OUTLINE

- Concepts of Vectors
- Linear Operation on Vectors
- Rectangular Coordinate System
- Vectors in Plane and Space
- Norm, Direction Angles and Projection of Vectors

1.Vectors

Scalars

Some of the quantities these we measure are determined by their magnitudes. we need only write down a number and name and appropriate unit of measure. The associated real numbers are **scalars (数量或标量)**.

Example length, area, mass, temperature, etc. .

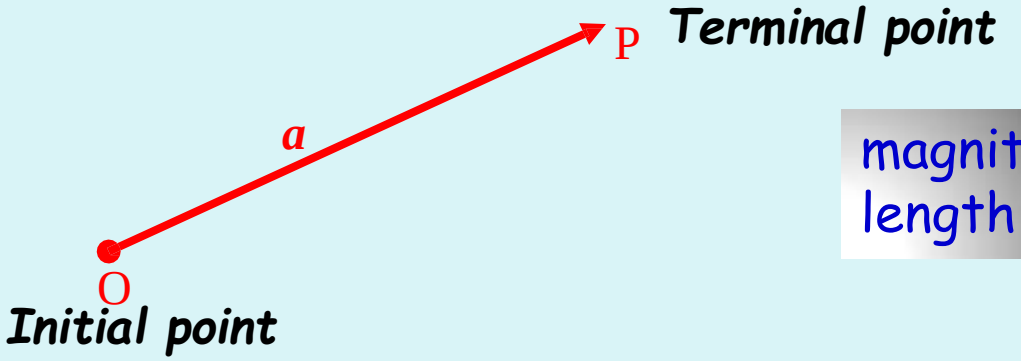
Vectors

Some of the quantities these we measure are determined by their directions and magnitudes. They are represented by **directed line segments (有向线段)**. These quantities are called **vectors (向量或矢量).**

Example force, displacement, etc. .

1. Vectors

Definition Vectors are quantities that possess both magnitude and direction.



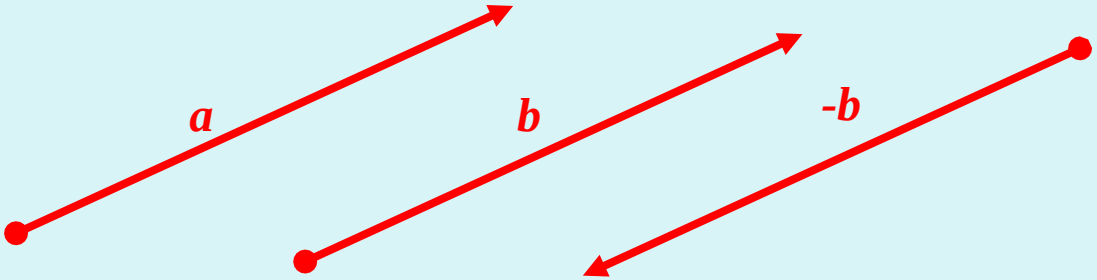
magnitude (or length) & direction

A vector is usually represented by a directed line segment, denoted by or ***A.***

Definition The magnitude of a vector is called the **length** or **norm (模)** of the vector.

The length of vectors $\overrightarrow{OP}$ and $\boldsymbol{a}$ is written as $|\overrightarrow{O}$ and $|\boldsymbol{a}|$, respectively.

1. Vectors



Definition Two vectors are **equal** (相等的) or **the same** if they have the same magnitude and direction.

Definition The **negative vector** (负矢量) is a vector which has the same magnitude with the original ***b***, and has an opposite direction with ***b*** **,** denoted by **–*b*.**

1.Vectors

Definition If two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ have same or opposite direction, we say that they are **parallel** (平行) or **collinear** (共线), denoted by $\boldsymbol{a}//\boldsymbol{b}$.

Definition If the directions of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are orthogonal, we say that they are **orthogonal** (正交) or **perpendicular** (垂直), denoted by $\mathbf{a} \perp \mathbf{b}$.

Definition Suppose that $\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_k (k>2)$ are $\boldsymbol{k}$ vectors with a common initial point. If they line in the same plane, then we say that these vectors are **coplanar** (共面).

1.Vectors

Definition A vector whose length is *1* is called a **unit vector** (单位向量), a unit vector whose direction is the same as that of $\boldsymbol{a}$ is written as $\boldsymbol{a}^{\circ}$.

Definition A vector whose length is *0* is called the **zero vector** (零向量), and is written as $\boldsymbol{0}$.

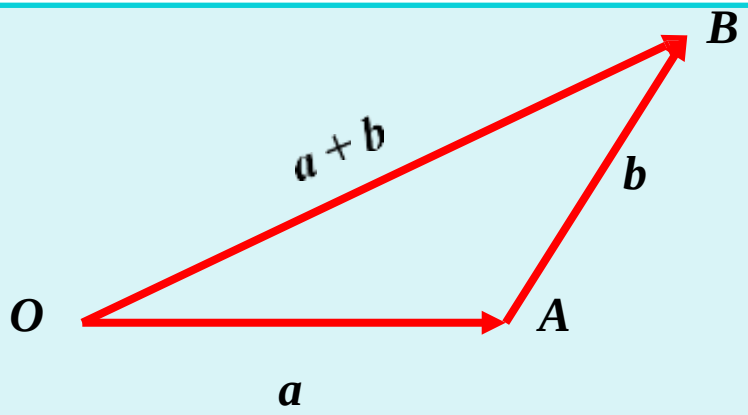
Note

In textbooks, vectors are usually written in lowercase, boldface letters, for example, **u,v** and **w.** Sometimes we use uppercase boldface letters, such as **F** to denote a force vector. In handwritten form, it is customary to draw small arrows above the letters, for example $\vec{u},\vec{v}$ and $\vec{F}$.

