3 \times \frac{1}{2} - 2 \times 3 \times \frac{1}{2} - 2 \times 3 \times \frac{1}{2} + 3^2 = 0,$$

$$\text{所以 } \cos \langle \overrightarrow{BC'}, \overrightarrow{CA'} \rangle = \frac{\overrightarrow{BC'} \cdot \overrightarrow{CA'}}{|\overrightarrow{BC'}| |\overrightarrow{CA'}|} = 0.$$

$$\text{所以 } \overrightarrow{BC'} \text{ 与 } \overrightarrow{CA'} \text{ 所成角的余弦值为 } 0.$$

3. 证明 设 $\overrightarrow{AB} = a, \overrightarrow{AD} = b, \overrightarrow{AA'} = c$, 则 $\{a, b, c\}$ 构成空间的一个正交基底.

$$\text{所以 } \overrightarrow{AO} = \overrightarrow{AD} + \overrightarrow{DO} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC} + \frac{1}{2}\overrightarrow{DD'} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AA'} = \frac{1}{2}a + b + \frac{1}{2}c,$$

$$\overrightarrow{CD'} = \overrightarrow{DD'} - \overrightarrow{DC} = \overrightarrow{AA'} - \overrightarrow{AB} = -a + c,$$

$$\text{所以 } \overrightarrow{AO} \cdot \overrightarrow{CD'} = \left(\frac{1}{2}a + b + \frac{1}{2}c\right) \cdot (-a + c) = -\frac{1}{2}a \cdot a + \frac{1}{2}a \cdot c - a \cdot b + b \cdot c - \frac{1}{2}a \cdot c + \frac{1}{2}c \cdot c = 0,$$

$$\text{所以 } \overrightarrow{AO} \perp \overrightarrow{CD'}, \text{ 所以 } AO \perp CD'.$$

◆习题 1.2

复习巩固

1. 解析 因为向量 a, b 与任何向量都不能构成空间的一个基底, 所以向量 a, b 共线或 a, b 都是零向量.

2. C 因为 $\{a, b, c\}$ 构成空间的一个基底, 所以向量 a, b, c 不共面.

$$\text{对于 A, 因为 } b = \frac{1}{2}(b + c) + \frac{1}{2}(b - c),$$

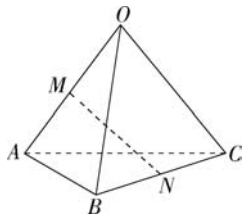
$$\text{所以 } b + c, b, b - c \text{ 三个向量共面;}$$

$$\text{对于 B, 同理可得, 向量 } a, a + b, a - b \text{ 共面;}$$

$$\text{对于 C, 若 } a + b, a - b, c \text{ 共面, 则 } c = x(a + b) + y(a - b) = (x + y)a + (x - y)b, \text{ 则 } a, b,$$

c 共面,与向量 a, b, c 不共面矛盾,所以 $a+b, a-b, c$ 不共面,所以 C 正确;
对于 D,因为 $a+b+c=(a+b)+c$,所以 $a+b, a+b+c, c$ 三个向量共面.故选 C.

3. 解析 如图,四面体 $OABC$ 中, M, N 分别是 OA, BC 的中点,



$$\begin{aligned}\overrightarrow{MN} &= \overrightarrow{MA} + \overrightarrow{AB} + \overrightarrow{BN} = \frac{1}{2}\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OA} + \\ &\quad \frac{1}{2}(\overrightarrow{OC} - \overrightarrow{OB}) \\ &= -\frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB} + \frac{1}{2}\overrightarrow{OC} = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} + \\ &\quad \frac{1}{2}\mathbf{c}.\end{aligned}$$

4. 解析 由已知得 $\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM} = \overrightarrow{AB} + \frac{1}{2}(\overrightarrow{BC} + \overrightarrow{BB'}) = \overrightarrow{AB} + \frac{1}{2}(\overrightarrow{AC} - \overrightarrow{AB}) + \frac{1}{2}\overrightarrow{AA'}$

$$\begin{aligned}&= \frac{1}{2}\overrightarrow{AA'} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} + \\ &\quad \frac{1}{2}\mathbf{c}, \\ \overrightarrow{AN} &= \overrightarrow{AC} + \overrightarrow{CN} = \overrightarrow{AC} + \overrightarrow{CC'} + \frac{1}{2}\overrightarrow{C'B'} = \\ &\quad \overrightarrow{AC} + \overrightarrow{AA'} + \frac{1}{2}(\overrightarrow{AB} - \overrightarrow{AC}) \\ &= \overrightarrow{AA'} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC} = \mathbf{a} + \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c}.\end{aligned}$$

