Chapter 4

Linear Programming

1. Introduction

Many management decision issues

to seek the firm's objective subject to some c

can be solved by Mathematical Programming

Mathematical Programming is a set of techniques of finding a solution which optimizes the certain objective(s) under given constraints.

Linear Programming

- *one category of the techniques under the banner of Mathematical Programming.*

Common Properties of Mathematical Programming Problems

All MP problems have three properties in common:

Seek to maximize or minimize some

We refer to the mathematical expression of this quantity as the

Get some restrictions or

They limit the degree to which we can pursue our

Several alternative courses of action to choose from

If there were no alternatives to select from, we would not need MP.

If the fourth property

The fourth

All objective and constraints are expressed in terms of linear equations or

A mathematical programming problem becomes a linear programming

In this chapter, we are going to discuss the linear programming.

2. Formulating LP Problem

Example: The Flair Furniture Company

- The Flair Furniture Company produces inexpensive tables and chairs.
- The production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labor hours in the painting and varnishing department.
- Each table takes 4 hours of carpentry and 2 hours in the painting and varnishing shop.
- Each chair requires 3 hours in carpentry and 1 hour in painting and varnishing.
- During the current production period, 240 hours of carpentry time are available and 100 hours of painting/varnishing department time are available.
- Each table sold yields a profit of $7; each chair produced may be sold for a $5 profit.

Product / Resource	Table	Chair	limitation
Carpentry hours	4 each	3 each	240
Painting and vanishing hours	2 each	1 each	100
profit	$7	$5	

Let X_1 = number of tables to be

Let X_2 = number of chairs to be

Decisio n variabl es

The Objective

$$Max.\ profit = 7X_1 + 5X_2$$

1st constraint – Carpentry time

$$4X_1 + 3X_2 \leq 240$$

2st constraint – Painting / varnishing time

$$2X_1 + X_2 \leq 100$$

3rd & 4th constraints – Non-negativity

$$X_1 \geq 0$$

$$X_2 \geq 0$$

The complete problem may now be restated as

$$\begin{aligned} &Max\ 7X_1 + 5X_2 \\ &s.t. \\ &4X_1 + 3X_2 \le 240 \\ &2X_1 + X_2 \le 100 \\ &X_1 \ge 0 \\ &X_2 \ge 0 \end{aligned}$$

3. Basic Assumptions of LP

The conditions of certainty exist, that is, numbers in the objective function and constraints do not change during the period being studied.

$$Max \quad 7X_1 + 5X_2$$

$$s.t.$$

$$4X_1 + 3X_2 \leq 240$$

$$2X_1 + 1X_2 \leq 100$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

3. Basic Assumptions of LP

The conditions of certainty exist, that is, numbers in the objective function and constraints do not change during the period being studied.

The proportionality exists in the objective function

This means: If production of 1 unit of a product uses 3 hrs of a particular scarce resources, then making 10 units uses 30 hrs of the resources.

3. Basic Assumptions of LP

The conditions of certainty exist, that is, numbers in the objective function and constraints do not change during the period being studied.

The proportionality exists in the objective function

The additivity exist, meaning that the total of all activities equals the sum of each individual

If 1 unit of each product is actually produced, then the profit contribution of $8 and $3 must add up to produce a sum of $11.

3. Basic Assumptions of LP

The ... *in* ... *cha*...

Instead, they are divisible and may take any fractional value.

If a fraction of a product cannot be produced, an Interger Programming problem exists.

The p... ...*nction*

The additivity exis... ...*at the total of all activities equals the su*... *of each individual*

The divisibility exists, that means, the solutions needs not be in integers.

3. Basic Assumptions of LP

The conditions of certainty exist, that is, numbers in the objective function and constraints do not change during the period being studied.

The proportionality exists in the objective function

The additivity exist, meaning that the total of all activities equals the sum of each individual

The divisibility exists, that means, the solutions needs not be in integers.

All variables are non-negative.