2.Operations on Vectors

Definition Triangle law of addition of vectors **(向量加法的三角形法则)**

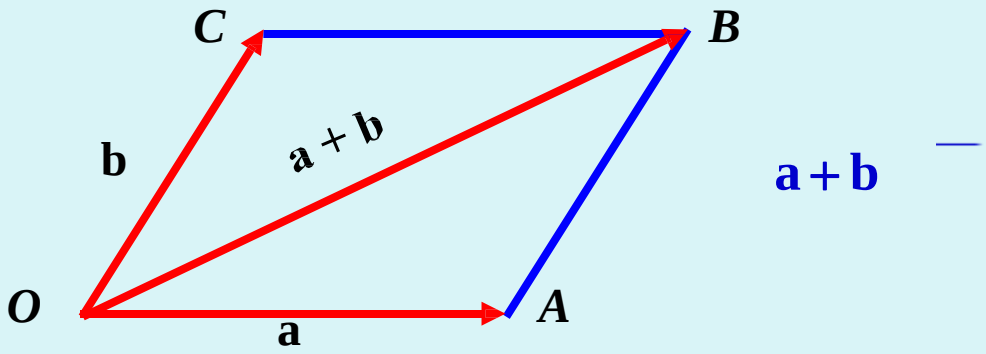
Suppose ***a*** and ***b*** are two vectors. If we draw a vector, which is equal to ***b***, from the initial point of **a**, then the **sum** ***a*+*b*** of ***a*** and ***b*** is the vector extending from the terminal point of ***a*** to the initial point of ***b***.



$$\mathbf{a}+\mathbf{b}=\overrightarrow{OB}$$

2.Operations on Vectors

Parallelogram law of addition of vectors (向量加法的平行四边形法则)



2.Operations on Vectors

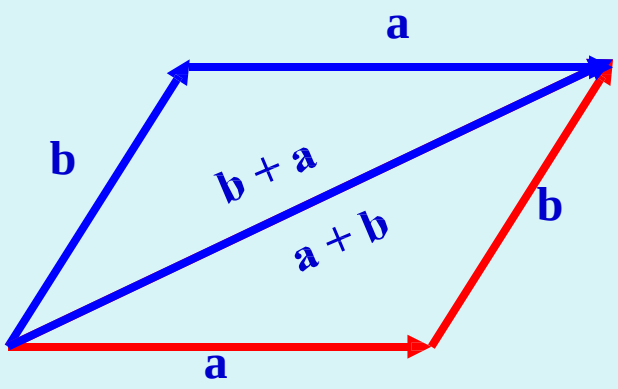
The addition of vectors satisfies the following laws:

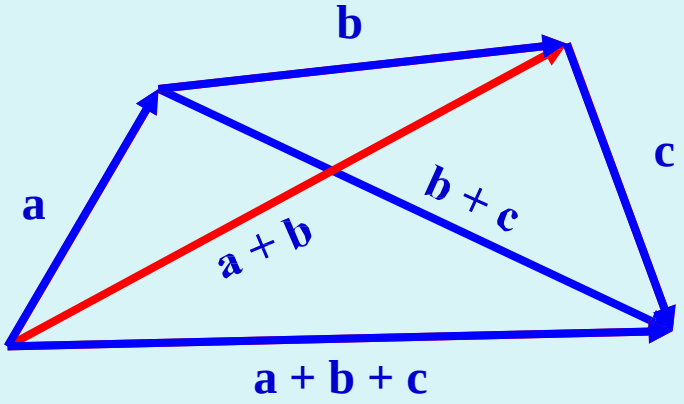
(1) Commutative law $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a};$

(2) Associative law $(\mathbf{a}+\mathbf{b})+\mathbf{c}=\mathbf{a}+(\mathbf{b}+\mathbf{c});$

(3) $\mathbf{a}+\mathbf{0}$

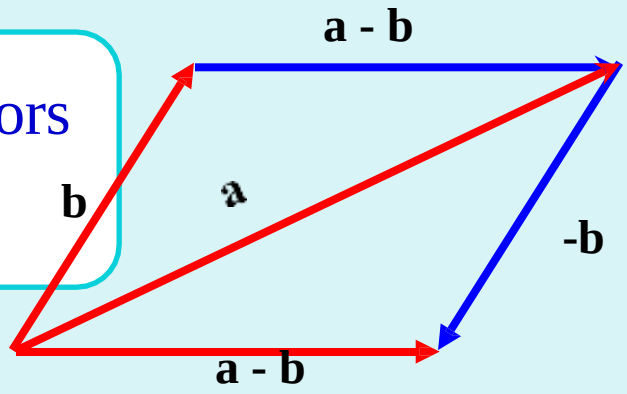
(4) $\mathbf{a}+(-\mathbf{a})=\mathbf{0}.$