综合运用

5. 解析 设 $\overrightarrow{D_1A_1} = \mathbf{a}, \overrightarrow{D_1C_1} = \mathbf{b}, \overrightarrow{D_1D} = \mathbf{c}$, 则 $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ 构成空间的一个正交基底.

$$\begin{aligned}\text{所以 } \overrightarrow{B_1M} &= \overrightarrow{B_1B} + \overrightarrow{BM} = \overrightarrow{B_1B} + \frac{1}{2}\overrightarrow{BD} = \overrightarrow{B_1B} \\ &+ \frac{1}{2}\overrightarrow{BA} + \frac{1}{2}\overrightarrow{BC} = \overrightarrow{D_1D} - \frac{1}{2}\overrightarrow{D_1C_1} - \frac{1}{2}\overrightarrow{D_1A_1} \\ &= -\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} + \mathbf{c},\end{aligned}$$

$$\begin{aligned}\text{所以 } |\overrightarrow{B_1M}|^2 &= \left(-\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} + \mathbf{c}\right)^2 = \frac{1}{4}\mathbf{a}^2 \\ &+ \frac{1}{4}\mathbf{b}^2 + \mathbf{c}^2 + \frac{1}{2}\mathbf{a} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{c} - \mathbf{b} \cdot \mathbf{c} = 11, \\ \text{所以 } |\overrightarrow{B_1M}| &= \sqrt{11}, \text{ 即 } B_1M \text{ 的长为 } \sqrt{11}.\end{aligned}$$

6. 证明 设 $\overrightarrow{CD} = \mathbf{a}, \overrightarrow{CB} = \mathbf{b}, \overrightarrow{CC_1} = \mathbf{c}$, 则 $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ 构成空间的一个基底.

由题意知,平行六面体的所有棱长都相等,设棱长为 m ,

$$\begin{aligned}\overrightarrow{CA_1} &= \overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{AA_1} = \mathbf{a} + \mathbf{b} + \mathbf{c}, \overrightarrow{BD} = \overrightarrow{CD} - \overrightarrow{CB} \\ &= \mathbf{a} - \mathbf{b}, \overrightarrow{BC_1} = \overrightarrow{CC_1} - \overrightarrow{CB} = -\mathbf{b} + \mathbf{c}, \\ \text{所以 } \overrightarrow{CA_1} \cdot \overrightarrow{BD} &= (\mathbf{a} + \mathbf{b} + \mathbf{c}) \cdot (\mathbf{a} - \mathbf{b}) \\ &= \mathbf{a}^2 - \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{b} - \mathbf{b}^2 + \mathbf{a} \cdot \mathbf{c} - \mathbf{b} \cdot \mathbf{c} \\ &= 0, \\ \overrightarrow{CA_1} \cdot \overrightarrow{BC_1} &= (\mathbf{a} + \mathbf{b} + \mathbf{c}) \cdot (-\mathbf{b} + \mathbf{c}) = -\mathbf{a} \cdot \mathbf{b} \\ &+ \mathbf{a} \cdot \mathbf{c} - \mathbf{b}^2 + \mathbf{b} \cdot \mathbf{c} - \mathbf{b} \cdot \mathbf{c} + \mathbf{c}^2 = 0, \\ \text{所以 } \overrightarrow{CA_1} &\perp \overrightarrow{BD}, \overrightarrow{CA_1} \perp \overrightarrow{BC_1}, \\ \text{所以 } CA_1 &\perp BD, CA_1 \perp BC_1, \\ \text{因为 } BD &\subset \text{平面 } C_1BD, BC_1 \subset \text{平面 } \\ C_1BD, BD \cap BC_1 &= B, \\ \text{所以 } CA_1 &\perp \text{平面 } C_1BD.\end{aligned}$$

拓展探索

7. 解析 (1) 证明: 设 $\overrightarrow{DA} = \mathbf{a}, \overrightarrow{DC} = \mathbf{b}, \overrightarrow{DD_1} = \mathbf{c}$, 则 $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ 构成空间的一个单位正交基底.

