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APPLICATION OF GPS IN A HIGH PRECISION ENGINEERING SURVEY NET WORK

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ABSTRACT .A GPS satellite survey was carried out with the Macrometer to sup-port construction at the Stanford Linear Accelerator Center(SLAC).The networkconsists of 16 stations of which 9 stations were part of the Macrometer network.The horizontal and vertical accuracy of the GPS survey is estimated to be l-2 m m and2-3 m m respectively.The horizontal accuracy of the terrestrial survey,consisting of angles and distances,equals that of the GPS survey only in the“loop”portion ofthe network.All stations are part of a precise level network.The ellipsoidal heightsobtained from the GPS survey and the orthometric heights of the level network are used to compute geoid undulations.A geoid profile along the linac was computed by the National Geodetic Survey in 1963.This profile agreed with the observed geoid within the standard deviation of the GPS survey.Angles and distances were adjusted together(TERRA),and all terrestrial observations were combined with the GPS vector observations in a combination adjustment(COMB).A comparison of C O M B and TERRA revealed systematic errors in the terrestrial solution.A scale factor of 1.5 ppm f.8 ppm was estimated.This value is of the same magnitude as the over-all horizontal accuracy of both networks.

INTRODUCTION

At the Stanford Linear Accelerator Center a new project is under construction,the Stanford Linear Collider(SLC).The shape of the completed SLC will be like a tennis racket with the handle being the existing linac and the curved parts being the new North and South collider arcs.The diameter formed by the loop will be about 1 km.To

position the approximately 1000 magnets in the arc tunnels,a network of nearby reference marks is necessary(Pietryka 1985).An error analysis has shown that a tunnel traverse cannot supply reference points with the required accuracy.Therefore,a control.network with vertical-penetrations will support the tunnel traverses.-The required absolute positional accuracy of a control point is f 2 m m(Friedsam -1984).

This two-dimensional surface net must be oriented to the same datum as defined by the design coordinate system.This design coordinate system is used to express the theoretical positions of all beam guiding elements.Since this coordinate system defines the direction of the existing two mile long linear accelerator(linac)as its Z-axis,the SLC coordinate system must integrate points along the linac in order to pick up its direction.Therefore,three linac stations have been added to the SLC net.Figure 1 shows the resulting network configuration.

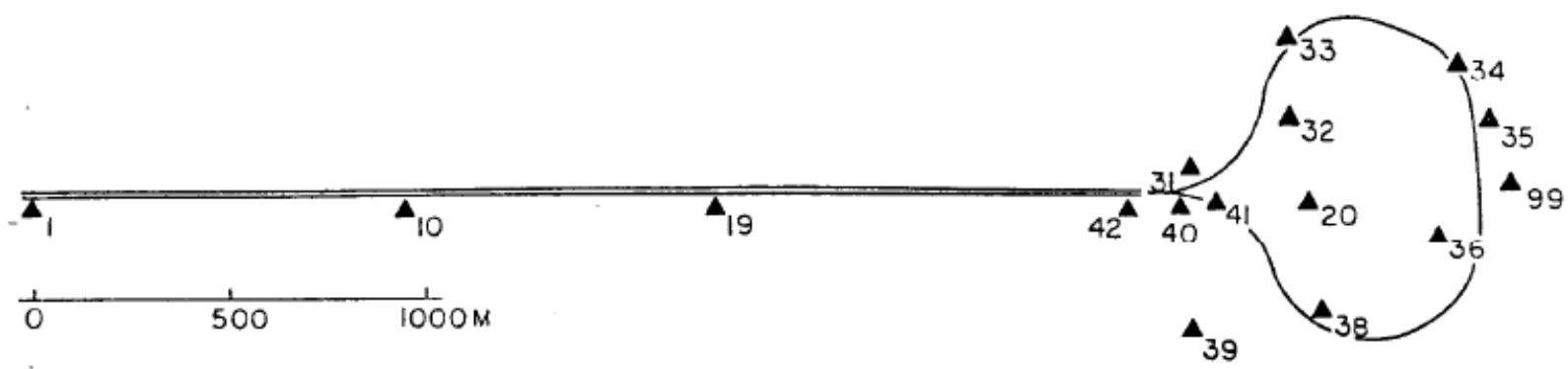

Figure 1 Network Configuration

The disadvantageous configuration is obvious,especially since there is no intervisibility between linac stations 1,10 and 19 to stations other than to 42 and 20.To improve this configuration,one would have to add stations northerly and southerly of the linac.However,due to local topography,doing that would have tripled the survey costs.

