

各章基本题详解

习题一

一、选择题

1. (A) $A \subset B \longrightarrow A \cup B = B$;

(B) $\bar{B} \subset \bar{A} \longrightarrow A \subset B \longrightarrow A \cup B = B$;

(C) $A\bar{B} = \phi \longrightarrow A \subset B \longrightarrow A \cup B = B$;

(D) $\bar{A}B = \phi \longrightarrow B \subset A$ 不一定能推出 $A \cup B = B$ (除非 $A = B$)

所以 选 (D)

2. $P(A - B) = P(A\bar{B}) = P(A\bar{B}) - P(A) - P(\bar{B}) + P(A) + P(\bar{B})$

$$= P(A) + P(\bar{B}) - P(A \cup \bar{B})$$

所以 选 (C)

3. $A \subset B \longrightarrow P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)} \longrightarrow \frac{P(A)}{P(B)} \geq P(A)$

所以 选 (B)

4. $P(A) = P(AB) = P(A)P(B) \longrightarrow P(A) = 0$ 或 $P(B) = 1$

所以 选 (B)

5. (A) 若 $A = \bar{B}$, 则 $AB = \phi$, 且 $\bar{A}\bar{B} = \bar{A}A = \phi$, 即 $\bar{A}, \bar{B}$ 不相容

(B) 若 $A \supset B \neq \phi$, 且 $A \neq \Omega$, 则 $AB \neq \phi$, 且 $\bar{A}\bar{B} = \bar{A} \neq \phi$, 即 $\bar{A}, \bar{B}$ 相容

(C) 若 $A = \phi, B \neq \phi$, 则 $AB = \phi$, 且 $\bar{A}B = B \neq \phi$, 即 $\bar{A}, B$ 相容

(D) 若 $AB \neq \phi$, 不一定能推出 $\bar{A}B = \phi$

所以 选 (D)

6. (A) 若 $AB \neq \phi$, 不一定能推出 $P(AB) = P(A)P(B)$

(B) 若 $P(A) = 1$, 且 $A \supset B \neq \phi$, 则 $P(AB) = P(B) = P(A)P(B)$, 即 A,B 独立

(C) 若 $AB = \phi$, $0 < P(A) < 1, 0 < P(B) < 1$, 则 $P(AB) \neq P(A)P(B)$

(D) 若 $P(A) = 1$, 则 A 与任何事件都相互独立

所以 选 (B)

7. 射击 n 次才命中 k 次, 即前 $n-1$ 次射击恰好命中 $k-1$ 次, 且第 n 次射击时命中目标, 所以 选 (C)

二、填空题

$$\begin{aligned} 8. (A \cup \bar{C})(\bar{A} \cup \bar{C}) \cup \overline{A \cup C} \cup \overline{\bar{A} \cup C} &= (A \cap \bar{A}) \cup \bar{C} \cup \overline{A \cup C} \cup \overline{\bar{A} \cup C} \\ &= \bar{C} \cup (\bar{A} \cap \bar{C}) \cup (A \cap \bar{C}) = \bar{C} \cup ((\bar{A} \cup A) \cap \bar{C}) = \bar{C} \cup \bar{C} = \bar{C} \end{aligned}$$

所以 $C = \bar{B}$

9. 共有 4×4 种基本事件, 向后两个邮筒投信有 2×2 种基本事件, 故所求概率为 $\frac{2 \times 2}{4 \times 4} = \frac{1}{4}$

10. 设事件 A 表示两数之和大于 $\frac{1}{2}$, 则

样本空间 $\Omega = \{(x, y) \mid 0 < x < 1, 0 < y < 1\}$,

$$A = \{(x, y) \mid x + y > \frac{1}{2}, 0 < x < 1, 0 < y < 1\}$$

$$P = \frac{S_A}{S_\Omega} = 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{7}{8}$$

11. 由 $P(A) = 0.8, P(A - B) = 0.1$, 得 $P(AB) = 0.7$, 故 $P(\overline{AB}) = 0.3$

12. 由 $P(A) = 0.2, P(B) = 0.3, P(A \cup B) = 0.4$, 得 $P(AB) = 0.1$,

故 $P(\overline{BA}) = P(B) - P(AB) = 0.2$

13. $P(AB) = P(A)P(B|A) = 0.2$, 故 $P(B) = \frac{P(AB)}{P(A|B)} = 0.8$

$$\begin{aligned} 14. P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC) \\ &= P(A) + P(B) + P(C) - P(A)P(B) - P(B)P(C) - P(C)P(A) + P(A)P(B)P(C) \\ &= \frac{19}{27} \end{aligned}$$

15. 由于 A,B 相互独立, 可得 $P(\overline{A}\overline{B}) = P(\overline{A})P(\overline{B}) = \frac{1}{9}$, $P(A\overline{B}) = P(\overline{A}B)$, 于是

$$P(\overline{A}) = P(\overline{B}) = \frac{1}{3}, \text{ 故 } P(B) = \frac{2}{3}$$

三、计算题

16. (1) $\Omega = \{(H, H, H), (H, H, T), (H, T, H), (H, T, T),$
 $(T, H, H), (T, H, T), (T, T, H), (T, T, T)\}$;

(2) $\Omega = \{0, 1, 2, 3\}$;

(3) $\Omega = \{(x, y) | x^2 + y^2 \leq 1\}$;

(4) $\Omega = \{5:0, 5:1, 5:2, 5:3, 5:4, 4:5, 3:5, 2:5, 1:5, 0:5\}$

17. (1) $A \cup B \cup C$;