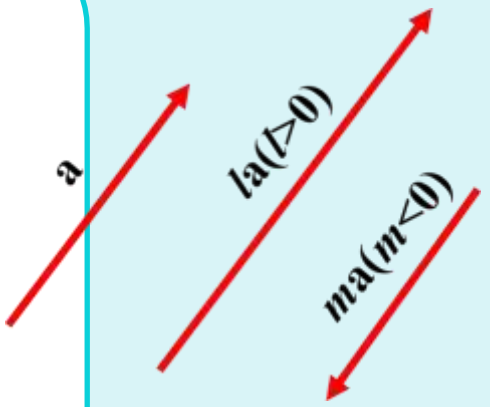
2.Operations on Vectors

Definition The **difference(减法)** of two vectors a and b is given by $\mathbf{a}-\mathbf{b}=\mathbf{a}+(-\mathbf{b})$.



Definition Scalar Multiplication **(数乘向量)**

The **product** of a scalar m and a vector $\boldsymbol{a}$ expressed by $m\boldsymbol{a}$ is a vector. Its length is $|m\boldsymbol{a}|$. Its direction is the same as that of $\boldsymbol{a}$ if m is positive and is opposite to that of $\boldsymbol{a}$ if m is negative.



2.Operations on Vectors

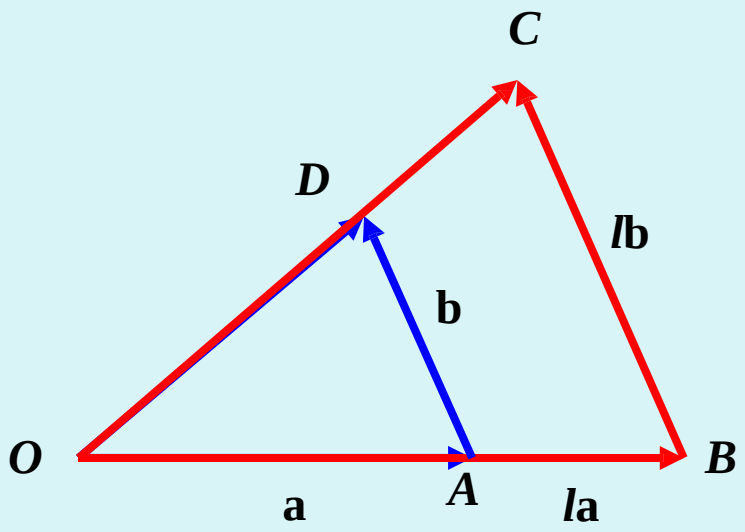
The scalar product satisfy the following laws:

(1) Associative law $l(m\mathbf{a})=(lm)\mathbf{a}$

(2) Distributive law $l(\mathbf{a}+\mathbf{b})=l\mathbf{a}+l\mathbf{b}$

$(l+m)\mathbf{a}=l\mathbf{a}+m\mathbf{a}$

(3) $1\mathbf{a}=\mathbf{a}$.



2.Operations on Vectors

Addition and scalar product of vectors are called by a joint name **linear operation (线性运算)** on vectors.

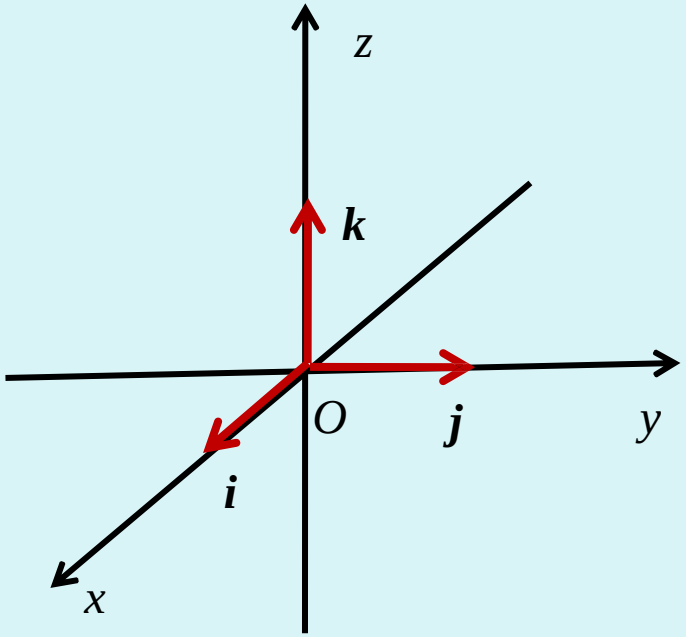
By the above discussion we known that the length of a vector has the following basic properties:

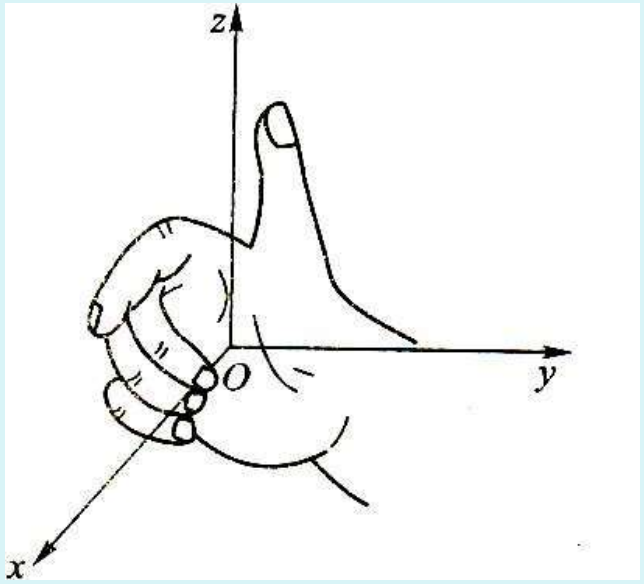
(1) Nonnegativity $|\mathbf{a}| \geq \mathbf{0}$, and $|\mathbf{a}| = \mathbf{0} \Leftrightarrow \mathbf{a} = \mathbf{0}$;

(2)Absolute homogeneity $|\lambda \mathbf{a}| = |\lambda| |\mathbf{a}|$;

(3) Triangle inequality $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$, where the sign of equality holds iff $\boldsymbol{a}$ and $\boldsymbol{b}$ have the same direction.

3.Rectangular Coordinate System

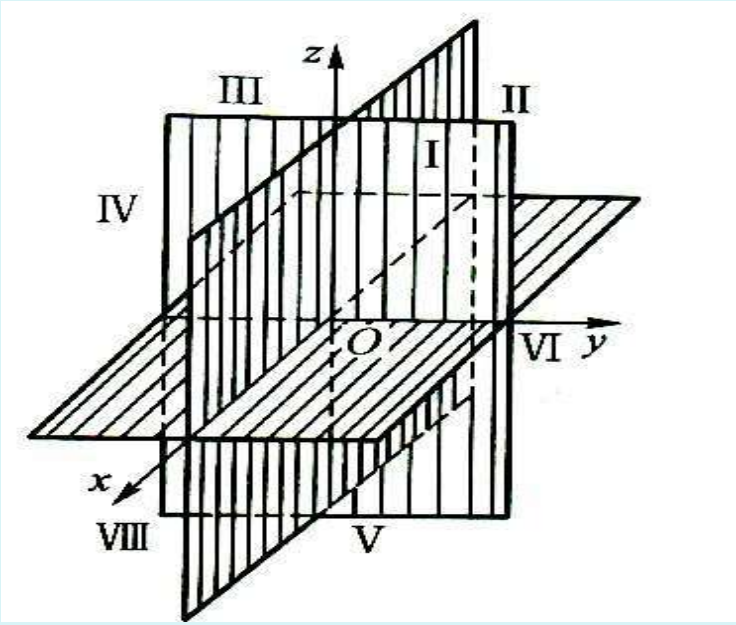




These three coordinate axes make a rectangular coordinate system in space, denoted by $Oxyz$.

Their positive direction is determined by the right-hand rule.

3.Rectangular Coordinate System

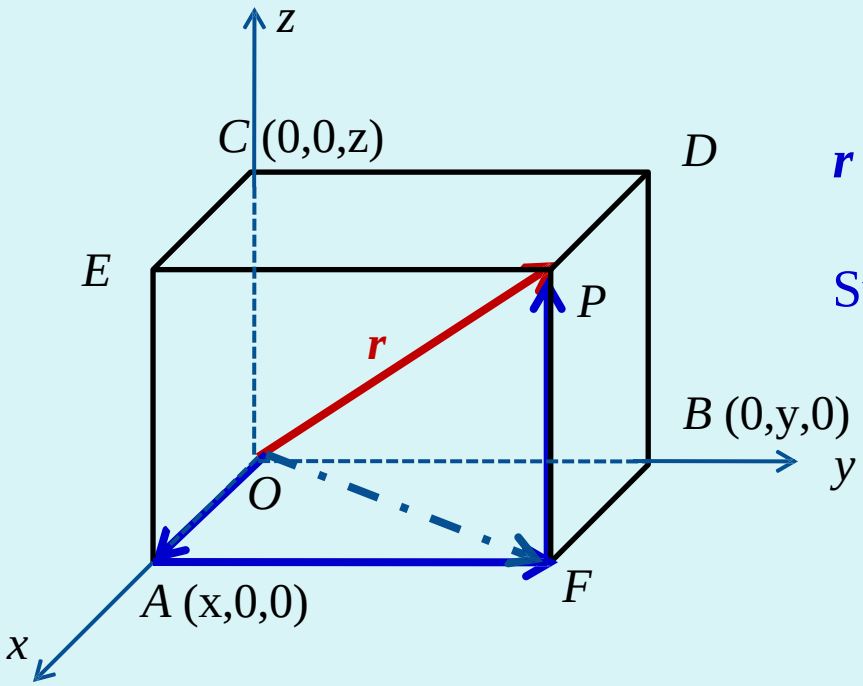


Three coordinate planes (坐标面): ***xOy***, ***yOz*** and ***xOz***

Eight octants (卦限):

Where the octants I, II, III, IV lie over the quadrants 1,2,3,4 of the ***xOy*** plane, respectively. The octants V, VI, VII, VIII lie below the ***xOy*** plane.