This was the situation when it was decided to try GPS technology,although it was at that time not yet proven that the required 2 m m standard deviation positional accuracy could be obtained.

SURVEY DESIGN

The horizontal control network consists of 16 stations,12 in the„loop“,and 4 along the linac.Because of financial considerations,not all 16 stations have been included in the GPS survey.Only the 4 linac and 5„loop“stations were occupied by the GPS survey.The intent was to determine the coordinates of the loop stations,including

station 42,by conventional means,i.e.triangulation and trilateration,followed by an inner constraint adjustment.Then the GPS information would be used to orient the net to the direction of the linac(Ruland 1985).

Conventional Horizontal Net

All monuments are equipped with forced centering systems and built either as massive concretears or steel frame towers,both with independant observation platforms.The observation schedule consists of directions and distances with standard deviations of 0.3 mgon and 2 mm,respectively.

Conventional V ertical Net

All 16 stations are part of a high precision level network.To minimize errors and simplify repeated leveling,both benchmarks and turning points are permanently monumented.Doublerunning the entire net requires about 700 setups.The standard deviation for a 1 km double-run
line is 0.3 mm.

GPS Survey

The GPS survey,which utilized the five available satellites,was carried out in August 1984 by Geo-Hydro Inc.The whole observation window was used for each station.In general three Macrometers were put to use.

Linac Laser Alignment System

For the frequent realignment of the linear accelerator,the linac laser alignment system was designed and installed.This system is capable of determining positions perpendicular to the axis of the linac(X and Y)to better than f.l m m over the total length of 3050 m.To do so,a straight line is defined between a point source of light and a detector.At each of the 274 support points,a target is supported on a remotely actuated hinge.To check the alignment at a desired point,the target at that point is inserted into the lightbeam by actuating the hinge mechanism.The target is actually a rectangular Fresnel lens with the correct focal length so that an image of the light source is formed on the plane of the detector.This image is then scanned by the

detector in both the vertical and the horizontal directions to determine the displacement of the target from the predetermined line.The targets are mounted in a 60 cm diameter aluminum pipe which is the basic support girder for the accelerator.The support girder is evacuated to about 10/.Lof Hg to prevent air refraction effects from distorting or deflecting the alignment image(Hermannsfeldt 1965).

Using this system it was possible to determine the X-coordinates of the four linac stations,independant of terrestrial or GPS survey techniques,to better than ±0.l mm.

ANALYSIS OF LEVELING DATA

To check for blunders,the L-l norm adjustment technique was applied(FUCHS 1983).Several blunders have been identified and cleared.A L-2 norm adjustment was then carried out with CATGPS(Collins 1985)in a minimally constrained fashion by fixing the height of station 41 to its published value of 64.259m.The choice of this particular station as well as the specific numerical value is,of course arbitrary for the purpose of the adjustment.CATGPS is suitable for adjusting leveling data if the latitudes and longitudes of the stations are fixed.The results of the level adjustment are summarized in Table 1(Column Level).

ANALYSIS OF GPS DA TA

All GPS vectors and their respective(3x3)covariance matrices as received from Geo-Hydro were subjected to an inner constraint least squares solution for the purpose of blunder detection and to get an unconstraint estimate of the obtained accuracy.