4. Graphical Solution to an LP Problem

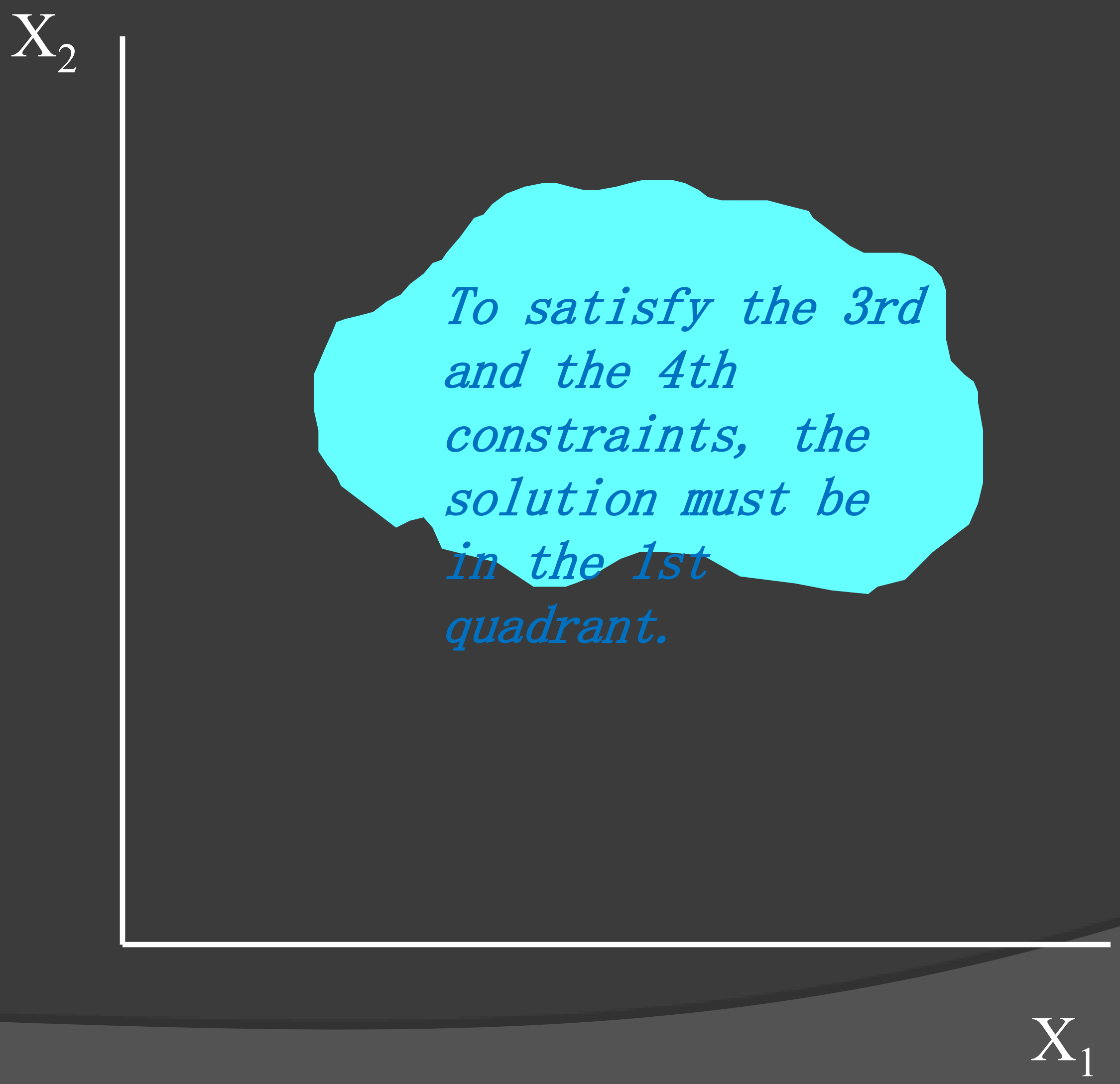
The easiest way to solve a small LP problem is the graphical solution approach.

The procedure of the graphical

- *graph each of the constraints*
- *find the feasible region that satisfies all of the constraints simultaneously*
- *use iso-profit line solution method or corner solution method*

graph each of the constraints

$$\begin{aligned}
&Max\ 7X_1 + 5X_2 \\
&s.t. \\
&4X_1 + 3X_2 \leq 240 \\
&2X_1 + X_2 \leq 100 \\
&X_1 \geq 0 \\
&X_2 \geq 0
\end{aligned}$$



To satisfy the 3rd and the 4th constraints, the solution must be in the 1st quadrant.

graph each of the constraints

$$\text{Max } 7X_1 + 5X_2$$

$$s.t.$$

$$4X_1 + 3X_2 \leq 240$$

$$2X_1 + X_2 \leq 100$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

X_2

To graph the 1st and the 2rd constraints, we first convert the inequalities in to equations:

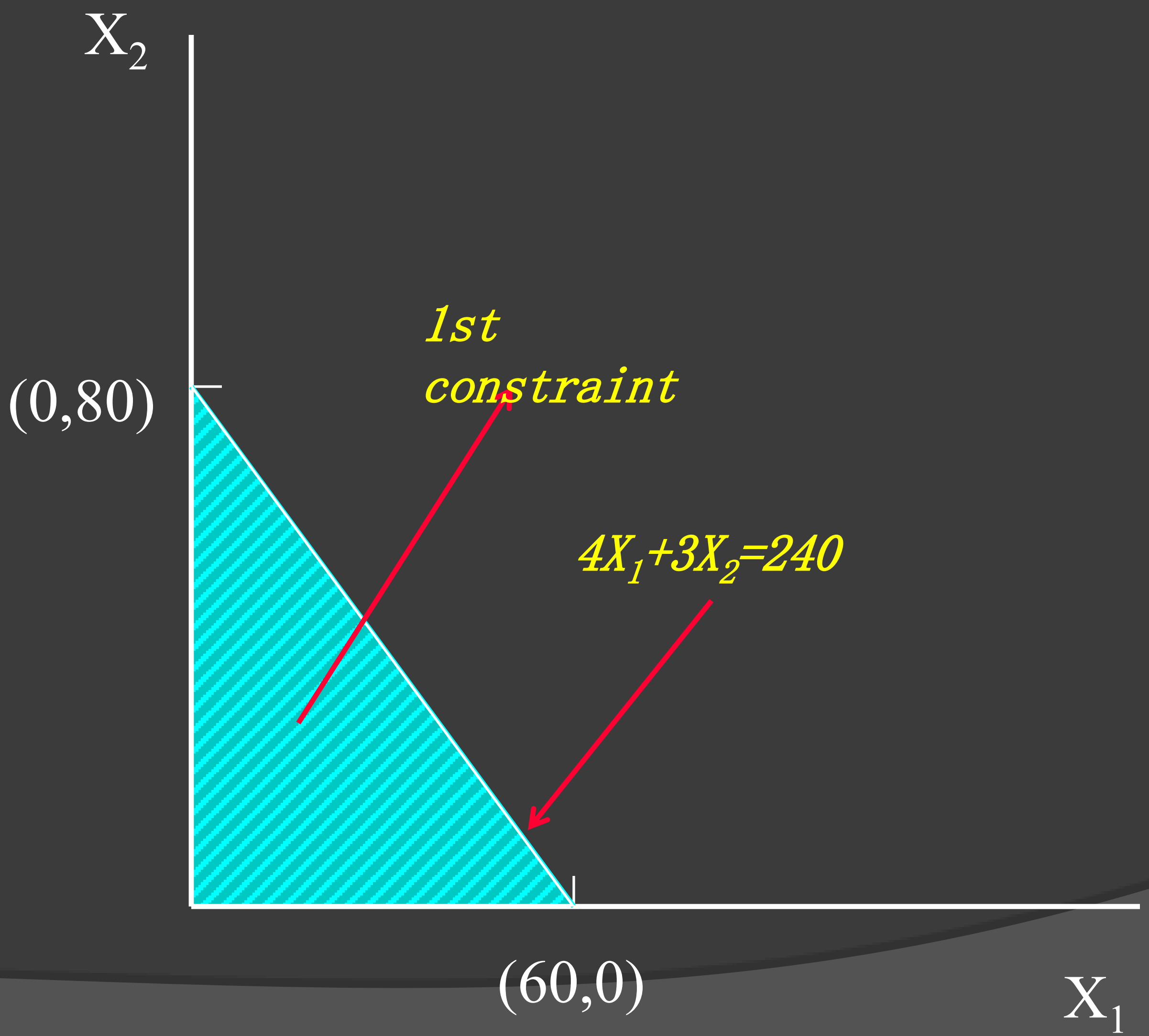
$$4X_1+3X_2=240$$

$$2X_1+1X_2=100$$

X_1

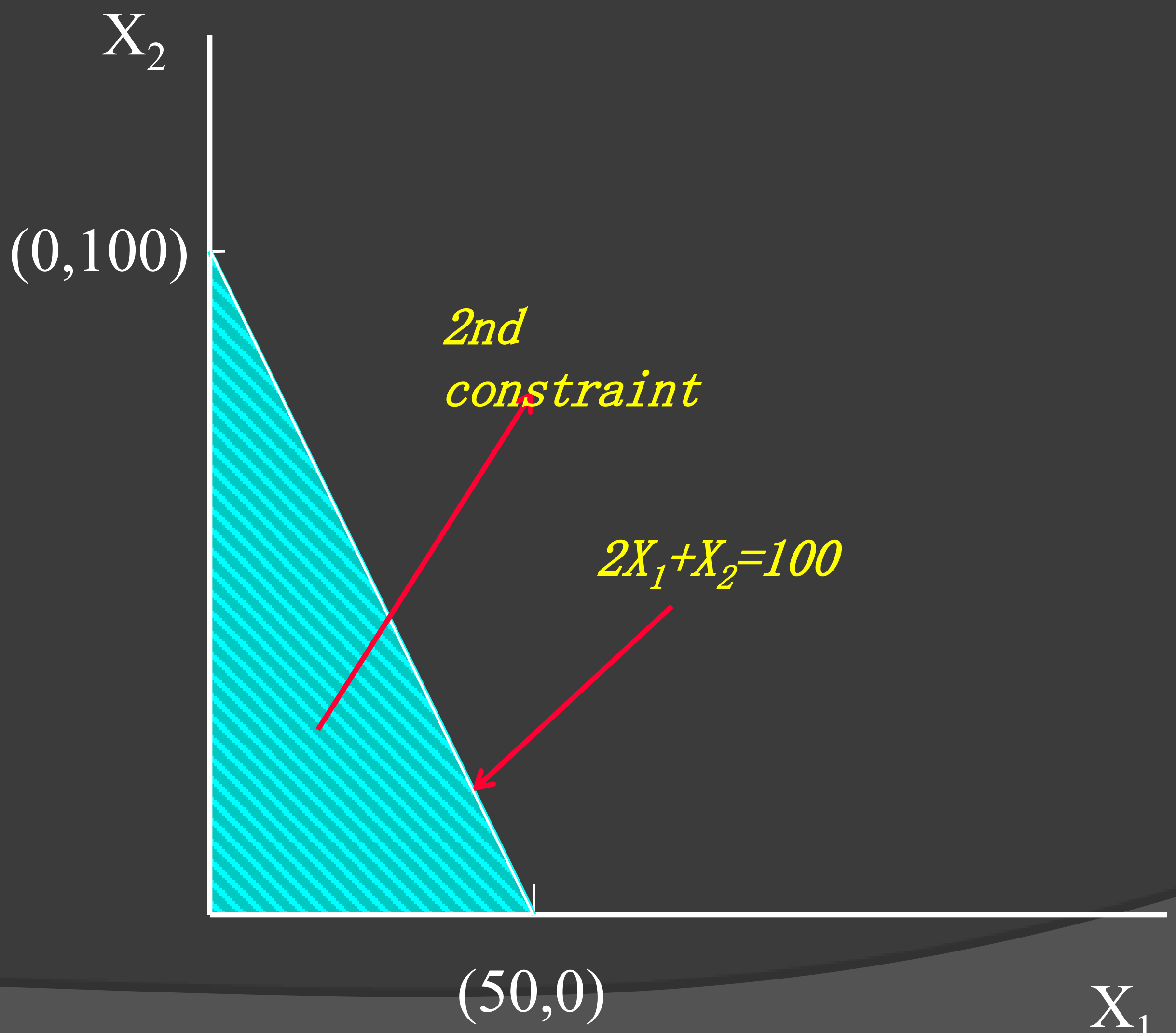
graph each of the constraints

$$\begin{aligned}
&Max\ 7X_1 + 5X_2 \\
&s.t. \\
&4X_1 + 3X_2 \le 240 \\
&2X_1 + X_2 \le 100 \\
&X_1 \ge 0 \\
&X_2 \ge 0
\end{aligned}$$