(2) $\overline{A}(B \cup C)$;

(3) $\overline{A}\overline{B}\overline{C} \cup \overline{A}B\overline{C} \cup \overline{A}B\overline{C}$;

(4) $AB \cup BC \cup AC$;

(5) $\overline{A}\overline{B}\overline{C}$;

(6) $\overline{A} \cup \overline{B} \cup \overline{C}$;

(7) $\overline{ABC}$

18. 法一, 由古典概率可知, 所求概率为: $\frac{C_{20}^4 \cdot 9^{16}}{10^{20}}$;

法二, 由伯努利定理可知, 所求概率为: $C_{20}^4 \cdot 0.1^4 \cdot 0.9^{16}$

19. 只有唯一的一个六位数号码能打开锁。因此由古典概率可知, 试开一次就能打开锁的

概率是 $\frac{1}{10^6}$; 如果要求这 6 个数字全不相同, 试开一次就能打开锁的概率是 $\frac{1}{A_{10}^6}$

20. 一枚均匀硬币抛掷三次, 共有 $2^3 = 8$ 种不同情况

(1)至少连续两次出现正面有: (正, 正, 正), (正, 正, 反), (反, 正, 正)3 种情况, 故其概率为 $\frac{3}{8}$

(2)恰好出现两次正面有: (正, 正, 反), (正, 反, 正), (反, 正, 正)3 种情况, 故其概率为 $\frac{3}{8}$

(3) 正面与反面都出现的概率为 $1 - P(\text{仅出现反面或仅出现正面}) = \frac{3}{4}$

$$21. (1) \frac{1}{7^6}; \quad (2) \left(\frac{6}{7}\right)^6; \quad (3) 1 - \frac{1}{7^6}$$

22. 设事件 A 表示两数之积小于 $\frac{2}{9}$, 则

$$\text{样本空间 } \Omega = \{(x, y) \mid 0 < x < 1, 0 < y < 1\},$$

$$A = \{(x, y) \mid xy < \frac{2}{9}, 0 < x < 1, 0 < y < 1\}$$

$$P = \frac{S_A}{S_\Omega} = 1 - \int_{\frac{2}{9}}^1 dx \int_{\frac{2}{9x}}^1 dy = \frac{2}{9} + \frac{2}{9} \ln \frac{9}{2}$$

23. 设事件 A 表示方程有实根, 则

$$\text{样本空间 } \Omega = \{(u, v) \mid 0 < u < 1, 0 < v < 1\},$$

$$A = \{(u, v) \mid u \leq v^2, 0 < u < 1, 0 < v < 1\}$$

$$P = \frac{S_A}{S_\Omega} = \int_0^1 dv \int_0^{v^2} du = \frac{1}{3}$$

$$24. P(A - B) = P(A) - P(AB) = P(A) - 1 + P(\overline{AB}) = P(A) - 1 + P(\overline{A} \cup \overline{B}) = 0.7$$

$$25. (1) P(\overline{A} \cup \overline{B}) = P(\overline{AB}) = 1 - P(AB) = 1 - c$$

$$(2) P(\overline{AB}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - a - b + c$$

$$(3) P(\overline{A}B) = P(B - AB) = P(B) - P(AB) = b - c$$

$$(4) P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A}B) = 1 - a + b - b + c = 1 - a + c$$

$$26. 1 - \frac{P(\overline{B})}{P(A)} = \frac{P(A) + P(B) - 1}{P(A)} \leq \frac{P(A) + P(B) - P(A \cup B)}{P(A)} = \frac{P(AB)}{P(A)} = P(A|B)$$

27. 设 A_i : 第 i 次调试能调试好, $i=1,2,3$, B : 调 3 次能调试好

$$\text{则 } P(A_1) = \frac{1}{3}, P(A_2 | \overline{A}_1) = \frac{3}{8}, P(A_3 | \overline{A}_1 \overline{A}_2) = \frac{9}{10}, B = A_1 \cup \overline{A}_1 A_2 \cup \overline{A}_1 \overline{A}_2 A_3$$

$$\begin{aligned}
P(B) &= P(A_1) + P(\bar{A}_1 A_2) + P(\bar{A}_1 \bar{A}_2 A_3) \\
&= P(A_1) + P(\bar{A}_1)P(A_2 | \bar{A}_1) + P(\bar{A}_1)P(\bar{A}_2 | \bar{A}_1)P(A_3 | \bar{A}_1 \bar{A}_2) \\
&= \frac{1}{3} + \frac{2}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{5}{8} \times \frac{9}{10} = \frac{23}{24} = 0.958
\end{aligned}$$

28. 设 A_i : 10 个球中有 i 个红球, $i=0,1,\dots,10$, B : 任取一球是红球

$$\text{则 } P(A_i) = \frac{1}{11}, P(B | A_i) = \frac{i+1}{11}, i = 0, 1, \dots, 10,$$

$$P(B) = \sum_{i=0}^{10} P(A_i)P(B | A_i) = \frac{6}{11}$$

29. 设 A : 邻居记得浇水, B : 树已死去

$$\text{则 } P(A) = 0.9, P(B | A) = 0.15, P(\bar{A}) = 0.1, P(B | \bar{A}) = 0.8$$

$$P(\bar{B}) = 1 - P(B) = 1 - [P(A)P(B | A) + P(\bar{A})P(B | \bar{A})] = 1 - 0.215 = 0.785$$

$$P(\bar{A} | B) = \frac{P(\bar{A}B)}{P(B)} = \frac{P(\bar{A})P(B | \bar{A})}{1 - P(\bar{B})} = 0.372$$