	SUMMARY of ADJUSTMENTS						
	LEVEL	**GPS**	**ANGLES**	**DIST**	**TERRA (A)**	**TERRA (B)**	**COMB**
Incl 1,10,19	YES	YES	NO	NO	NO	YES	YES
Hz Angles	—	—	85	—	85	93	93
Slant dist	—	—	—	94	94	106	106
Leveled ΔH	54	—	—	—	—	—	—
GPS Vectors	—	18	—	—	—	—	18
Observed h	—	—	12	12	12	15	15
Fixed Coordinates	H_{41}	$(\varphi, \lambda, h = H)_{41}$	$(\varphi, \lambda, h)_{41}$ $(\varphi, \lambda)_{35}$	$(\varphi, \lambda, h)_{41}$ φ_{35}	$(\varphi, \lambda, h)_{41}$ φ_{35}	$(\varphi, \lambda, h)_{41}$ φ_{35}	$(\varphi, \lambda)_{41}$ $(\varphi_{10}, h_{33}, h_{39})$
Parameters	44	24	34	35	35	44	46
DF	10	30	63	71	156	170	207
σ_0^2	1.10	1.13	1.11	1.17	1.26	1.30	1.30
VPV	2	34	70	83	196	204	269
$\chi^2_{DF,0.05}$	18	43	84	94	188	205	242
Transformation Parameters	NA	NA	NA	NA	NA	NA	$\alpha = 0.03 \pm 0.3$ sec $\xi = -0.1 \pm 0.9$ sec $\eta = 0.02 \pm 0.3$ sec $s = 1.50 \pm 0.8$ ppm

Table 1 Summary of Adjustment Results

Inner Constraint GPS Solution

Applying data snooping(Baarda 1976)on the residuals the vector observation(39-42)was suspected-of containing a blunder of about 1.3 cm.A recomputation was carried out at GeoHydro and,indeed,the time bias was not fixed in the original computation.Fixing the time biasin the case of short vectors is the standard procedure in Macrometer vector computation.The components of the recomputed vector agreed within 2 m m with the adjusted values of the original network solution.Upon implementing the corrected observations the residuals did not suggest the existence of other blunders.The inner constraint solution was carried out with MAC(Leick 1984);the results are documented in Table 1,Table 2,and Fig.2.The quality and homogeneity of the GPS network is well documented by the tables and the figure.The standard deviations for the horizontal positions are between 1 and 2 m m and for the vertical positions between 2 and 3 mm respectively.

If one computes the standard deviations and the adjusted length for all observed vectors and their ratios,then the average ratio is 1:690000.This value yields another

characterization of the horizontal accuracy achieved in this GPS survey.

Minimum Constraint GPS Solution

This solution defines the reference datum.The most simple set of minimal constraints are i imposed by fixing one station to account for the translatory component of the GPS polyhedron. The rotation and the scale are inherent in the Macrometer vector measurement and processing technique.The published geodetic latitude and longitude(NAD 1927)are adopted for station 41.The ellipsodial height for this station is equated to its orthometric height given above.Thus_the defined ellipsoid differs only slightly from the classical definition of a local reference ellipsoid(At the initial point the geodetic latitude and longitude equal astronomical latitude and longitude respectively;one geodetic and one astronomical azimuth are equated,and the ellipsodial height is taken as zero.)This classical definition makes the ellipsoid tangent to the equipotential surface at the initial point.Since the choice of the numerical values for station 41 are totally immaterial as far as the adjustment of GPS vectors is concerned,the classical definition of the local reference ellipsoid could have been used as well.The deflections of the vertical happen to be known in his adequate for this project as long as the correction of the measured horizontal angles due to deflections of the vertical are negligible since no attempt is made to apply these corrections.

STAT.	NORTHING [mm]	EASTING [mm]	ELL. HEIGHT [mm]
1	1.0	1.5	3.1
10	0.8	1.2	2.4
19	0.8	1.5	2.6
42	0.7	1.5	2.4
41	0.8	1.2	2.2
20	1.0	1.4	2.6
39	0.9	1.3	2.0
33	1.0	1.3	2.6
35	1.5	2.1	3.4