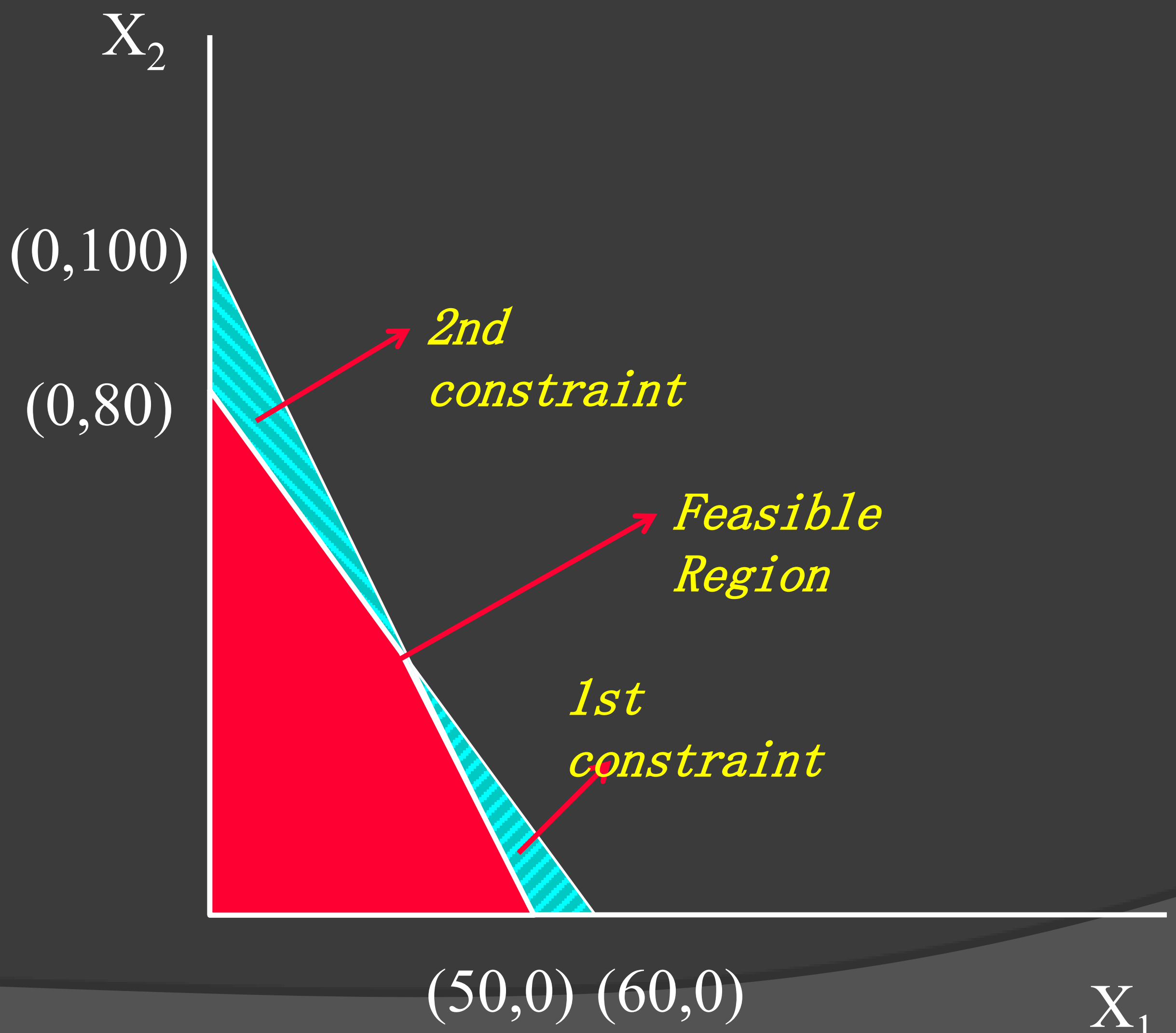
graph each of the constraints

$$\begin{aligned}&Max\ 7X_1+5X_2\\&s.t.\\&4X_1+3X_2\le 240\\&2X_1+X_2\le 100\\&X_1\ge 0\\&X_2\ge 0\end{aligned}$$



find the feasible region that satisfies all of the constraints simultaneously

$$\begin{aligned}
&Max\ 7X_1 + 5X_2 \\
&s.t. \\
&4X_1 + 3X_2 \le 240 \\
&2X_1 + X_2 \le 100 \\
&X_1 \ge 0 \\
&X_2 \ge 0
\end{aligned}$$



Now the feasible region has been graphed.

The optimal solution is the point lying in the feasible region that produces the highest profit.