30. 设 A : 某人患肺癌, B : 某人吸烟

$$\text{则 } P(A) = 0.001, P(B | A) = 0.9, P(\bar{A}) = 0.999, P(B | \bar{A}) = 0.2$$

$$P(A | B) = \frac{P(AB)}{P(B)} = \frac{P(A)P(B | A)}{P(A)P(B | A) + P(\bar{A})P(B | \bar{A})} = \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.2} = 0.00448$$

$$P(A | \bar{B}) = \frac{P(A\bar{B})}{P(\bar{B})} = \frac{P(A)P(\bar{B} | A)}{1 - P(B)} = \frac{0.001 \times 0.1}{1 - 0.001 \times 0.9 - 0.999 \times 0.2} = 0.000125$$

31. 设每次击中目标的概率为 p , 则由 $1 - (1 - p)^3 = \frac{7}{8}$, 可得 $p = \frac{1}{2}$

32. 设应选出 n 件产品, 则由 $1 - (1 - 0.01)^n \geq 0.95$, 可得 $n \geq 299$

33. (1) p ; (2) $(1 - p)^{k-1} p$;

(3) 即前 $k - 1$ 次射击恰好命中 $r - 1$ 次, 且第 k 次射击时命中目标, 故所求概率为

$$C_{k-1}^{r-1} p^{r-1} (1 - p)^{k-r};$$

(4) 即共射击 $k + r$ 次, 前 $k + r - 1$ 次射击恰好命中 $k - 1$, 且最后一次射击时命中目标,

故所求概率为 $C_{k+r-1}^r p^k (1 - p)^r$

习题二

一. 选择题

1. 因为 $P(X \geq 1) = 1 - P(X = 0) = 1 - (1 - p)^3 = 19/27$, 故 $p = 1/3$, 选择 (A)
2. 由正态分布密度关于 $x = \mu$ 对称知, $P(X < 0) = [1 - P(0 < X < 4)]/2 = 0.35$, 选择 (C)
3. 因为密度为偶函数, 所以 $F(-a) = 1/2 - \int_{-a}^0 f(x)dx = 1/2 - \int_0^a f(x)dx$, 选择 (B)
4. 因为 $P(|X - \mu| < \sigma) = P(|\frac{X - \mu}{\sigma}| < 1) = 2\Phi(1) - 1$, 所以选择 (D)
5. 由严格单调条件下的定理 2.4.1 可知 $f_Y(y) = f_X(\frac{1-y}{3})|-\frac{1}{3}|$, 选择 (A)

二. 填空题

6. 代入泊松分布律可解得 λ , 填: 2
7. $P(a/2 < X < b) = F(b-0) - F(a/2) = 0.4$, 填: 0.4
8. $P(X > 2) = \frac{a-2}{a+2} = 0.6$ 或者 $P(X \leq 2) = \frac{4}{a+2} = 0.4$, 填: 8
9. 由 $\int_0^1 (kx+2)dx = 1$ 得出 $k = -2$ 的值, 然后计算 $P(X < 0.5) = \int_0^{0.5} (2-2x)dx = 3/4$,
填: 3/4
10. $P(120 < X \leq 200) = P(|\frac{X-160}{\sigma}| < \frac{40}{\sigma}) = 2\Phi(\frac{40}{\sigma}) = 0.8$, 查表, 填: 31.25
11. $P(Y=1) = P(X=1) + P(X=2)$, 填: 0.72

三. 计算题

12. 1 颗骰子掷 2 次总共有 36 种可能, 将这些可能排除二维表格形式,

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

易得分布律

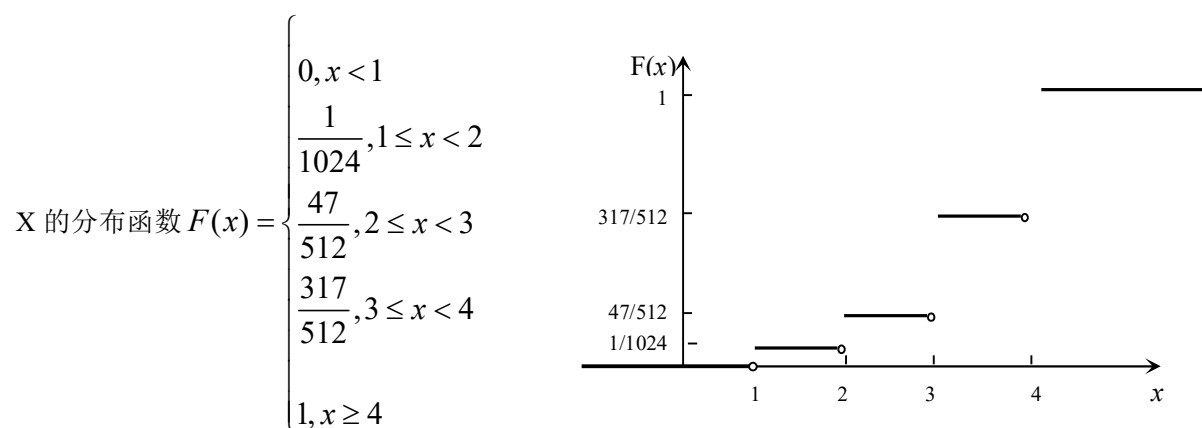
X	2	3	4	5	6	7	8	9	10	11	12
P	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$13. \quad P(X=1) = \frac{C_4^1}{4^6} = \frac{1}{4^5} = \frac{1}{1024}, \quad P(X=2) = \frac{C_4^2(2^6-2)}{4^6} = \frac{93}{4^5} = \frac{93}{1024}$$

$$P(X=3) = \frac{C_4^3(3^6 - C_3^2(2^6-2) - 3)}{4^6} = \frac{540}{4^5} = \frac{540}{1024}, \quad P(X=4) = 1 - \sum_{k=1}^3 P(X=k) = \frac{390}{1024}$$