Table 2 Standard Deviations of GPS Solution

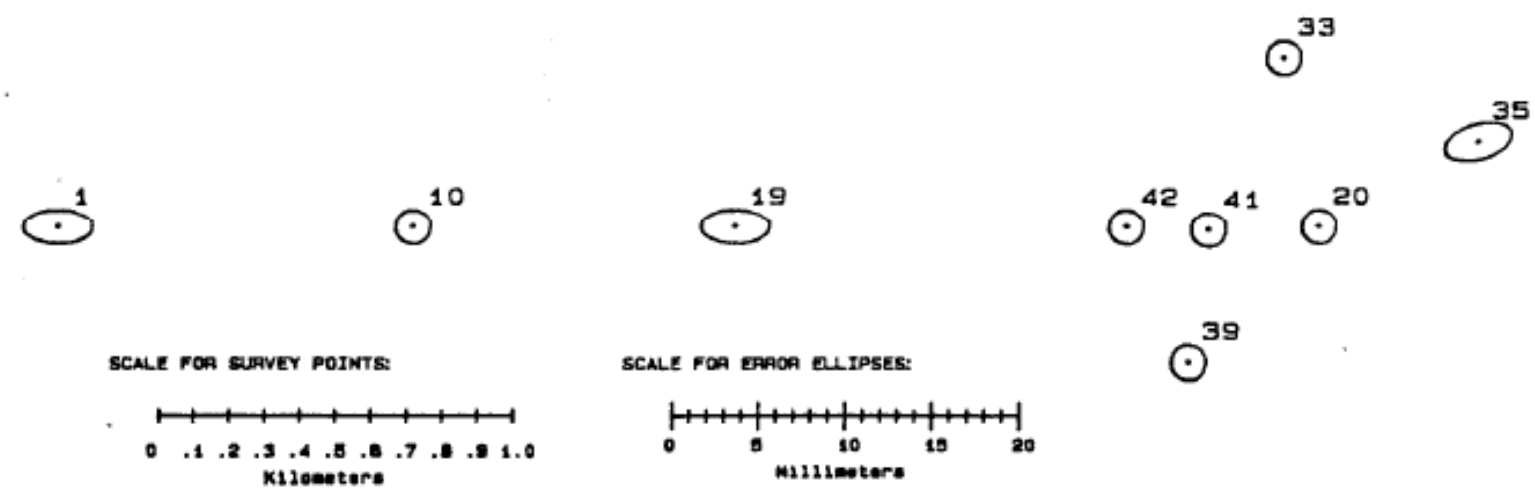

Figure 2 Error Ellipses from GPS Inner Constraint Solution

SHAPE OF THE GEOID

The shape of the geoid in the area of the survey follows readily from a comparison of the ellipsoidal and orthometric heights according to

$$H=h-N$$

Figure3 shows the geoidal profile along the linear accelerator.

The figure shows an unexpected dip of the-observed geoid at station 20.It so happens that this station required an observation tower of 20 m for the terrestrial measurements and that the height above the ground monument was measured trigonometrically.Assuming that the geoid follows the dashed line one can deduce an error in the height of the tower platform of about 8mm.In the context of an earlier survey for the construction of the linear accelerator the Coast and Geodetic Survey computed a geoid profile between stations 1 and 42.The report(Rice 1966)lists the components of the deflection of the vertical for stations 1 and 42,and for a non-existing station halfway between stations 10 and 19.From these values the Coast and Ge9detic Survey computed a function for the undulation.All linear values are in feet.The variable z is measured from station 1. It is stated in the report that this function gives undulations with an accuracy estimate of better than 0.001 ft.No procedure is given as to how this accuracy estimate was obtained.The undulation curve,derived from the following function,is shown in Fig.3.:

$$N = 11.102 * 10^{-6}(x) - 11.4331 * 10^{-10}(x)^2 - 6.0629 * 10^{-14}(x)^3$$

The.deviation between this curve and the observed geoid just barely exceeds,at station

10,the standard deviation for the Macrometer determined height difference from 1 to 10,and is within the standard deviation at stations 19 and 42.