We now proceed to find the optimal solution to the problem.

use iso-profit line solution method or corner solution method

Iso-profit Line Solution

Corner Solution Method

use iso-profit line solution method or corner solution method

Iso-profit Line Solution

Corner Solution Method

start by letting profits equal some arbitrary amount

For the Flair Furniture problem, we may choose a profit of $210.

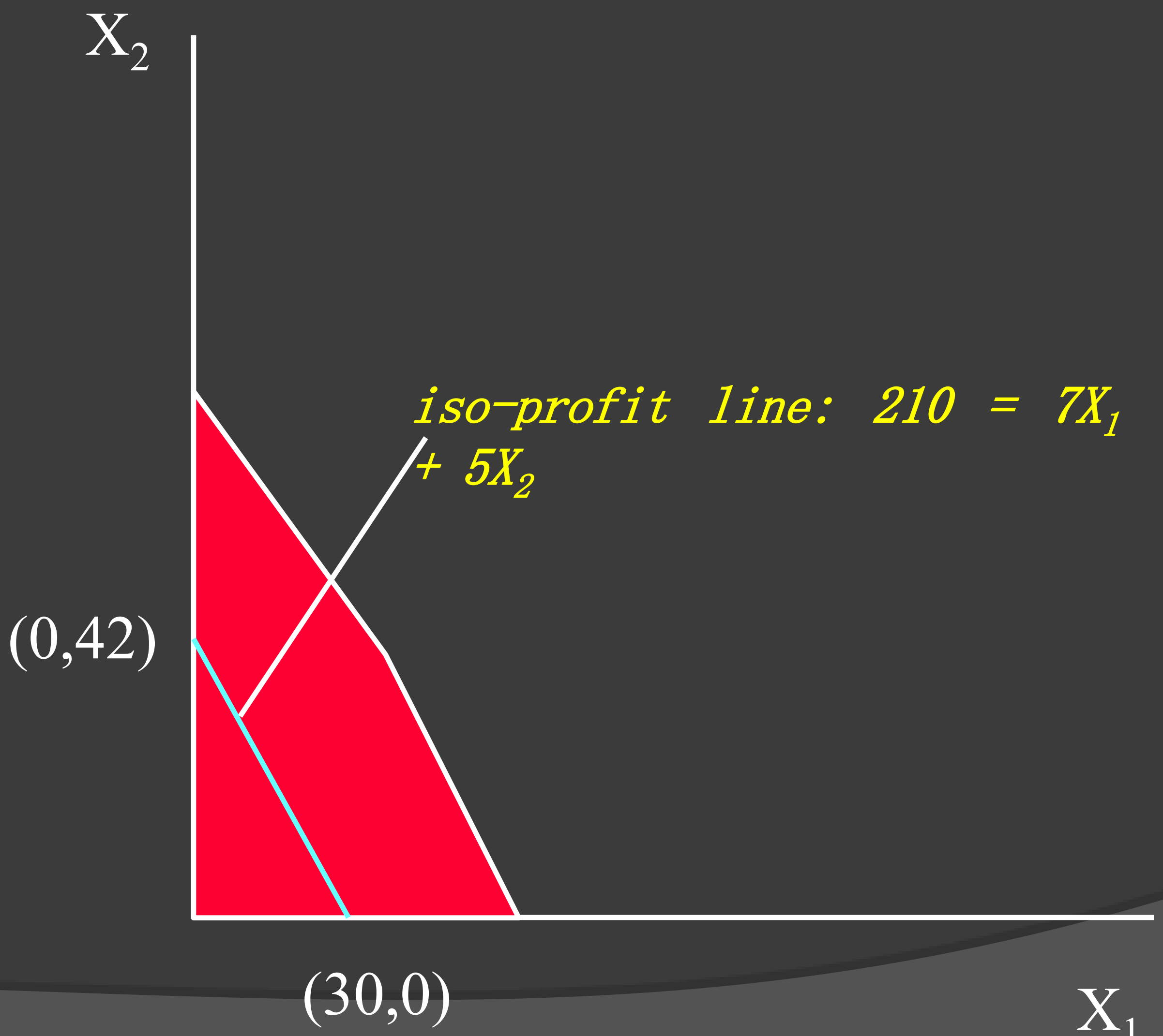
Then, the objective function will be

$$210 = 7X_1 + 5X_2$$

This is called iso-profit line since it represents all combinations of (X_1, X_2) *that would yield a total profit of $210.*