所以 X 的分布列为

X	1	2	3	4
P	$\frac{1}{1024}$	$\frac{93}{1024}$	$\frac{540}{1024}$	$\frac{390}{1024}$



$$14. \quad \text{一次打开} \quad P(X=1) = \frac{1}{n}, \quad P(X=2) = \frac{n-1}{n} \frac{1}{n-1} = \frac{1}{n}, \quad \text{以此类推得}$$

$$P(X=k) = \frac{1}{n}, \quad k=1, 2, \dots, n$$

15. 因为 $F(\pi/2) < F(0)$ 不满足单调不减性, 或者 $F(-k\pi) = 1, k \in \mathbb{Z}^+, \rightarrow +\infty$, 所以不是某个变量的分布函数

$$16. \quad (1) \quad \sum_{k=1}^N \frac{k}{C_1} = \frac{N(N+1)}{2C_1} = 1 \quad \text{得} \quad C_1 = \frac{N(N+1)}{2}$$

$$(2) \quad C_2 \left(\frac{2}{3}\right)^1 + C_2 \left(\frac{2}{3}\right)^2 + C_2 \left(\frac{2}{3}\right)^3 = C_2 \frac{38}{27} = 1, \quad \text{得} \quad C_2 = \frac{27}{38}$$

$$(3) \quad \sum_{k=1}^{+\infty} C_3 \frac{\lambda^k}{k!} = C_3 \left(\sum_{k=0}^{+\infty} \frac{\lambda^k}{k!} - 1 \right) = C_3 (e^\lambda - 1) = 1, \quad \text{得} \quad C_3 = \frac{1}{e^\lambda - 1}$$

17. 易知 $F(x)$ 的间断点为 $-1, 1, 3$, 再由离散分布函数的性质可得

$$P(X=-1) = F(-1) - F(-1-0) = 0.4, \quad P(X=1) = F(1) - F(1-0) = 0.8 - 0.4 = 0.4$$

$$P(X=3) = F(3) - F(3-0) = 1 - 0.8 = 0.2$$

X	-1	1	3
P	0.4	0.4	0.2

18. 已知 $X \sim P(4)$, 则 $(1) P(X=6) = e^{-4} \frac{4^6}{6!} = F(6) - F(5) = 0.1042$ 查表得

(2) $P(X > 8) = 1 - P(X \leq 8) = 1 - F(8) = 0.0214$ 查表得

19. 设患者中病的人数为 X , 则由题意得 $X \sim B(5000, 0.001)$, 由于 n 较大, p 较小,

所以取 $\lambda = np = 5$

所求概率为 $P(X \leq 6) \approx \sum_{k=0}^6 e^{-5} \frac{5^k}{k!} = 0.7622$ (泊松近似, 查表得)

20. 设发生故障的设备数为 X , 维修人员数为 m , 则由题意可构建概率方程:

$$P(X > m) < 0.01$$

因为 $X \sim B(300, 0.01)$, $P(X > m) = 1 - P(X \leq m) \approx 1 - \sum_{k=0}^m e^{-3} \frac{3^k}{k!} < 0.01$

查表可得 $m \geq 8$, 即至少配备 8 个维修人员才能满足题目要求。

$$21. (1) \int_{-1}^1 \frac{C}{\sqrt{1-x^2}} dx = C \arcsin(x) \Big|_{-1}^1 = \pi C = 1 \Rightarrow C = \frac{1}{\pi}$$

$$(2) F(x) = \begin{cases} 0, & x \leq -1 \\ \int_{-1}^x \frac{1}{\pi \sqrt{1-x^2}} dx = \frac{1}{2} + \frac{1}{\pi} \arcsin(x), & -1 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

$$22. F(x) = \begin{cases} 0, & x < 0 \\ \int_0^x x dx = \frac{x^2}{2}, & 0 \leq x < 1 \\ \int_0^1 x dx + \int_1^x (2-x) dx = 2x - \frac{x^2}{2} - 1, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

$$23. (1) \int_0^2 (Cx^2 + x) dx = \frac{Cx^3}{3} + \frac{x^2}{2} \Big|_0^2 = \frac{8C}{3} + 2 = 1 \Rightarrow C = -\frac{3}{8}$$