3.Rectangular Coordinate System



$$\boldsymbol{r} = \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AF} + \overrightarrow{FP} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$

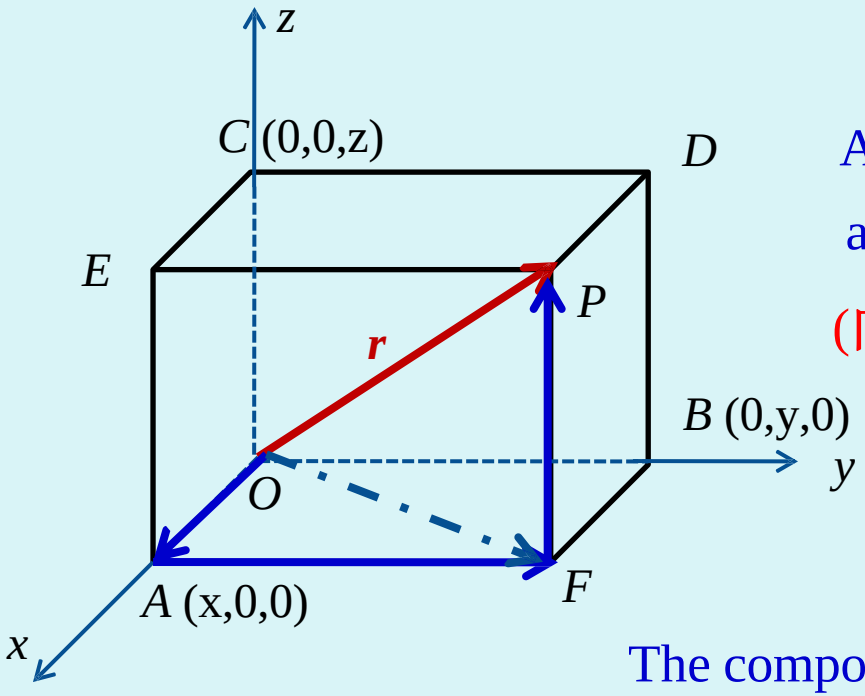
Suppose $\overrightarrow{OA} = x\boldsymbol{i}, \overrightarrow{OB} = y\boldsymbol{j}, \overrightarrow{OC} = z\boldsymbol{k}$, then

$$\boldsymbol{r} = \overrightarrow{OP} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

the **coordinate (**or **component)** **decomposition** (坐标分解式) of the vector **r**

point $P \leftrightarrow \boldsymbol{r} = \overrightarrow{OP} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} \leftrightarrow (x, y, z)$ (Page 88 Theorem 8.1.14)

3.Rectangular Coordinate System



Any nonzero vector $\overrightarrow{OP}$ with initial point at the origin O is called the radius vector (向径) of the point P, or radius vector $\overrightarrow{OP}$.

The components (x, y, z) of the radius vector $\overrightarrow{OP}$ are defined as the components of the vector $\boldsymbol{r}$, denotes by

$$\boldsymbol{r} = (x, y, z).$$

4.Vectors in Plane and Space

- Component Representation of Linear Operations on Vectors

Let

$$\mathbf{a} = x_1\mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}, \quad \mathbf{b}$$

By means of linear operations on vectors we get

$$\mathbf{a} \pm \mathbf{b} = (x_1 \pm x_2)\mathbf{i} + (y_1 \pm y_2)\mathbf{j} + (z_1 \pm z_2)\mathbf{k}$$

$$\lambda\mathbf{a} = (\lambda x_1)\mathbf{i} + (\lambda y_1)\mathbf{j} + (\lambda z_1)\mathbf{k}$$

or

$$\begin{cases} \mathbf{a} \pm \mathbf{b} = (x_1 \pm x_2, y_1 \pm y\ , z \\ \lambda\mathbf{a} = (\lambda \end{cases}$$

$$\mathbf{a} = \mathbf{b} \Leftrightarrow x_1 = x_2, y_1 = y_2, z_1 = z_2$$

4.Vectors in Plane and Space

Example 1 If $A = 4i + 3j - 5k$ and $B = -2i + j + 4k$ find

$3A, -4B, 3A - 4B$ and $3A + 4B$.