$$\begin{aligned}\text{所以 } \overrightarrow{EF} &= \overrightarrow{DF} - \overrightarrow{DE} = \frac{1}{2}(\mathbf{a} + \mathbf{b} - \mathbf{c}), \overrightarrow{B_1C} = \\ \overrightarrow{C_1C} - \overrightarrow{C_1B_1} &= -(\mathbf{a} + \mathbf{c}), \\ \text{所以 } \overrightarrow{EF} \cdot \overrightarrow{B_1C} &= -\frac{1}{2}(\mathbf{a} + \mathbf{b} - \mathbf{c}) \cdot (\mathbf{a} + \mathbf{c}) \\ &= -\frac{1}{2}(\mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{c} + \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} - \mathbf{a} \cdot \mathbf{c} - \mathbf{c} \cdot \mathbf{c} \\ &\quad - \mathbf{c} \cdot \mathbf{c}) = 0, \\ \text{所以 } \overrightarrow{EF} &\perp \overrightarrow{B_1C}, \text{ 所以 } EF \perp B_1C.\end{aligned}$$

$$\begin{aligned}(2) \text{ 由 (1), 知 } \overrightarrow{EF} &= \frac{1}{2}(\mathbf{a} + \mathbf{b} - \mathbf{c}), \text{ 所以} \\ |\overrightarrow{EF}| &= \frac{1}{2}|\mathbf{a} + \mathbf{b} - \mathbf{c}| = \frac{1}{2}\sqrt{1^2 + 1^2 + 1^2} \\ &= \frac{\sqrt{3}}{2},\end{aligned}$$

$$\text{因为 } CG = \frac{1}{4}CD,$$

$$\begin{aligned}\text{所以 } \overrightarrow{C_1G} &= \overrightarrow{C_1C} + \frac{1}{4}\overrightarrow{CD} = -\left(\frac{1}{4}\mathbf{b} + \mathbf{c}\right), \\ \text{所以 } |\overrightarrow{C_1G}| &= \left|-\left(\frac{1}{4}\mathbf{b} + \mathbf{c}\right)\right| = \\ &\sqrt{\left(\frac{1}{4}\right)^2 + 1^2} = \frac{\sqrt{17}}{4},\end{aligned}$$

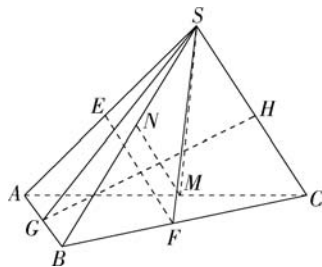
$$\begin{aligned}\text{所以 } \overrightarrow{EF} \cdot \overrightarrow{C_1G} &= -\frac{1}{2}(\mathbf{a} + \mathbf{b} - \mathbf{c}) \\ &\cdot \left(\frac{1}{4}\mathbf{b} + \mathbf{c}\right) \\ &= -\frac{1}{8}\mathbf{a} \cdot \mathbf{b} - \frac{1}{2}\mathbf{a} \cdot \mathbf{c} - \frac{1}{8}\mathbf{b}^2 - \frac{1}{2}\mathbf{b} \cdot \mathbf{c} + \\ &\quad \frac{1}{8}\mathbf{b} \cdot \mathbf{c} + \frac{1}{2}\mathbf{c}^2 = \frac{3}{8}.\end{aligned}$$

$$\text{所以 } \cos \langle \overrightarrow{EF}, \overrightarrow{C_1G} \rangle = \frac{\overrightarrow{EF} \cdot \overrightarrow{C_1G}}{|\overrightarrow{EF}| \cdot |\overrightarrow{C_1G}|} =$$

$$\frac{\frac{3}{8}}{\frac{\sqrt{3}}{2} \times \frac{\sqrt{17}}{4}} = \frac{\sqrt{51}}{17},$$

所以 EF 与 C_1G 所成角的余弦值为 $\frac{\sqrt{51}}{17}$.

8. 证明 如图,已知四面体 $SABC$, E, F, G, H, M, N 分别是 SA, BC, AB, SC, AC, SB 的中点,且 $|EF| = |HG| = |NM|$.



设 $\overrightarrow{SA} = \mathbf{a}, \overrightarrow{SB} = \mathbf{b}, \overrightarrow{SC} = \mathbf{c}$, 则 $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ 构成空间的一个基底.