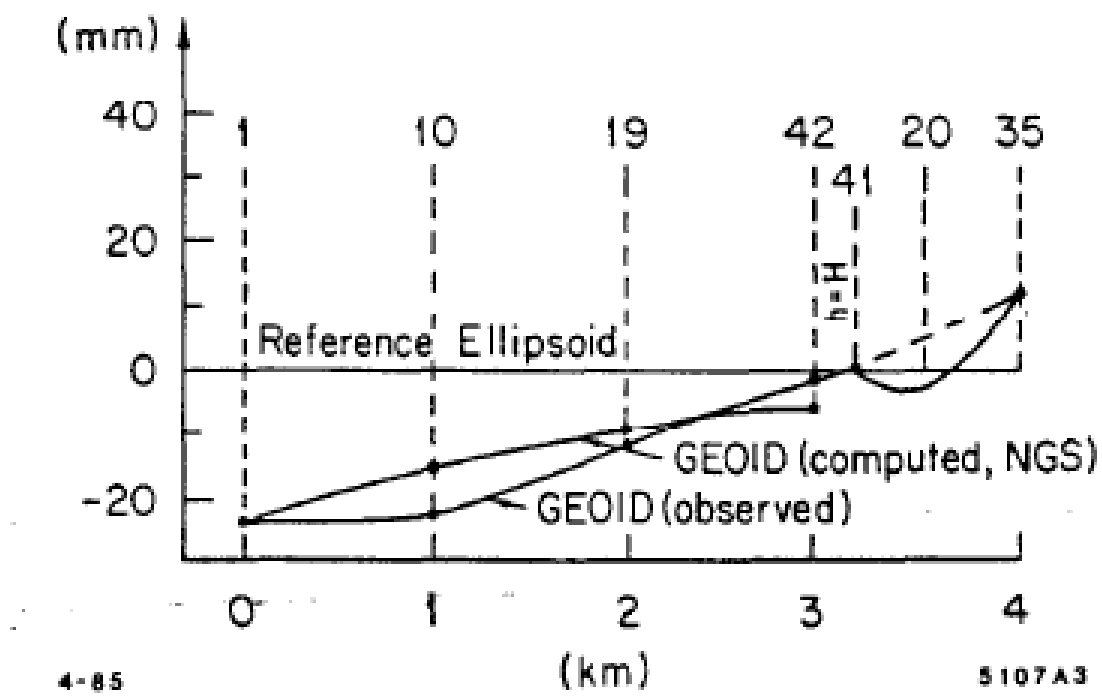

Figure 3 Geoid Profile

Incidentally,the over-all slope of the observed geoid is a consequence of adopting geodetic rather than astronomic positions as minimal constraints at station 41.The east-west component of the deflection of the vertical at station 42 is 1.84 arcsec which accounts for 27 m m of the 22 mm geoidal slope between stations 1 and 42.

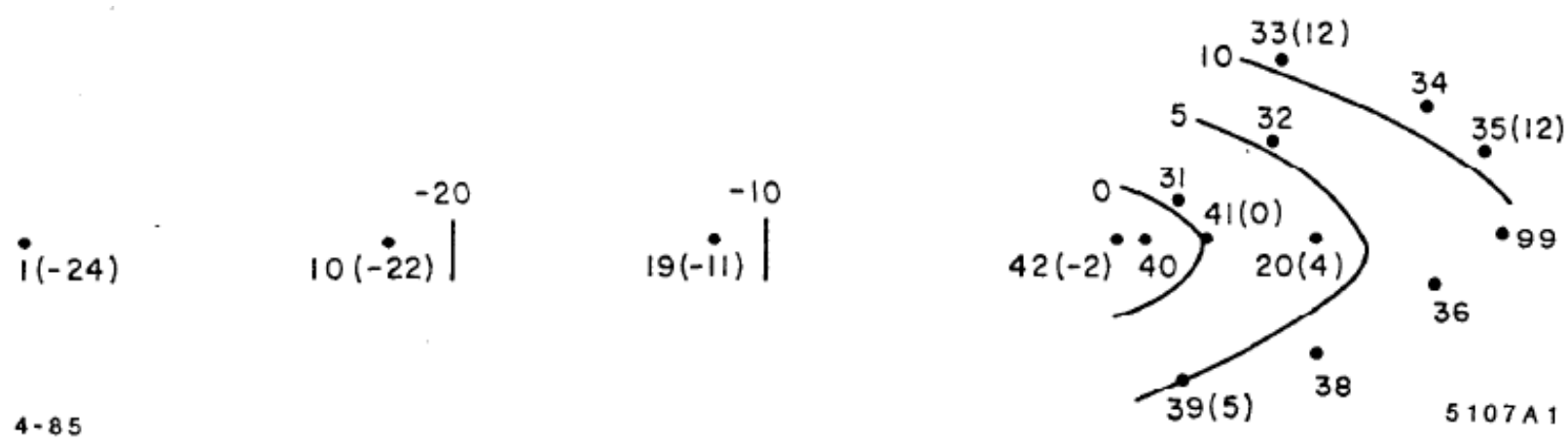

Figure 4 Geoid Undulation Contours

Figure 4 shows an attempt to draw contours of equal geoid heights.The small number of G P S stat&rs and their area1 distribution effects the accuracy of the contours.