$$(2) P(X \geq 1) = 1 - \int_0^1 (x - \frac{3}{8}x^2) dx = \frac{5}{8}$$

$$24. (1) F(+\infty) = \frac{A}{1} = 1 \Rightarrow A = 1$$

$$(2) f(x) = F'(x) = \frac{e^{-x}}{(1+e^{-x})^2}, -\infty < x < +\infty$$

$$(3) P(X \leq 0) = F(0) = 1/2$$

25. 已知 $X \sim E(0.2)$, 指数分布具有无记忆特性, 所以

$$(1) P(X > 15 | X > 5) = P(X > 10) = e^{-0.2 \times 10} = e^{-2}$$

$$(2) \text{ 因为 } P(X > 10) = e^{-0.2 \times 10} = e^{-2}$$

假设 Y 表示三次等待不到服务而离开窗口的次数, 由题意得 $Y \sim B(3, e^{-2})$

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1 - e^{-2})^3$$

26. 已知 $X \sim E(1/1000)$, 3 个元件的使用相互独立, 故所求概率为

$$[P(X > 1000)]^3 = [e^{-1000 \times 1/1000}]^3 = e^{-3}$$

27. 已知 $X \sim E(1/2)$, 则 (1) $P(X > 2) = e^{-2 \times 1/2} = e^{-1}$

(2) 由指数分布的无记忆特性得, $P(X > 10 | X > 9) = P(X > 1) = e^{-1/2}$

$$28. (1) P(2 < X \leq 5) = \Phi\left(\frac{5-3}{2}\right) - \Phi\left(\frac{2-3}{2}\right) = \Phi(1) - \Phi\left(-\frac{1}{2}\right) = \Phi(1) + 1 - \Phi\left(\frac{1}{2}\right) = 0.5328$$

$$(2) P(-4 < X \leq 10) = \Phi\left(\frac{10-3}{2}\right) - \Phi\left(\frac{-4-3}{2}\right) = 2\Phi(3.5) - 1 = 0.9996$$

$$(3) P\{|X| > 2\} = 1 - P(-2 \leq X \leq 2)$$

$$= 1 - \Phi\left(\frac{2-3}{2}\right) - \Phi\left(\frac{-2-3}{2}\right) = \Phi\left(\frac{1}{2}\right) + 1 - \Phi(2.5) = 0.6977$$

$$(4) P\{|X| < 3\} = \Phi(0) - \Phi(-3) = \Phi(3) - 0.5 = 0.4987$$

(5) 有正态分布的特性, 知 C 等于 μ 的时候正好满足题目要求, 所以 $C = 3$

29. 由题意得 次品率为 $1 - P(|X - 10.05| \leq 0.12) = 2 - 2\Phi(2) = 0.0456$

30. 由 $P(X > 96) = 1 - \Phi\left(\frac{96-72}{\sigma}\right) = 0.023$ 得 $\sigma = 12$

$$\text{则 } P(60 \leq X \leq 84) = 2\Phi\left(\frac{12}{12}\right) - 1 = 0.6826$$

31. (1) 因为 $y = e^x$ 为严格单调函数, 且 $y > 0$, 所以

$$f_Y(y) = \begin{cases} f_X(\ln y) \frac{1}{y} = \frac{1}{\sqrt{2\pi}} e^{-\frac{\ln^2 y}{2}}, & y > 0 \\ 0, & \text{其他} \end{cases}$$

$$(2) F_Y(y) = P(|X| \leq y) = \begin{cases} P(-y \leq X \leq y) = F_X(y) - F_X(-y), & y > 0 \\ 0, & \text{其他} \end{cases}$$

$$f_Y(y) = \begin{cases} f_X(y) + f_X(-y) = \frac{2}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}, & y > 0 \\ 0, & \text{其他} \end{cases}$$

$$32. (1) y = 1 - 2x \text{ 严格单调, 所以 } f_Y(y) = \begin{cases} f_X(\frac{1-y}{2}) \frac{1}{2} = \frac{1}{8}, & -5 \leq y \leq 2 \\ 0, & \text{其他} \end{cases}$$

$$(2) F_Y(y) = P(X^2 \leq y) = \begin{cases} P(-\sqrt{y} \leq X \leq \sqrt{y}) = F_X(\sqrt{y}) - F_X(-\sqrt{y}), & y > 0 \\ 0, & \text{其他} \end{cases}$$

$$f_Y(y) = \begin{cases} f_X(\sqrt{y}) \frac{1}{2\sqrt{y}} + f_X(-\sqrt{y}) \frac{1}{2\sqrt{y}} = \frac{1}{4\sqrt{y}}, & 0 < y \leq 1 \\ f_X(\sqrt{y}) \frac{1}{2\sqrt{y}} = \frac{1}{8\sqrt{y}}, & 1 < y \leq 9 \\ 0, & \text{其他} \end{cases}$$

$$33. \text{ 圆片面积 } Y = \pi \left(\frac{X}{2}\right)^2 = \frac{\pi X^2}{4}, \text{ 函数 } y = \frac{\pi x^2}{4} \text{ 在区间 } [5, 6] \text{ 上严格单调, 所以}$$

$$f_Y(y) = \begin{cases} f_X\left(\frac{2\sqrt{y}}{\sqrt{\pi}}\right) \frac{2}{2\sqrt{\pi y}} = \frac{1}{\sqrt{\pi y}}, & y \in [25\pi/4, 9\pi] \\ 0, & \text{其他} \end{cases}$$

习题三

一、 选择题

$$1. P(X=Y) = P(X=-1, Y=-1) + P(X=1, Y=1) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}, \text{ 故选 (A)}$$

$$2. \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = \int_0^2 \int_0^2 c x dx dy = 4c = 1, \text{ 则 } c=1/4 \text{ 故选 (A)}$$

3. 选 (D)

$$4. \text{ 由题意可得 } X+Y \sim N(1, 2), \text{ 所以 } P(X+Y \leq 1) = \frac{1}{2} \text{ 故选 (B)}$$