因为 E, F, G, H, M, N 分别是 SA, BC, AB, SC, AC, SB 的中点,

$$\begin{aligned}\text{所以 } \overrightarrow{SE} &= \frac{1}{2}\mathbf{a}, \overrightarrow{SF} = \frac{1}{2}(\mathbf{b} + \mathbf{c}), \overrightarrow{SG} = \frac{1}{2}(\mathbf{a} \\ &+ \mathbf{b}), \overrightarrow{SH} = \frac{1}{2}\mathbf{c}, \overrightarrow{SM} = \frac{1}{2}(\mathbf{a} + \mathbf{c}), \overrightarrow{SN} = \\ &\quad \frac{1}{2}\mathbf{b},\end{aligned}$$

$$\text{所以 } \overrightarrow{EF} = \overrightarrow{SF} - \overrightarrow{SE} = \frac{1}{2}(\mathbf{b} + \mathbf{c} - \mathbf{a}), \overrightarrow{HG} = \overrightarrow{SG}$$

$$- \overrightarrow{SH} = \frac{1}{2}(\mathbf{a} + \mathbf{b} - \mathbf{c}), \overrightarrow{NM} = \overrightarrow{SM} - \overrightarrow{SN} = \frac{1}{2}(\mathbf{a} + \mathbf{c} - \mathbf{b}).$$

因为 $|EF| = |HG| = |NM|$,

$$\begin{aligned}\text{所以 } \left(\frac{\mathbf{b} + \mathbf{c} - \mathbf{a}}{2}\right)^2 &= \left(\frac{\mathbf{a} + \mathbf{b} - \mathbf{c}}{2}\right)^2 \\ &= \left(\frac{\mathbf{a} + \mathbf{c} - \mathbf{b}}{2}\right)^2,\end{aligned}$$

整理,得 $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{a} \cdot \mathbf{c}$,

所以 $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = 0$,

因为 $\mathbf{a} \neq \mathbf{0}, \mathbf{b} - \mathbf{c} \neq \mathbf{0}$, 所以 $\mathbf{a} \perp (\mathbf{b} - \mathbf{c})$,

即 $\overrightarrow{SA} \perp \overrightarrow{CB}$, 所以 $SA \perp CB$.

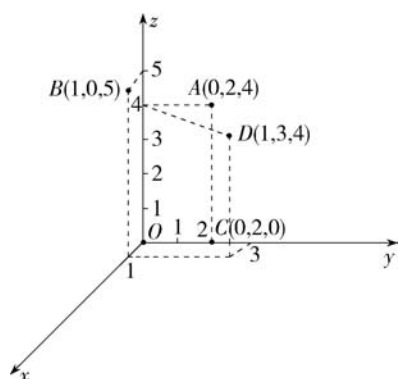
同理,可得 $SB \perp AC, SC \perp AB$.

1.3 空间向量及其运算的坐标表示

1.3.1 空间直角坐标系

练习

1. 解析 建立如图所示的空间直角坐标系,表示各点如图.



2. 解析 (1) 在空间直角坐标系 $Oxyz$ 中, Oyz 平面与 x 轴垂直, Oxz 平面与 y 轴垂直, Oxy 平面与 z 轴垂直.

(2) 点 $P(2,3,4)$ 在 Oyz 平面内的射影坐标为 $P_1(0,3,4)$, 在 Oxz 平面内的射影坐标为 $P_2(2,0,4)$, 在 Oxy 平面内的射影坐标为 $P_3(2,3,0)$.

(3) 点 $P(1,3,5)$ 关于原点成中心对称的点的坐标为 $P'(-1,-3,-5)$.

3. 解析 (1) $C(0,4,0)$, $B'(3,4,3)$, $P(\frac{3}{2}, 2, 3)$.

(2) $\overrightarrow{BB'} = \overrightarrow{OD} = (0,0,3)$, $\overrightarrow{A'C'} = \overrightarrow{A'D'} + \overrightarrow{D'C'} = (-3,4,0)$.

4. 解析 因为点 B 是点 $A(3,4,5)$ 在坐标平面 Oxy 内的投影, 所以 $B(3,4,0)$, 所以 $\overrightarrow{OB} = (3,4,0)$, 所以 $|\overrightarrow{OB}| = \sqrt{3^2+4^2} = 5$.

1.3.2 空间向量运算的坐标表示

练习

1. 解析 (1) $\mathbf{a} + \mathbf{b} = (-3+1, 2+5, 5-1) = (-2, 7, 4)$.
(2) $6\mathbf{a} = (6 \times (-3), 6 \times 2, 6 \times 5) = (-18, 12, 30)$.
(3) $3\mathbf{a} - \mathbf{b} = (-9, 6, 15) - (1, 5, -1) = (-10, 1, 16)$.
(4) $\mathbf{a} \cdot \mathbf{b} = (-3, 2, 5) \cdot (1, 5, -1) = -3 + 10 - 5 = 2$.