ANALYSIS OF THE TERRES TRIAL OBSERV ATIONS

The triangulation and trilateration data were also checked for blunders applying the L-l norm technique(Fuchs 1980).The terrestial observations are then adjusted using the S-dimensional model of CATGPS.The reference ellipsoid is the one defined

above for the minimal constraint G P S vector solution,i.e.the same numerical values for station 41 are held fixed.The orientation in azimuth is achieved by holding the latitude of station 35 fixed to the numerical value computed for the minimal constraint GPS solution.The height of station 41 is constrained to the GPS solution as well.A consequence of this definition is that the terrestrial system(U)and the satellite system(S)coincide.Since the triangulation and trilateration observations do not contain much information in the third dimension,the ellipsoidal heights of the remaining stations are introduced as observed parameters.The heights are shown in Table 3.

	H [m]	h [m]	N [mm]	
1	76.161	76.137	−24	Observed
10	71.011	71.001	−22	
19	66.492	66.481	−11	
42	60.971	60.969	−2	
41	64.2594	64.2594	0	
20	78.196	78.193	3	
39	46.581	46.586	5	
33	47.469	47.481	12	
35	67.115	67.127	12	
31	76.278	76.279	1	Interpolated
32	69.169	69.175	7	
34	49.646	49.659	13	
36	72.335	72.341	6	
38	53.597	53.603	6	
99	77.074	77.085	11	
40	64.971	64.970	−1	

Table 3 Orthometric Height H and Ellipsidal Height H

The elliposidal heights for the GPS stations follow immediately from the&iinrmal c&straint GPS vector adjustment,whereas the ellipsoidal heights of the remaining points are computed from the orthometric heights and the interpolated geoid undulations.The standard deviations for the latter set of heights are derived from a guess for the accuracy of the geoid interpolations.

In order to investigate the relative weighting of theles and the distances,two separate adjustments are ried out with CATGPS,each having only one type observation.The result is shown in Table 1.The le for the angle adjustment is provided by fixing the gitude of station 35.The stations 1,10,and 19 are luded from these adjustments because of the weak of that part of the network.In the next step angles

distances are combined in a common ustment which excludes(TERRA A)and includes(TE

CATGPS is finally used to adjust the terrestrial observations and the GPS vectors together.The minimal constraints are implemented by assigning to the latitude and longitude of station 41,to the latitude of station 35,and to the ellipsoidal heights of stations 1,33,and 39 the minimum constraint GPS results as constants.In this way the GPS vector observations will determine the heights of all stations,i.e.the leveled orthometric heights do not enter this adjustment at all.Table 1 shows that the estimated rotation parameters differ only insignificantly from zero.Their theoretical value is zero because of the specific choice of the numerical values of the coordinates held fixed.A different selection for the fixed coordinate values at station 41,e.g.astronomical positions,would have resulted in estimated rotation parameters significantly different from zero.The estimated scale factor is 1.5 ppm which is about twice its estimated standard deviation.

INTERPRETA TION

Table 1 shows the a-posteriori variances of unit weight for all adjustments.It is seen that these values for the adjustments GPS,ANGLES,and DIST are all slightly above one,but are acceptable at a significance level of.05.Since the three variances of unit weight(1.13,1.11.1.17) are of nearly the same size,one could scale the variance of the GPS vectors,the angles,and the distances by a common scale.This would formally reduce the a-posteriori variances for TERRA(A),TERRA(B),and COMB,but would not change the outcome..of the adjustments.There appears to be no need to scale the variance for the GPS vector observation,the terrestrial angles and distances by separate(different)factors.