4.Vectors in Plane and Space

Let $\mathbf{a}=(x_1,y_1,z_1),\mathbf{b}=(x_2,y_2,z_2)$. It is not difficult to obtain that, ***a*** and ***b*** are collinear (or parallel) if and only if there is a real number λ, such that

$$\mathbf{a}=\lambda\mathbf{b}.$$

That is,

$$(x_1,y_1,z_1)=\lambda(x_2,y_2,z_2)=(\lambda x_2,\lambda y_2,\lambda z_2)$$

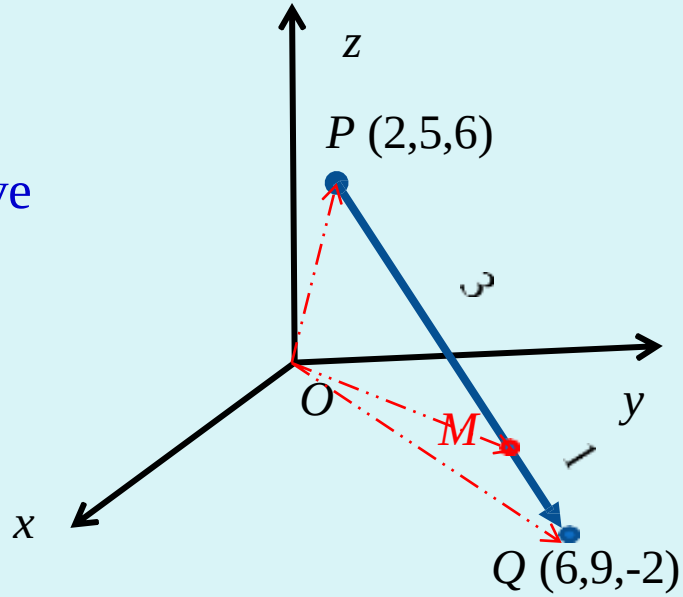
Then

$$\frac{x_1}{x}=\frac{y_1}{}$$

This expression implies that two nonzero vectors to be parallel iff their corresponding components are proportional.

4.Vectors in Plane and Space

Example 2 Given points $P(2,5,6)$ and $Q(6,9,-2)$, please find the point $M(x, y, z)$ on the line segment PQ such that $\overrightarrow{PM} = 3\overrightarrow{MQ}$.



Solution

Since $\overrightarrow{PM} = \overrightarrow{OM} - \overrightarrow{OP}$, $\overrightarrow{MQ} = \overrightarrow{OQ} - \overrightarrow{OM}$, we have

$\overrightarrow{OM} - \overrightarrow{OP} = 3(\overrightarrow{OQ} - \overrightarrow{OM})$.

Then $\overrightarrow{OM} = \frac{1}{4}(\overrightarrow{OP} + 3\overrightarrow{OQ})$.

$\overrightarrow{OP} = 2\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}$, $\overrightarrow{OQ} = 6\boldsymbol{i} + 9\boldsymbol{j} - 2\boldsymbol{k}$, thus

$$\overrightarrow{OM} = \frac{1}{4}[(2\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}) + 3(6\boldsymbol{i} + 9\boldsymbol{j} - 2\boldsymbol{k})]$$
$$= 5\boldsymbol{i} + 8\boldsymbol{j} + 0\boldsymbol{k}.$$

Hence , the coordinates of point M are (5, 8, 0).

Finish.

point of division in definite proportion (定比分点)

4. Vectors in Plane and Space

Exercise Suppose that $A(x, y$ and $B(x_2, y_2, z_2)$ are two given points.

Find a point $C(x, y, z)$ on the line segment AB such that

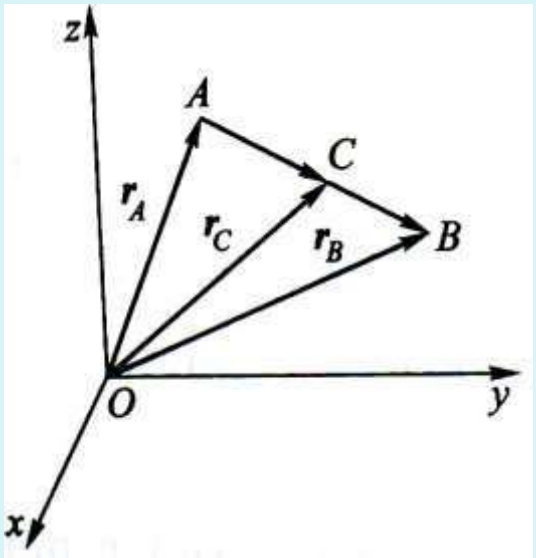
$$\overrightarrow{AC} = \lambda \overrightarrow{CB} \quad (\lambda \neq -1).$$

Solution Let r_A, r_B, r_C be the radius vectors of the points A, B, C. Then

$$\overrightarrow{AC} = r - r,$$

and hence $r_c - r_A = \lambda(r_B - r_C)$. Therefore

$$r_C = \frac{1}{1+\lambda}(r_A + \lambda r_B).$$



Thus the coordinates of the point C are

$$(x, y, z) = \left(\frac{x_1 + \lambda x_2}{1+\lambda}, \frac{y_1 + \lambda y_2}{1+\lambda}, \frac{z_1 + \lambda z_2}{1+\lambda} \right).$$

Finish.

5.Length, Direction Angles and Projection

Let $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ be a vector; then $\mathbf{a} = \overrightarrow{OP}$

$$|\overrightarrow{OP}|^2 = |\overrightarrow{OP'}|^2$$

$$= |\overrightarrow{OA}|^2 + |\overrightarrow{O}|^2$$

$$= x^2 + \quad ^2$$

Hence,

$$|\mathbf{a}| = \sqrt{x^2 + y^2 + z^2},$$

and this is the **component representation of the length of vector a**.