2. 解析 因为 $\mathbf{a} \perp \mathbf{b}$, 所以 $\mathbf{a} \cdot \mathbf{b} = 0$, 所以 $-8 - 2 + 3x = 0$, 解得 $x = \frac{10}{3}$.

3. 解析 由点 M 在 z 轴上, 可设 $M(0,0,a)$, 又因为 $A(1,0,2)$, $B(1,-3,1)$, $|\overrightarrow{MA}| = |\overrightarrow{MB}|$,

所以 $\sqrt{(1-0)^2 + (0-0)^2 + (2-a)^2} = \sqrt{(1-0)^2 + (-3-0)^2 + (1-a)^2}$, 解得 $a = -3$. 所以 $M(0,0,-3)$.

4. 解析 因为正方体的棱长为 a , 所以 $A(a,0,0)$, $B(a,a,0)$, $C(0,a,0)$, $C'(0,a,a)$, 设 $M(x_1, a, z_1)$, $N(x_2, y_2, 0)$, 因为 $BM = 2MC'$, 所以 $\overrightarrow{BM} = \frac{2}{3}\overrightarrow{BC'}$,

所以 $(x_1 - a, 0, z_1) = \frac{2}{3}(-a, 0, a)$,

所以 $\begin{cases} x_1 = \frac{1}{3}a, \\ z_1 = \frac{2}{3}a, \end{cases}$ 所以 $M(\frac{1}{3}a, a, \frac{2}{3}a)$.

因为 $AN = 2CN$, 所以 $\overrightarrow{AN} = \frac{2}{3}\overrightarrow{AC}$,

所以 $(x_2 - a, y_2, 0) = \frac{2}{3}(-a, a, 0)$,

所以 $\begin{cases} x_2 = \frac{1}{3}a, \\ y_2 = \frac{2}{3}a, \end{cases}$

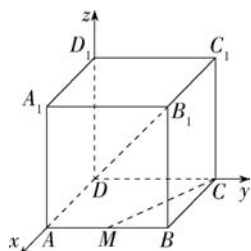
所以 $N(\frac{1}{3}a, \frac{2}{3}a, 0)$.

所以 $|\overrightarrow{MN}| =$

$$\sqrt{\left(\frac{1}{3}a - \frac{1}{3}a\right)^2 + \left(a - \frac{2}{3}a\right)^2 + \left(\frac{2}{3}a - 0\right)^2} = \frac{\sqrt{5}}{3}a.$$

所以 MN 的长为 $\frac{\sqrt{5}}{3}a$.

5. 解析 设正方体的棱长为 2, 建立如图所示的空间直角坐标系 $Dxyz$.



则 $D(0,0,0)$, $C(0,2,0)$, $B_1(2,2,2)$, $M(2,1,0)$, 所以 $\overrightarrow{DB_1} = (2,2,2)$, $\overrightarrow{CM} = (2,-1,0)$, 设 DB_1 与 CM 所成的角为 θ

$(0 \leq \theta \leq \frac{\pi}{2})$, 则 $|\cos \theta| = \frac{|\overrightarrow{DB_1} \cdot \overrightarrow{CM}|}{|\overrightarrow{DB_1}| |\overrightarrow{CM}|} =$

$$\frac{|(2,2,2) \cdot (2,-1,0)|}{\sqrt{3 \times 2^2} \times \sqrt{2^2 + (-1)^2}} = \frac{\sqrt{15}}{15}.$$

所以 DB_1 与 CM 所成角的余弦值为 $\frac{\sqrt{15}}{15}$.

◆ 习题 1.3

复习巩固

1. 解析 若向量 $\mathbf{a} \parallel x$ 轴, 则向量 $\mathbf{a} = (x, 0, 0)$ ($x \neq 0$); 向量 $\mathbf{b} \parallel y$ 轴, 则向量 $\mathbf{b} = (0, y, 0)$ ($y \neq 0$); 向量 $\mathbf{c} \parallel z$ 轴, 则向量 $\mathbf{c} = (0, 0, z)$ ($z \neq 0$).

2. 解析 (1) 与点 M 关于 x 轴的对称点为 $M_1(x, -y, -z)$.