COORDINATE COMPARISON

STATION	GPS φ			COMB. φ			TERRA φ			Δ(GPS-COMB)		Δ(GPS-TERRA)		Δ(COMB-TERRA)	
	deg	min	sec	deg	min	sec	deg	min	sec	10^{-5}sec	mm	10^{-5}sec	mm	10^{-5}sec	mm
1	37	24	46.00522	37	24	46.00527	37	24	46.00911	−5	−2	−389	−117	−384	−115
10	37	24	50.85612	37	24	50.85614	37	24	50.85738	−2	−1	−176	−53	−124	−52
19	27	24	55.27041	37	24	55.27044	37	24	55.27134	−3	−1	−93	−31	−90	−30
42	37	25	0.65662	37	25	0.65665	37	25	0.65665	−3	−1	−3	−1	0	0
41	37	25	1.62420	37	25	1.62420	37	25	1.62420	—	—	—	—	—	—
20	37	25	3.49515	37	25	3.49508	37	25	3.49507	7	2	8	3	1	0
39	37	24	49.26119	37	24	49.26116	37	24	49.26114	3	1	4	1	2	0
33	37	25	18.39270	37	25	18.39266	37	25	18.39268	4	1	2	0	−2	0
35	37	25	13.49168	37	25	13.49168	37	25	13.49168	—	—	—	—	—	—
STATION	GPS λ			COMB. λ			TERRA λ			Δ(GPS-COMB)		Δ(GPS-TERRA)		Δ(COMB-TERRA)	
	deg	min	sec	deg	min	sec	deg	min	sec	10^{-5}sec	mm	10^{-5}sec	mm	10^{-5}sec	mm
1	237	45	44.00905	237	45	44.00925	237	45	44.00845	−20	−5	60	15	80	20
10	237	46	24.43642	237	46	24.43653	237	46	24.43611	−11	−3	31	8	42	11
19	257	47	1.20160	237	47	1.20171	237	47	1.20151	−11	−3	9	2	20	5
42	237	47	46.15328	237	47	46.15330	237	47	46.15327	−2	0	1	0	3	0
41	237	47	55.67460	237	47	55.67460	237	47	55.67460	—	—	—	—	—	—
20	237	48	8.23223	237	48	8.23222	237	48	8.23209	1	0	14	3	13	3
39	237	47	55.67696	237	47	55.67702	237	47	55.67704	−6	−1	−8	−2	−2	0
33	237	48	1.23324	237	48	1.23318	237	48	1.23318	6	1	6	1	0	0

Table 4 Compilation of Adjustment Results

Table 4 shows the adjusted coordinates for the GPS vector adjustment,the combined angle and distance adjustment TERRA(B),and the combination solution –TERRA”shows for each coordinate the discrepancies in milhmeters between the cornbinedmsolution and the terrestrial solution.The comparison is permissable since solutions in the same terrestrial system(U)are compared.There is a large discrepancy in latitude at station1.However,this discrepancy can be readily explained by a weakness of the terrestrial solution TERRA.The lateral position(with respect to the linac)is only determined by the angles(33-20-1)and(20-N-l).Note that the separation of stations 20-l and 10-l is 3500m and 2500m respectively.The discrepancies COMB-TERRA(B)are shown in Fig.5.There appears to be a systematic effect along the linac in the ter-I I Irestrial observations.The deviation definitely exceeds what can be expected from the formal standard deviations of the terrestrial solution TERRA(B).Several partial solutions were carried out and the residuals were inspected in all cases.No evidence could be found for the existance of blunders in the data.If one excludes the stations 1,10,and

A verification of whether either the GPS or the terrestrial observations along the linac are systematically debased could finally be obtained through utilizing the linac laser alignment system.A comparison of the X-coordinates of the linac stations from the TERRA and COMB solution with those determined using the linac alignment system was done by means of a seven parameter transformation after the ellipsoidal coordinates had been converted into Cartesian coordinates.The results are shown in table 5.Looking at the(LINAC-COMB)CO~UIIUI,the values of the differences are insignificant with respect to the standard deviations of the COMB-solution.In other words,the COMB-solution reflects the correct geometry of the linac;whereas the significant differences in the(LINAC-TERRA)column indicate that the geometry of the stations in the systems is not congruent.