start by letting profits equal some arbitrary amount

graph the iso-profit line

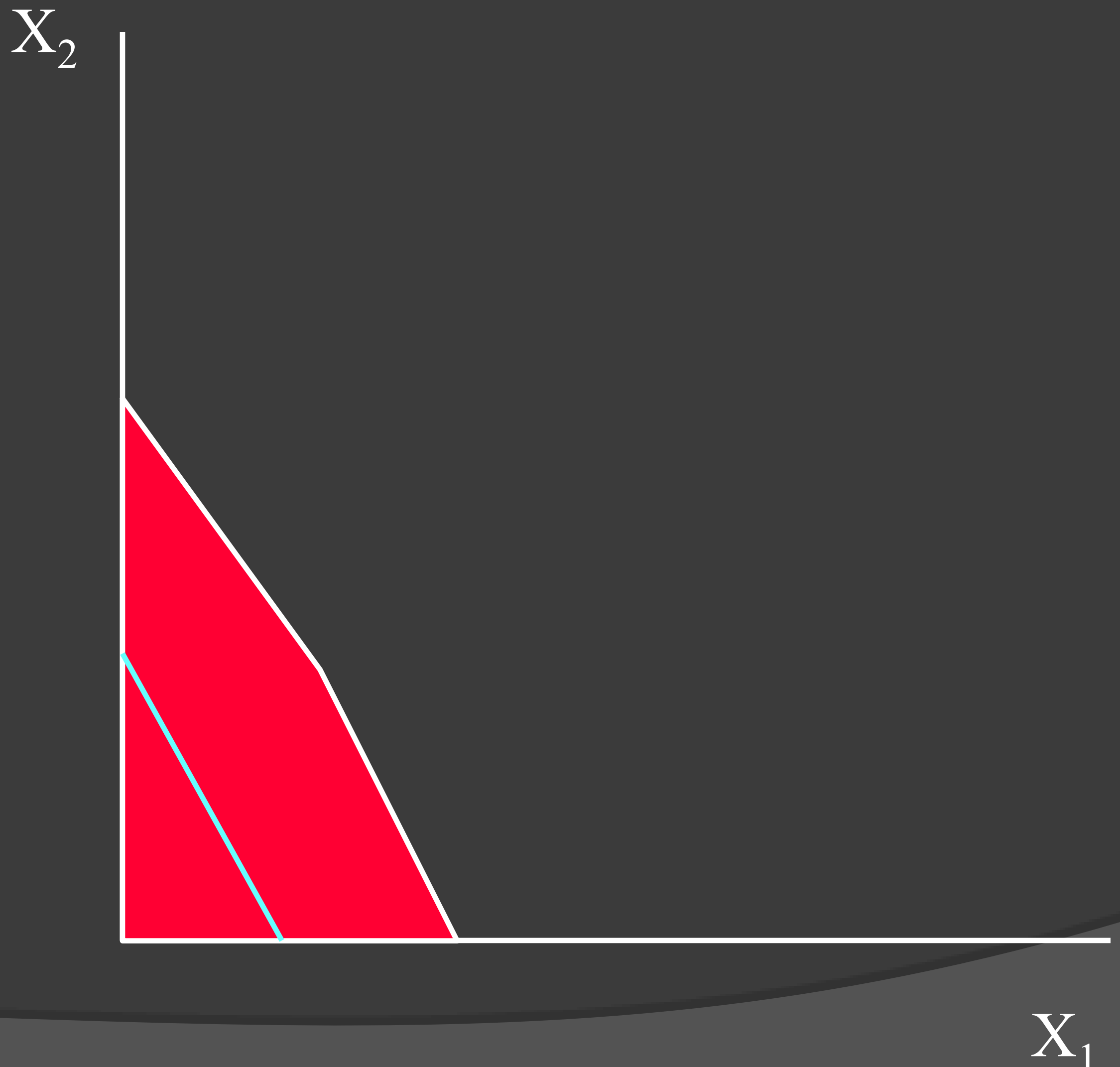


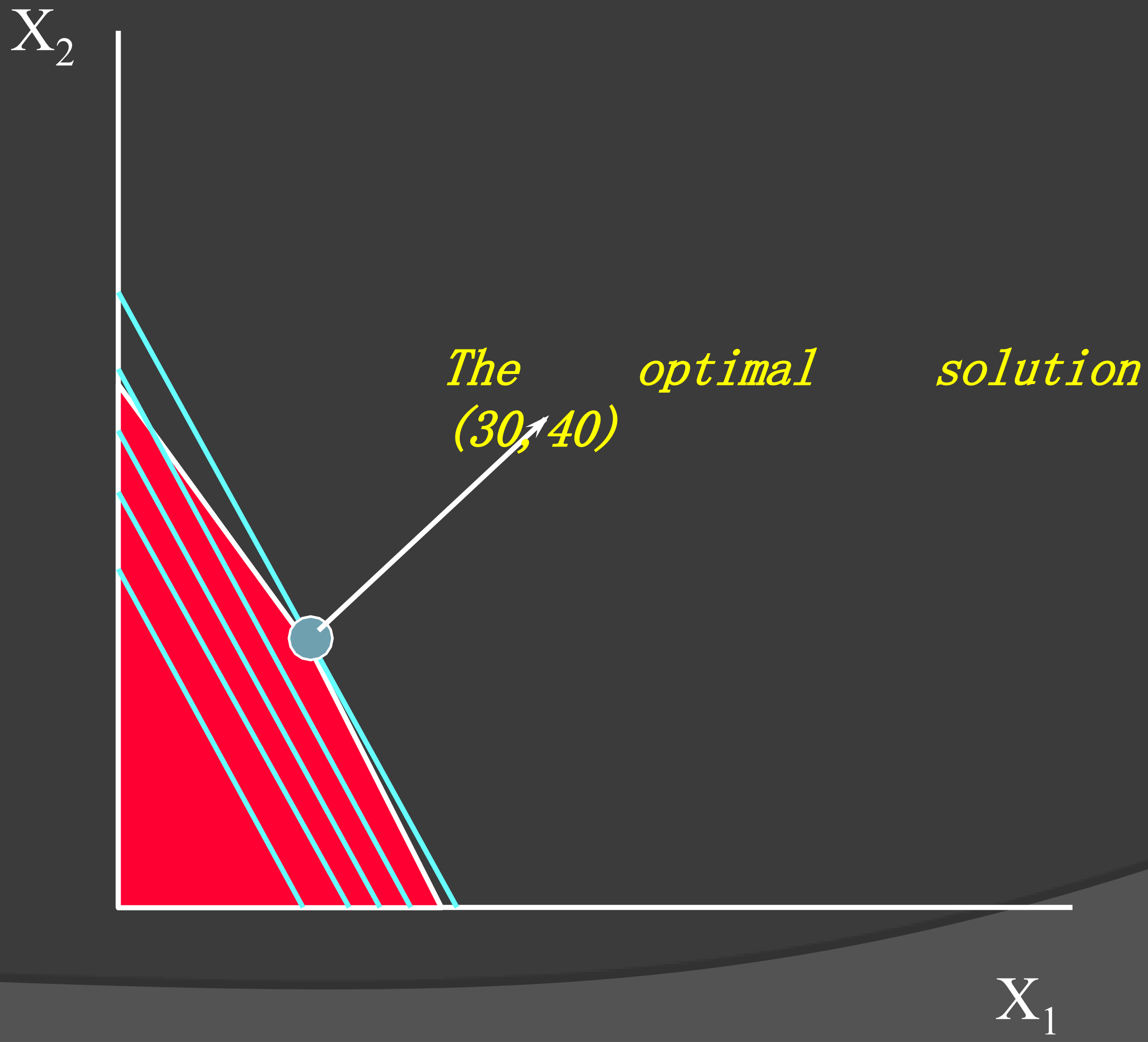
X2
iso-profit line: 210 = 7X1 + 5X2
(0,42)
(30,0)
X1