(2) 与点 M 关于 y 轴的对称点为 $M_2(-x, y, -z)$.

(3) 与点 M 关于 z 轴的对称点为

$M_3(-x, -y, z)$.

(4) 与点 M 关于原点的对称点为 $M_4(-x, -y, -z)$.

3. 解析 因为正方体的棱长为 a , E, F, G, H, I, J 分别是相应棱的中点,

所以 $E(0, \frac{a}{2}, a)$, $F(\frac{a}{2}, 0, a)$,

$G(a, 0, \frac{a}{2})$, $H(a, \frac{a}{2}, 0)$, $I(\frac{a}{2}, a, 0)$,

$J(0, a, \frac{a}{2})$.

4. 解析 图略.

(1) $|\overrightarrow{AB}| =$

$$\sqrt{(3-2)^2 + (1-3)^2 + (4-5)^2} = \sqrt{6}.$$

(2) $|\overrightarrow{AB}| =$

$$\sqrt{(3-6)^2 + (5-0)^2 + (7-1)^2} = \sqrt{70}.$$

5. 解析 (1) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = (2, -3, 1) \cdot (2, 0, 5) = 4 + 0 + 5 = 9$.

(2) $\mathbf{a} + 6\mathbf{b} - 8\mathbf{c} = (2, -3, 1) + (12, 0, 18) - (0, 0, 16) = (14, -3, 3)$.

综合运用

6. 证明 由已知, 可得 $|\overrightarrow{AB}| =$

$$\sqrt{(10-4)^2 + (-1-1)^2 + (6-9)^2} = 7,$$

$$|\overrightarrow{AC}| = \sqrt{(2-4)^2 + (4-1)^2 + (3-9)^2} = 7,$$

$$|\overrightarrow{BC}| = \sqrt{(2-10)^2 + [4-(-1)]^2 + (3-6)^2} = 7\sqrt{2},$$

$$\text{因为 } 7^2 + 7^2 = (7\sqrt{2})^2,$$

$$\text{所以 } |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = |\overrightarrow{BC}|^2,$$

$$\text{又 } |\overrightarrow{AB}| = |\overrightarrow{AC}|,$$

所以以 $A(4,1,9)$, $B(10,-1,6)$, $C(2,4,3)$ 为顶点的三角形是等腰直角三角形.

7. 解析 因为 $A(3,5,-7)$, $B(-2,4,3)$, 所以 $\overrightarrow{AB} = (-5,-1,10)$, $\overrightarrow{BA} = (5,1,-10)$,

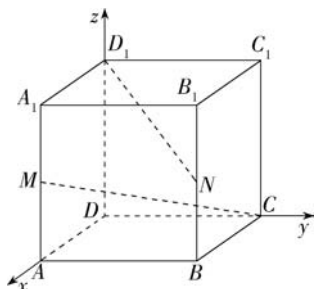
AB 的中点坐标为 $(\frac{3-2}{2}, \frac{5+4}{2}, \frac{-7+3}{2})$,

即 $(\frac{1}{2}, \frac{9}{2}, -2)$,

$$|\overrightarrow{AB}| = \sqrt{(-5)^2 + (-1)^2 + 10^2} = 3\sqrt{14}.$$

拓展探究

8. 解析 设正方体的棱长为 a , 以 D 为原点, 建立如图所示的空间直角坐标系.



则 $C(0,a,0)$, $M(a,0,\frac{a}{2})$, $D_1(0,0,$

$a)$, $N(a,a,\frac{a}{2})$,

$$\text{所以 } \overrightarrow{CM} = \left(a, -a, \frac{a}{2}\right),$$

$$\overrightarrow{D_1N} = \left(a, a, -\frac{a}{2}\right),$$

$$\text{所以 } \overrightarrow{CM} \cdot \overrightarrow{D_1N} = -\frac{a^2}{4}, |\overrightarrow{CM}| = \frac{3a}{2}, |\overrightarrow{D_1N}| = \frac{3a}{2}.$$

$$\text{所以 } \cos \langle \overrightarrow{CM}, \overrightarrow{D_1N} \rangle = \frac{\overrightarrow{CM} \cdot \overrightarrow{D_1N}}{|\overrightarrow{CM}| |\overrightarrow{D_1N}|} = -\frac{1}{9},$$

所以 CM 与 D_1N 所成角的余弦值为 $\frac{1}{9}$.