The column GPS-COMB shows only small discrepancies.The latitudinal differences are all smaller than 2 mm.The discrepancies in the east-west direction are somewhat larger.A proper interpretation of these discrepancies requires that one distinguish between the two coordinate systems involved.The combination solution C O M B(as well as TERRA)refers to the terrestrial coordinate system(U).B ecause of the specific choice of the coordinates of the fixed station 41 and the futed latitude of station 10,the terrestial coordinate system(U)and the satellite system(S)are parallel.This is confirmed by the estimates of the rotation angles listed in Table 1.However,the same table lists a scale of±l.5 ppm.Going back to the definition of these transformation parameters it is seen that a positive scale estimate implies that the polyhedron determined by GPS observations(satellite system)is bigger than the one determined from the terrestrial observations.This is readily confirmed by comparing the longitudes of stations 1,41,and 35 for the GPS and the C O M B solutions in Table 4.The scale factor is,of course,also present in the latitudinal discrepancies,but to a lesser extent,because of the predominently east-west extension of the whole network.The longitudinal effect of the scale factor onaation 1 relative to station 41 is 1.5 ppm*3200 m=5.4 mm.This is the value by which the longitudinal separation of stations 1 and 41 should be increased in COMB.In fact,the effect of the

millimeters.Differencing these values with those listed in Table 4 under

–COMB”yields the discrepancies in which the effect of the scale is eliminated.The values are(O,O,-l,O,-,-l,-l,O,-3) in millimeters.These values and those listed for the latitude are of the same size.They reflect the“non–scale”discrepancies between the GPS solution and the combination solution.Their smallness reflects the dominance of the GPS vector observations in the combination solution.

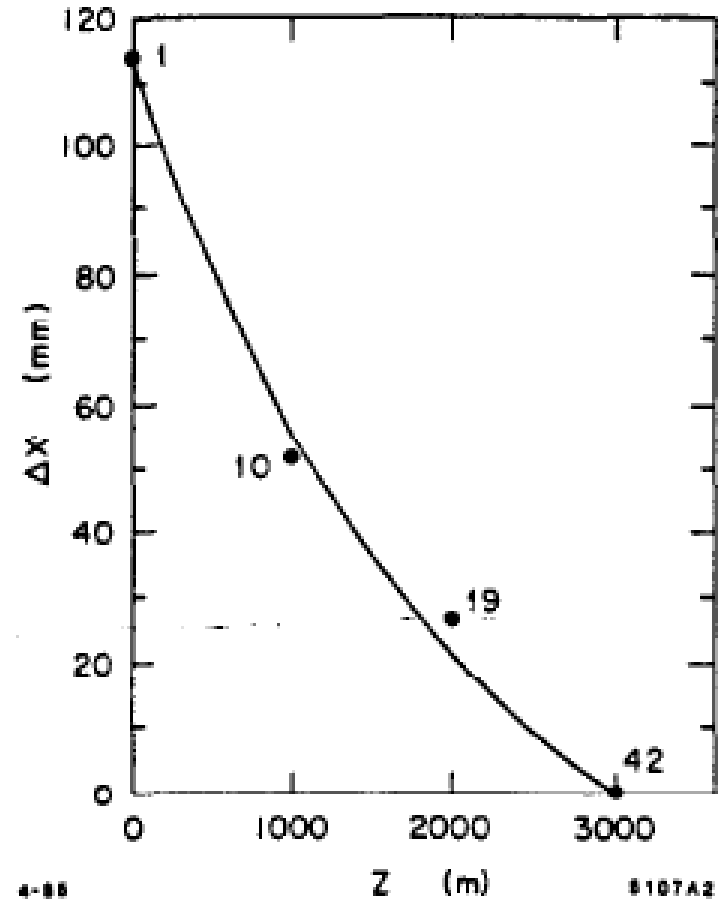

Figure 5 Linac Discrepancies

STATION	ΔX LINAC-COMB [mm]	ΔX LINAC-TERRA [mm]
1	0	−7
10	−1	11
19	0	−1
42	+1	−3

Table 5 Linac Comparison

The leveling data were used only to compute(interpolate)the geoid