start by letting profits equal some arbitrary amount

graph the iso-profit line

move the iso-profit line in a parallel way. The highest iso-profit line that still touches some point of the feasible region will pinpoint the optimal


X_2
X_1



X_2
The optimal solution (30, 40)
X_1

use iso-profit line solution method or corner solution method

Iso-profit Line Solution

Corner Solution Method

use iso-profit line solution method or corner solution method

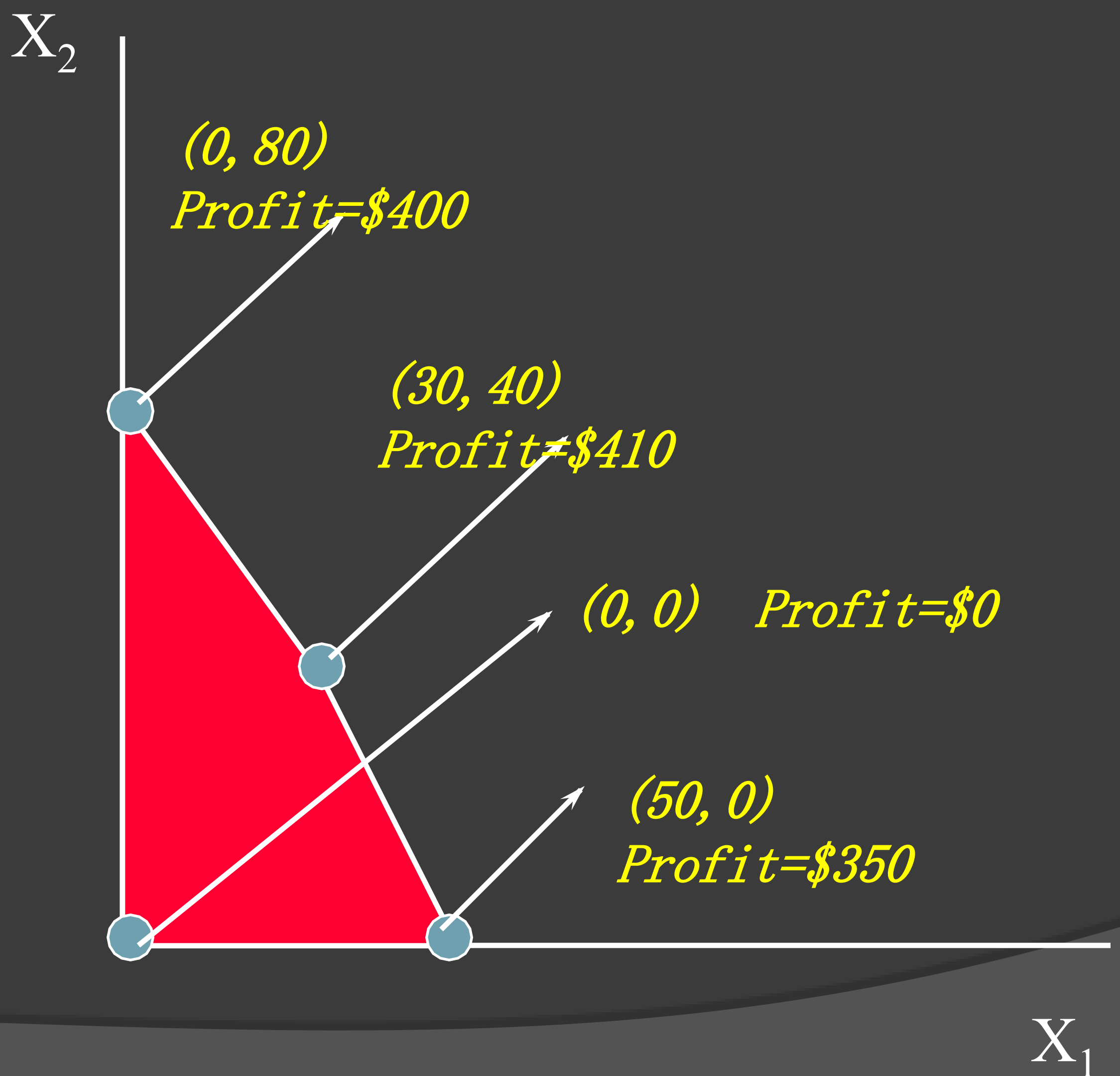
Iso-profit Line Solution Method

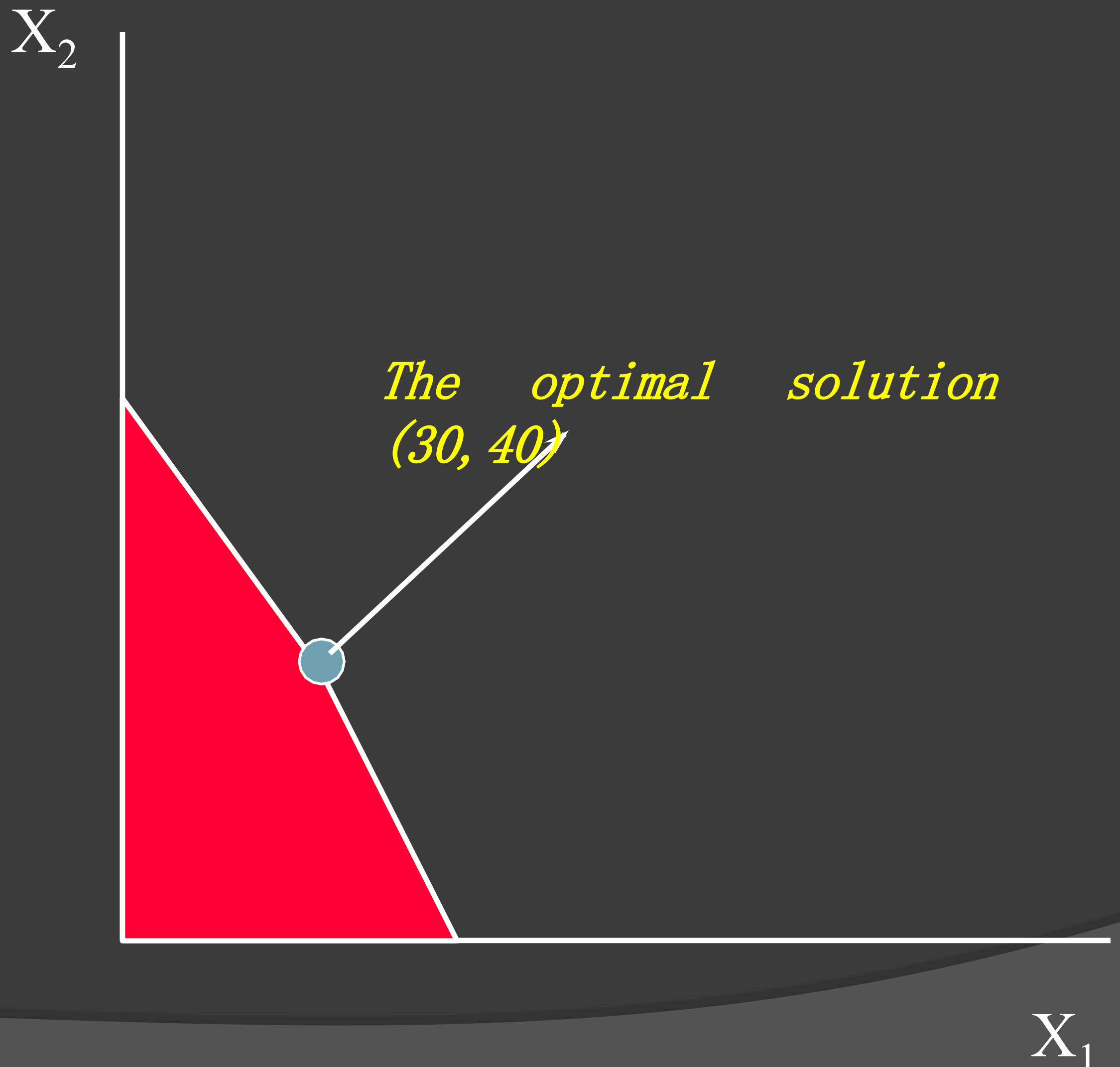
Corner Solution Method

From above, we saw that an optimal solution to any problem will lie at a corner point, or extreme point, of the feasible region.

It is only necessary to look at the profits at every corner point of the feasible region.

The corner point with the maximum profit is the


X_2
(0, 80)
Profit=$400
(30, 40)
Profit=$410
(0, 0) Profit=$0
(50, 0)
Profit=$350
X_1


X2
The optimal solution (30,40)
X1

5. Solving Minimization Problems

Example: The Holiday Meal Turkey Ranch

- The H.M.T.R is considering buying two different brands of turkey feed and blending them to provide a good, low-cost diet for its turkeys.
- Each feed contains, in varying proportions, some or all of the three nutritional ingredients essential for fattening turkeys.
- Each pound of Brand 1 contains 5 oz. of Ingredient A, 4 oz. of Ingredient B and 0.5 oz of Ingredient C.
- Each pound of Brand 2 contains 10 oz. of A, 3 oz of B, but no C.
- The Brand 1 feed costs 2c a pound, while the Brand 2 feed costs 3c a pound.
- The minimum monthly requirements per turkey for A, B and C are 90 oz., 48 oz. and 1.5 oz..
- The problem is to determine the lowest-cost diet that meets the