9. 解析 设向量 p 在基底 $\{a+b, a-b, c\}$ 下的坐标为 (x, y, z) , 则 $p = x(a+b) + y(a-b) + zc = (x+y)a + (x-y)b + zc$. 因为 $p = a + 2b + 3c$,

$$\text{所以 } \begin{cases} x+y=1, \\ x-y=2, \\ z=3, \end{cases} \text{ 解得 } \begin{cases} x=\frac{3}{2}, \\ y=-\frac{1}{2}, \\ z=3, \end{cases}$$

所以向量 p 用基底 $\{a+b, a-b, c\}$ 表示为 $p = \frac{3}{2}(a+b) - \frac{1}{2}(a-b) + 3c$.

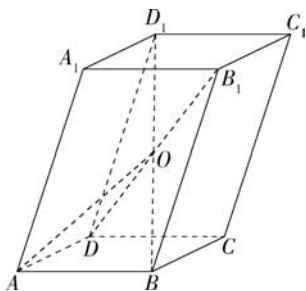
1.4 空间向量的应用

1.4.1 用空间向量研究直线、平面的位置关系

练习

1. 答案 (1) $\checkmark$ (2) $\times$ (3) $\checkmark$

2. 解析 如图,

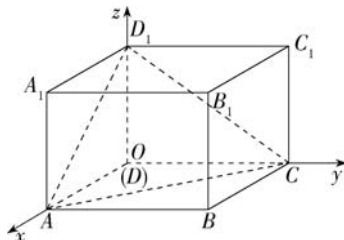


$$\begin{aligned} \overrightarrow{OA} &= \overrightarrow{OB} + \overrightarrow{BA} = \frac{1}{2}\overrightarrow{D_1B} - \overrightarrow{AB} = \frac{1}{2}(\overrightarrow{D_1D} + \overrightarrow{DA} \\ &+ \overrightarrow{AB}) - \overrightarrow{AB} = -\frac{1}{2}\overrightarrow{AA_1} - \frac{1}{2}\overrightarrow{AD} - \frac{1}{2}\overrightarrow{AB} \\ &= -\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}. \end{aligned}$$

所以直线 OA 的一个方向向量为 $\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$.

3. 解析 依题意, $A(3, 0, 0), C(0, 4, 0), D_1(0, 0, 2)$, 所以 $\overrightarrow{AC} = (-3, 4, 0), \overrightarrow{AD_1} =$

$(-3, 0, 2)$,



设 $n = (x, y, z)$ 是平面 ACD_1 的法向量, 则 $n \perp \overrightarrow{AC}, n \perp \overrightarrow{AD_1}$.

$$\text{所以 } \begin{cases} n \cdot \overrightarrow{AC} = -3x + 4y = 0, \\ n \cdot \overrightarrow{AD_1} = -3x + 2z = 0, \end{cases} \text{ 取 } x=4, \text{ 则 } y=3, z=6, \text{ 于是 } n = (4, 3, 6),$$

所以平面 ACD_1 的一个法向量为 $(4, 3, 6)$.

练习

1. 证明 已知直线 a, b , 平面 $\alpha, a \parallel b, a \not\subset \alpha, b \subset \alpha$.

求证: $a \parallel \alpha$.

证明: 设平面 α 的一个法向量为 n , 直线 a, b 的方向向量分别为 u, v , 因为 $b \subset \alpha$, 所以 $n \perp v$, 因为 $a \parallel b$, 所以 $u \parallel v$, 所以 $n \perp u$, 所以 $u \parallel \alpha$, 所以 $a \parallel \alpha$.

2. 解析 不存在. 在四面体 $ABCD$ 中, 设 $\overrightarrow{AB} = a, \overrightarrow{AC} = b, \overrightarrow{AD} = c$, 则 $\{a, b, c\}$ 构成一个基底,

因为 E 是 BC 的中点, 所以 $\overrightarrow{AE} = \frac{1}{2}a + \frac{1}{2}b$,

设 $\overrightarrow{AF} = mc (0 \leq m \leq 1)$, 则 $\overrightarrow{CF} = \overrightarrow{AF} - \overrightarrow{AC} = mc - b$,

若 $\overrightarrow{AE} \parallel \overrightarrow{CF}$, 则设 $\overrightarrow{AE} = n \overrightarrow{CF}$, 所以 $\frac{1}{2}a + \frac{1}{2}b = n(mc - b)$,

所以 $\frac{1}{2}a + \left(\frac{1}{2} + n\right)b - mnc = 0$, 因为 $\{a, b, c\}$ 构成一个基底,

$$\text{所以 } \begin{cases} \frac{1}{2} = 0, \\ \frac{1}{2} + n = 0, \\ mn = 0, \end{cases} \text{ 此方程组无解, 所以直}$$

线 AD 上不存在点 F , 使得 $AE \parallel CF$.