5. (X, Y) 服从区域 $G = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}$ 上的均匀分布,

$$P(X+Y>1)=\int_{0.5}^1 dx \int_{-x+1.5}^1 dy=0.125, \quad \text{故选 (A)}$$

二、填空题

$$6. \quad P(X=1)=\frac{1}{6}+\frac{1}{9}+\frac{1}{18}=\frac{1}{3}, \text{ 所以 } P(X=2)=\frac{1}{3}+\alpha+\beta=\frac{2}{3},$$

$$\text{且由 } X \text{ 与 } Y \text{ 独立可得 } P(X=1, Y=2)=P(X=1)P(Y=2), \text{ 因此 } P(Y=2)=\frac{1}{3}$$

$$\text{所以 } \alpha=P(X=2, Y=2)=P(X=2) \cdot P(Y=2)=\frac{2}{9}, \quad \beta=\frac{2}{3}-\frac{1}{3}-\frac{2}{9}=\frac{1}{9}$$

$$7. \quad P(\max(X, Y) \geq 0) = P(\{X \geq 0\} \cup \{Y \geq 0\}) = P(X \geq 0) + P(Y \geq 0) - P(X \geq 0, Y \geq 0)$$

$$= \frac{4}{7} + \frac{4}{7} - \frac{3}{7} = \frac{5}{7}$$

$$8. \quad F_X(x) = F(x, +\infty) = \begin{cases} 1 - e^{-3x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$\text{故当 } x > 0 \text{ 时, } (X, Y) \text{ 关于 } X \text{ 的边缘概率密度 } f_X(x) = F'_X(x) = 3e^{-3x}$$

$$9. \quad f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_{-1}^{+1} \frac{1}{4} dy = \frac{1}{2}, & |x| < 1 \\ 0, & \text{其他} \end{cases},$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_{-1}^{+1} \frac{1}{4} dx = \frac{1}{2}, & |y| < 1 \\ 0, & \text{其他} \end{cases}$$

$$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)} = \begin{cases} \frac{1}{2}, & |x| < 1 \\ 0, & \text{其他} \end{cases}$$

$$10. \quad \text{由正态分布的性质可得 } X-Y \sim N(0, 2), \text{ 所以 } Z = X - Y \text{ 的概率分布 } f(z) = \frac{e^{-\frac{z^2}{4}}}{2\sqrt{\pi}}$$

三、计算题

11.

$X \backslash Y$	1	2	3
1	0	$\frac{1}{6}$	$\frac{1}{12}$
2	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
3	$\frac{1}{12}$	$\frac{1}{6}$	0

12.

$$P(X=i, Y=j) = C_5^i C_{5-i}^j 0.5^i 0.3^j 0.2^{5-i-j} = \frac{5!}{i!j!(5-i-j)!} 0.5^i 0.3^j 0.2^{5-i-j}$$

$$(i, j = 0, 1, \dots, 5; i+j \leq 5)$$

13.

$X \backslash Y$	1	2	3	$p_{i \cdot}$
1	$\frac{1}{3}$	0	0	$\frac{1}{3}$
2	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{3}$
3	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$
$p_{\cdot j}$	$\frac{11}{18}$	$\frac{5}{18}$	$\frac{1}{9}$	

14. (1) $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = \int_0^{+\infty} dx \int_0^1 A y e^{-x} dy = \frac{A}{2} = 1$, 故得 $A=2$.

(2) $P(X < 1) = \int_0^1 dx \int_0^1 2 y e^{-x} dy = 1 - e^{-1}$

$$(3) F(x, y) = \int_{-\infty}^x ds \int_{-\infty}^y f(s, t) dt = \begin{cases} \int_0^x ds \int_0^y 2 t e^{-s} dt = y^2(1 - e^{-x}), & x > 0, 0 < y < 1 \\ \int_0^x ds \int_0^1 2 t e^{-s} dt = 1 - e^{-x}, & x > 0, y \geq 1 \\ 0, & \text{其它} \end{cases}$$

15. (1) 区域 G 的面积 $= \int_0^1 x - x^2 dx = \frac{1}{6}$, 故 $f(x, y) = \begin{cases} 6, & (x, y) \in G \\ 0, & \text{其它} \end{cases}$

(2) $f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_{x^2}^x 6 dy = 6(x - x^2), & 0 \leq x \leq 1 \\ 0, & \text{其它} \end{cases}$,

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_y^{\sqrt{y}} 6 dx = 6(\sqrt{y} - y), & 0 \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$$

16. (1) $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = \int_0^1 dx \int_0^1 C x y^3 dy = \frac{C}{8} = 1$, 故得 $C=8$.

(2) $P(Y > X) = \int_0^1 dx \int_0^x 8 x y^3 dy = \frac{2}{3}$

$$(3) \quad f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_0^1 8xy^3 dy = 2x, & 0 \leq x \leq 1 \\ 0, & \text{其它} \end{cases},$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_0^1 8xy^3 dx = 4y^3, & 0 \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$$

因为 $f(x, y) = f_X(x)f_Y(y)$, 故 X 与 Y 相互独立。

$$17. (1) \quad \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = \int_0^1 dx \int_0^x Cy(1-x) dy = \frac{C}{24} = 1, \text{故得 } C=24.$$

$$(2) \quad f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_0^x 24y(1-x) dy = 12x^2(1-x), & 0 \leq x \leq 1 \\ 0, & \text{其它} \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_y^1 24y(1-x) dx = 12y(1-y)^2, & 0 \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$$