For the vector $\overrightarrow{AB}$, where $A(x\ , y$ and $B(x\ , y$ the distance between two points A and B is

$$d(A,B) = |\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Direction cosines of a vector

Since the direction of a vector (that is, the included angles between the vector and the basic unit vectors **i**, **j**, **k**) is fixed when the vector is moved parallel to itself, when a vector **a** is moved to the corresponding radius vector ***OP*** vector, then the direction

α,β,γ of the radius vector ***OP***

are just the direction angles of the vector

a, and the cosines of the direction

angles $\cos\alpha,\cos\beta,\cos\gamma$ are called the

direction cosines of the vector a.

Direction cosines of a vector

Obviously, the direction of a vector **a** is determined completely by the direction cosines of the vector. Hence, to determine the direction of a vector **a**, we need only determine the direction cosines of **a**.

Component Representations of the Direction Cosines:

$$\cos\alpha = \frac{x}{|\mathbf{a}|} = \frac{x}{\sqrt{x^2+y^2+z^2}},$$

$$\cos\beta = \frac{y}{|\mathbf{a}|} = \frac{y}{\sqrt{x^2+y^2+z^2}},$$

$$\cos\gamma = \frac{z}{|\mathbf{a}|} = \frac{z}{\sqrt{x^2+y^2+z^2}},$$

Direction cosines of a vector

By the last formula, it is easy to see the following conclusions:

(1)The quadratic sum of the direction cosines of any nonzero vector is equal to 1, that is

$$\cos^2\alpha+\cos^2\beta$$

(2) The three components of the unit vector $\mathbf{a}$ are just its three direction cosines, that is

$$\mathbf{a}^{\circ}=\frac{\mathbf{a}}{|\mathbf{a}}=\left(\begin{array}{cc} x & y \end{array}\right.$$

$$=(\cos\alpha,\cos\beta,\cos\gamma).$$

Find the Length and Direction Cosines of a Vector

Example 3 Suppose $A=(1,-2,3)$ and $B=(3,2,1)$. Find the length and direction cosines of the vector AB.

Solution Since

$$AB=(3-1,2-(-2),1-3)=(2,4,-2),$$

so

$$|\overrightarrow{AB}|=\sqrt{2^2+4^2+(-2)^2}=2\sqrt{6}.$$

Therefore, the direction cosines of the vector A are

$$\cos\alpha=\frac{2}{2\sqrt{6}}=\frac{\sqrt{6}}{6},\cos\beta=\frac{4}{2\sqrt{6}}=\frac{\sqrt{6}}{3},\cos\gamma=\frac{-2}{2\sqrt{6}}=-\frac{\sqrt{6}}{6}.$$

Finish.

Direction number(方向数)

We know that the direction of a vector can be determined by its direction cosines.

$$\cos\alpha = \frac{x}{|\mathbf{a}|} = \frac{x}{\sqrt{x^2+y^2+z^2}},$$

$$\cos\beta = \frac{y}{|\mathbf{a}|} = \frac{y}{\sqrt{x^2+y^2+z^2}},$$

$$\cos\gamma = \frac{z}{|\mathbf{a}|} = \frac{z}{\sqrt{x^2+y^2+z^2}},$$

But sometimes we need only to consider the **azimuth(方位)** of the vector and are not interested in its **sense(指向)** (vector **a** and **-a** have the same azimuth but opposite sense).

Direction number

If we need only to consider the azimuth and are not interested in the sense

In this case, we need only know **three numbers proportional to the direction cosines** and it is not necessary to find the direction cosines of the vector.

Suppose that the three numbers ***A***, ***B***, ***C*** are proportional to the direction cosines of a vector **a**, namely

$$\frac{A}{\cos\alpha} = \frac{B}{\quad} = C$$

where ***k*** is a proportional constant.

Direction number

Then

$$A = k\cos\alpha, \quad B = k\cos\beta, \quad C = k\cos\gamma.$$

Thus

$$A^2 + B^2 + C^2 = k^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = k^2.$$

Thus, we have

$$\cos\alpha = \frac{A}{\pm\sqrt{A^2 + B^2 + C^2}}, \quad \cos\beta = \frac{B}{\pm\sqrt{A^2 + B^2 + C^2}},$$

$$\cos\gamma = \frac{C}{\pm\sqrt{A^2 + B^2 + C^2}}.$$

These express two parallel directions with opposite sense.

By the three numbers A, B, C, we may determine two sets of direction cosines.

Direction number



Three numbers which are proportional to the direction cosines of a vector **a** are called **direction numbers** of the vector **a** or that of the straight line on which the vector **a** lines.

Obviously, a set of direction cosines or the components of a vector can be regarded as a set of special direction numbers; two vectors are parallel iff they have a common set of direction numbers.

Application of the Direction of a Vector

Example 4 Suppose that a set of direction numbers of the vector $\mathbf{a}$ is $1, \sqrt{2}, -1,$ and the included angle between $\mathbf{a}$ and the Oz axis is an acute angle. Find the unit vector $\mathbf{a}^{\circ}$.