3. 证明 设正方体的棱长为 2, 以 D 为坐标原点, $\overrightarrow{DA}, \overrightarrow{DC}, \overrightarrow{DD_1}$ 的方向分别为 x 轴, y 轴, z 轴的正方向, 建立空间直角坐标系 $Dxyz$, 则根据题意, $A(2, 0, 0), C(0, 2, 0), D_1(0, 0, 2), E(2, 1, 1)$,

$F(1, 1, 2)$,

所以 $\overrightarrow{EF} = (-1, 0, 1), \overrightarrow{AC} = (-2, 2, 0)$,

$\overrightarrow{AD_1} = (-2, 0, 2)$,

设 $n = (x, y, z)$ 是平面 ACD_1 的一个法向量, 则 $n \perp \overrightarrow{AC}, n \perp \overrightarrow{AD_1}$,

$$\text{所以 } \begin{cases} n \cdot \overrightarrow{AC} = -2x + 2y = 0, \\ n \cdot \overrightarrow{AD_1} = -2x + 2z = 0. \end{cases} \text{ 取 } x=1, \text{ 则 } y=1, z=1, \text{ 所以 } n = (1, 1, 1),$$

又 $\overrightarrow{EF} \cdot n = (-1, 0, 1) \cdot (1, 1, 1) = -1 + 1 = 0$, 所以 $\overrightarrow{EF} \perp n$,

所以 $EF \parallel$ 平面 ACD_1 .

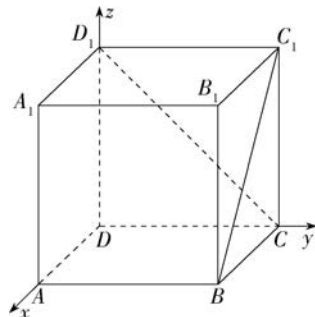
练习

1. 解析 (1) 由 $l \parallel \alpha$, 得 $u \perp n$, 所以 $u \cdot n = 0$, 即 $(3, a+b, a-b) \cdot (1, 2, 3) = 0$, 所以 $3 + 2a + 2b + 3a - 3b = 0$, 即 $5a - b + 3 = 0$.

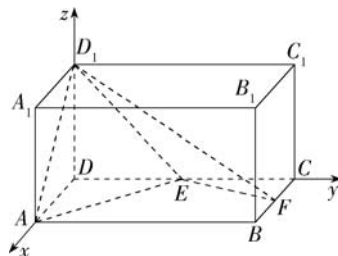
(2) 由 $l \perp \alpha$, 得 $u \parallel n$, 设 $u = tn (t \neq 0)$, 即 $(3, a+b, a-b) = t(1, 2, 3)$,

$$\text{所以 } \begin{cases} 3 = t, \\ a+b = 2t, \\ a-b = 3t, \end{cases} \text{ 解得 } \begin{cases} t = 3, \\ a = \frac{15}{2}, \\ b = -\frac{3}{2}. \end{cases}$$

2. 证明 如图, $A_1(1, 0, 1), B(1, 1, 0), C(0, 1, 0), C_1(0, 1, 1)$, 所以 $\overrightarrow{A_1C} = (-1, 1, -1), \overrightarrow{BC_1} = (-1, 0, 1)$, 所以 $\overrightarrow{A_1C} \cdot \overrightarrow{BC_1} = (-1, 1, -1) \cdot (-1, 0, 1) = 1 - 1 = 0$, 所以 $\overrightarrow{A_1C} \perp \overrightarrow{BC_1}$, 所以 $A_1C \perp BC_1$.



3. 证明 如图, 在长方体 $ABCD - A_1B_1C_1D_1$ 中, 以 D 为坐标原点, $\overrightarrow{DA}, \overrightarrow{DC}, \overrightarrow{DD_1}$ 的方向分别为 x 轴, y 轴, z 轴的正方向, 建立空间直角坐标系 $Dxyz$,



以上内容仅为本文档的试下载部分，为可阅读页数的一半内容。如要下载或阅读全文，请访问：<https://d.book118.com/108076132054006121>