故 X 与 Y 不独立。

$$18. (1) \quad \text{区域 } G \text{ 的面积} = 2\sqrt{2}, \quad \text{故 } f(x, y) = \begin{cases} \frac{\sqrt{2}}{4}, & (x, y) \in G \\ 0, & \text{其它} \end{cases}$$

$$(2) \quad P(X+Y < 1) = \frac{\sqrt{2} + \frac{1}{2}}{2\sqrt{2}} = \frac{1}{2} + \frac{\sqrt{2}}{8}$$

$$(3) \quad f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \begin{cases} \int_{-\sqrt{2}}^{\sqrt{2}} \frac{\sqrt{2}}{4} dy = 1, & 0 \leq x \leq 1 \\ 0, & \text{其它} \end{cases},$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \begin{cases} \int_0^1 \frac{\sqrt{2}}{4} dx = \frac{\sqrt{2}}{4}, & 0 \leq y \leq 1 \\ 0, & \text{其它} \end{cases}$$

因为 $f(x, y) = f_X(x)f_Y(y)$, 故 X 与 Y 相互独立。

$$19. \quad P(X = m) = \sum_{n=m+1}^{+\infty} p^2 q^{n-2} = pq^{m-1}, m = 1, 2, \dots$$

$$P(Y = n) = \sum_{m=1}^{n-1} p^2 q^{n-2} = (n-1)p^2 q^{n-2}, n = 2, 3, \dots$$

20. (1) 因为 X 与 Y 相互独立, 则(X,Y)的联合分布律为

$X \backslash Y$	-1	1	2	$p_{i\cdot}$
-1	0.06	0.09	0.15	0.3
0	0.08	0.12	0.2	0.4
1	0.06	0.09	0.15	0.3
$p_{\cdot j}$	0.2	0.3	0.5	1

(2)

Z_1	-2	-1	0	1	2	3
P_z	0.06	0.08	0.15	0.27	0.29	0.15

Z_2	-2	-1	0	1	2
P_z	0.15	0.15	0.4	0.15	0.15

21.(1)由题意

$$P(U=0, V=0) = P(\max\{X, Y\}=0, \min\{X, Y\}=0) = P(X=0, Y=0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(U=1, V=1) = P(\max\{X, Y\}=1, \min\{X, Y\}=1) = P(X=1, Y=1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$P(U=0, V=1) = P(\max\{X, Y\}=0, \min\{X, Y\}=1) = 0$$

故 $P(U=1, V=0) = \frac{1}{2}$, 则 (U,V) 的联合分布律为

$U \backslash V$	0	1	$p_{i\cdot}$
0	$\frac{1}{4}$	0	$\frac{1}{4}$
1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
$p_{\cdot j}$	$\frac{3}{4}$	$\frac{1}{4}$	1

(2) 由于 $P(U=0, V=1) = 0 \neq P(U=0) \cdot P(V=1)$, 所以 U 与 V 不独立。

22. (1) $Z=X+Y$, 由连续型卷积公式可得

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) f_Y(z-x) dx = \begin{cases} \int_0^z 1 dx = z, & 0 < z \leq 1 \\ \int_{z-1}^1 1 dx = 2-z, & 1 < z \leq 2 \\ 0, & \text{其它} \end{cases}$$

(2) $Z = \min\{X, Y\}$ 的分布函数为

$$\begin{aligned}
F_Z(z) &= P(Z \leq z) = 1 - P(Z > z) = 1 - P(\min\{X, Y\} > z) \\
&= 1 - P(X > z, Y > z) = 1 - P(X > z)P(Y > z) \\
&= 1 - (1 - F_X(z))(1 - F_Y(z)) \\
&= \begin{cases} 0, & z \leq 0 \\ 1 - (1 - z)^2, & 0 < z < 1 \\ 1, & z \geq 1 \end{cases}
\end{aligned}$$

所以 $Z = \min\{X, Y\}$ 的概率密度函数为 $f_Z(z) = \begin{cases} 2(1-z), & 0 < z < 1 \\ 0, & \text{其他} \end{cases}$

23. (X, Y) 的联合密度函数为 $f(x, y) = \begin{cases} \frac{1}{\pi R^2}, & x^2 + y^2 < R^2 \\ 0, & \text{其他} \end{cases}$

Z 的分布函数

$$F_Z(z) = P(Z \leq z) = P(\sqrt{X^2 + Y^2} \leq z) = \begin{cases} \iint_{\sqrt{x^2 + y^2} < z} \frac{1}{\pi R^2} dx dy = \frac{z^2}{R^2}, & 0 \leq z \leq R \\ 0, & z < 0 \\ 1, & z > R \end{cases}$$

所以 Z 的概率密度函数 $f_Z(z) = \begin{cases} 2z/R^2, & 0 \leq z \leq R \\ 0, & \text{其它} \end{cases}$

24. 当 $z < 0$ 时 $F(z) = P\{Z \leq z\} = P\{X - Y \leq z\} = 0$

当 $0 \leq z \leq 1$ 时

$$F(z) = P\{Z \leq z\} = P\{X - Y \leq z\} = \int_0^z dx \int_0^x 3x dy + \int_z^1 dx \int_{x-z}^x 3x dy = \frac{3}{2}z - \frac{1}{2}z^3$$

当 $z > 1$ 时 $F(z) = 1$

$$\text{故 } f(z) = \begin{cases} \frac{3}{2}z - \frac{1}{2}z^3, & 0 \leq z \leq 1 \\ 0, & \text{其它} \end{cases}$$

25.