Solution We know

$$\cos\gamma = \frac{-1}{\pm\sqrt{1+2+1}} = \frac{-1}{\pm 2}.$$

Since, γ is an acute angle, $\cos\gamma > 0$, so that the denominator of the above fraction is negative. Thus

$$\cos\alpha = -\frac{1}{\sqrt{2}}, \cos\beta$$

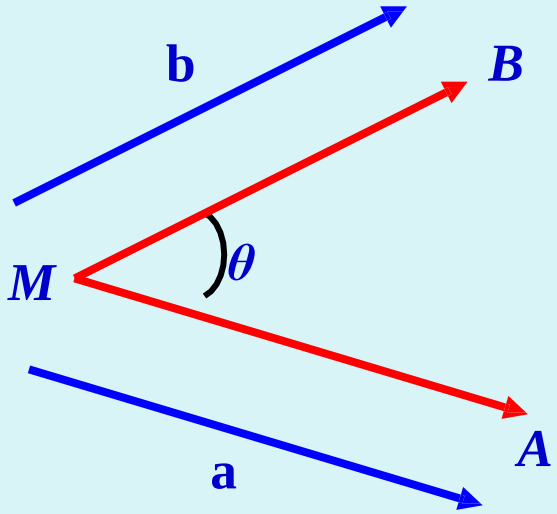
and

$$\mathbf{a}^{\circ} = \left(-\frac{1}{2}, -\frac{\sqrt{2}}{2}, \frac{1}{2}\right).$$

Finish.

Angle between two vectors

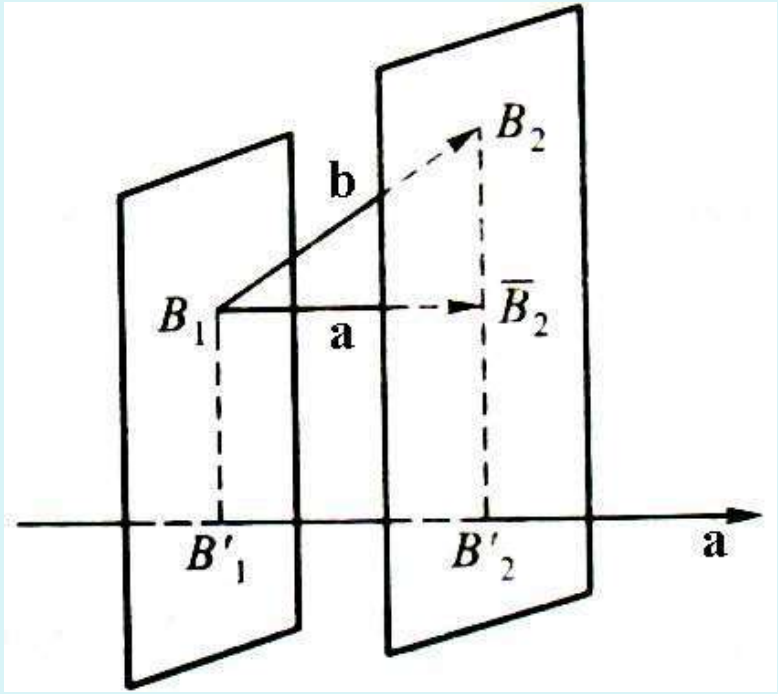
Suppose that **a** and **b** are any two nonzero vectors. Taking any point M in the space, we can draw vectors $\overrightarrow{MA} = \boldsymbol{a}$, $\overrightarrow{MB} = \boldsymbol{b}$. Then the angle $\angle AMB$ (not greater than π) is called the **included angle(夹角)** between the vectors **a** and **b**, denoted by $\langle \mathbf{a}, \mathbf{b} \rangle$.



If the included angle between **a** and **b** is $\frac{\pi}{2}$, then **a** and **b** are said to be perpendicular, denoted by $\mathbf{a} \perp \mathbf{b}$.

The Projection of Vectors（投影向量）

Suppose that **a** and **b** are any two nonzero vectors which are not perpendicular. Draw two planes through the initial point B_1 and the terminal point B_2 of the vector **b** respectively, such that both of them are perpendicular to the oriented straight line which the vector **a** lies on and has the same direction as **a**. Let B_1' and B be the intersection points of the planes and the straight line; then the vector $B_1'B_2'$ is called **the projection vector of b onto a,** denoted by $\mathbf{proj}_{\mathbf{a}}\mathbf{b}$. It is easy to see that $\mathbf{proj}_{\mathbf{a}}\mathbf{b} = |\mathbf{b}|\cos\theta\mathbf{a}^{\circ}$.



The Projection of Vectors

Definition **Orthogonal projection vector, orthogonal projection**

Suppose that the included angle between the vectors **a** and **b** is θ.

Then the vector

$$\mathbf{proj}_{\mathbf{a}}\mathbf{b} = |\mathbf{b}|\cos\theta\mathbf{a}^{\circ}$$

is called the orthogonal projection vector of **b** onto **a** (or onto the unit vector with the same direction as **a**), or simply the projection vector(投影向量).

The scalar

$$(\mathbf{b})_{\mathbf{a}} = |\mathbf{b}|\cos\theta$$

is called the orthogonal projection of **b** onto **a** (or onto the unit vector with the same direction as **a**), or simply the projection(投影).

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