$$\begin{aligned}
f_X(x) &= \int_{-\infty}^{+\infty} f(x, y) dy \\
&= \begin{cases} \int_0^{+\infty} x e^{-x(y+1)} dy = e^{-x}, & x > 0, \\ 0, & \text{其它} \end{cases}
\end{aligned}$$

$$\begin{aligned}
f_Y(y) &= \int_{-\infty}^{+\infty} f(x, y) dx \\
&= \begin{cases} \int_0^{+\infty} x e^{-x(y+1)} dx = \frac{1}{(1+y)^2}, & y > 0 \\ 0, & \text{其它} \end{cases}
\end{aligned}$$

$$\text{在 } Y=y (y>0) \text{ 条件下, } f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} \frac{xe^{-x(y+1)}}{(1+y)^2}, & x > 0 \\ 0, & \text{其他} \end{cases}$$

$$\text{在 } X=x (x>0) \text{ 条件下, } f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \begin{cases} xe^{-xy}, & y > 0 \\ 0, & \text{其他} \end{cases}$$

X 与 Y 不独立。

26. 已知随机变量 X 与 Y 独立同分布, 且 $P(X=k) = p(1-p)^{k-1}, k=1, 2, \dots$

$$\text{证明 } P(X=k | X+Y=n) = \frac{1}{n-1}, k=1, \dots, n-1.$$

证: 当 $n=2, 3, \dots$ 时,

$$\begin{aligned} P(X+Y=n) &= \sum_{k=1}^{n-1} P(X=k, Y=n-k) = \sum_{k=1}^{n-1} P(X=k)P(Y=n-k) \\ &= \sum_{k=1}^{n-1} p(1-p)^{k-1} \cdot p(1-p)^{n-k-1} = \sum_{k=1}^{n-1} p^2(1-p)^{n-1} = (n-1)p^2(1-p)^{n-1} \end{aligned}$$

$$\begin{aligned} \therefore P(X=k | X+Y=n) &= \frac{P(X=k, X+Y=n)}{P(X+Y=n)} = \frac{P(X=k, Y=n-k)}{P(X+Y=n)} \\ &= \frac{p(1-p)^{k-1} \cdot p(1-p)^{n-k-1}}{(n-1)p^2(1-p)^{n-1}} = \frac{1}{n-1}, k=1, \dots, n-1. \end{aligned}$$

习题四

一、选择题

1. 由 $E(XY) - E(X)E(Y) = 0$ 知 X, Y 不相关, 所以 $D(X+Y) = DX + DY$, 故选择 (B)
2. 独立难以判断, 所以考虑相关与否, 因为 $Cov(U, V) = DX - DY = 0$, 故选择 (D)
3. 由方差性质可选择 (D)
4. 因为 $E(X-C)^2 = D(X-C) + [E(X-C)]^2 = DX + [E(X-C)]^2$, $DX = E(X-\mu)^2$, 故选择 (D)
5. 试用钥匙次数设为 X , 则 $P(X=k) = \frac{1}{n}, k=1, 2, \dots, n$, 故选择 (C)

6. 因为 $EX = 1, DX = 2, EY = 2, DY = 4$ ，题目没有讲 X, Y 相互独立，故选择 (C)

二、填空题

7. 由期望和方差定义知，期望和方差分别，填：0.6，0.84

8. 由
$$\begin{cases} \sum_{k=0}^{+\infty} \frac{AB^k}{k!} = 1 \\ \sum_{k=0}^{+\infty} k \frac{AB^k}{k!} = \sum_{k=1}^{+\infty} \frac{AB^k}{(k-1)!} = B \sum_{k=0}^{+\infty} \frac{AB^k}{k!} = 2 \end{cases}$$
 两式相除，可得 $\begin{cases} B = 2 \\ A = e^{-2} \end{cases}$ ，填： e^{-2} ，2

或者直接套用泊松分布可容易解得 A, B

9. $X \sim P(1/2), EX = 1/2, E(2X+3) = 2 \cdot 1/2 + 3 = 4$ ，填：4

10. 由 $EX^2 = DX + (EX)^2 = 7$ ，则 $E(2X^2 - 6) = 3EX^2 - 6 = 15$ ，填：15

11. 由题可得 Y 的分布律为

Y	-1	0	1
P	$P(X < 0) = 1/3$	0	$P(X > 0) = 2/3$

所以 $DY = EY^2 - (EY)^2 = 8/9$ ，填：8/9

12. 由同分布及期望性质， $E(\bar{X}) = \frac{1}{n} \sum_{i=1}^n EX_i = \mu$ ，由独立同分布及方差性质，

$$D(\bar{X}) = \frac{1}{n^2} \sum_{i=1}^n DX_i = \frac{\sigma^2}{n} \quad , \quad \text{所以填：} \mu, \frac{\sigma^2}{n}$$

13. $P(5200 < X < 9400) = P(|X - 7300| < 2100) \geq 1 - \frac{700^2}{2100^2} = \frac{8}{9}$ ，填： $\frac{8}{9}$

14. 由方差性质可知， $D(X \pm Y) = DX + DY \pm 2\rho_{XY}\sqrt{DX}\sqrt{DY}$ ，所以，填：85，37

15. 因为 $Z = X - Y \sim N(0, 1)$ ，所以

$$E|Z| = \int_{-\infty}^{+\infty} |z| \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = 2 \int_0^{+\infty} z \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = \sqrt{2/\pi} \quad , \quad \text{填：} \sqrt{2/\pi}$$

三、计算题

16. 由期望定义知， $EX = -1 \times 0.2 + 1 \times 0.3 + 2 \times 0.2 = 0.5$ ，

$$EX^2 = 1 \times 0.2 + 1 \times 0.3 + 4 \times 0.2 = 1.3$$

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