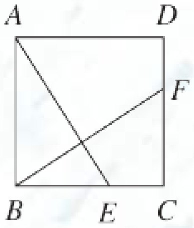
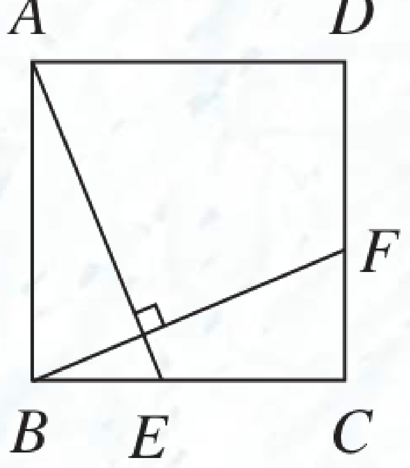
阶段拔尖专训3 正方形中的“十字架”模型

类型1 端点为顶点

图示	条件	结论
		

1.如图，四边形$ABCD$是正方形.若E，F分别是BC，CD上的点，且$AE \perp BF$.求证：$\triangle ABE \cong \triangle BCF$.



【证明】∵ 四边形$ABCD$是正方形，

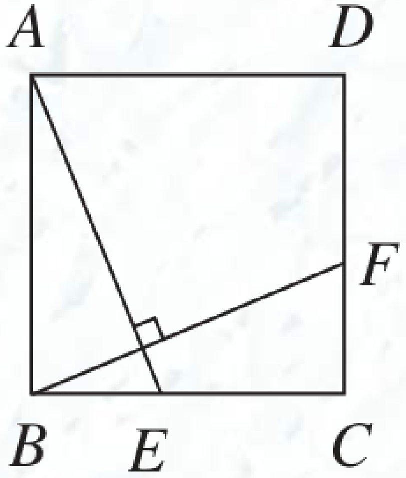
∴ $AB = BC$，$\angle ABC = \angle BCD = 90°$.

∵ $AE \perp BF$，∴ $\angle AEB + \angle CBF = 90°$.

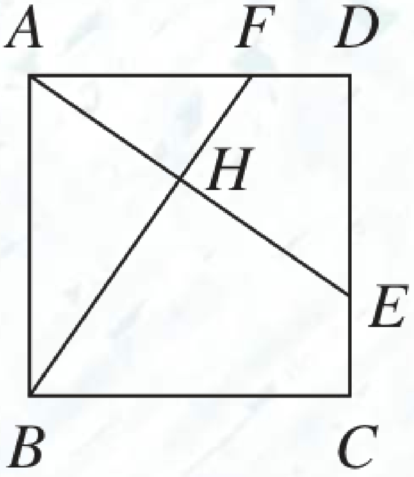
又∵ $\angle BFC + \angle CBF = 90°$ ，

∴ $\angle AEB = \angle BFC$.

∴ $\triangle ABE \cong \triangle BCF$(AAS).



2.[2024·泰安模拟] 如图，正方形$ABCD$中，点E，F分别为边CD，AD上的点，$CE=DF$，AE，BF交于点H.



（1）求证：$AE=BF$；

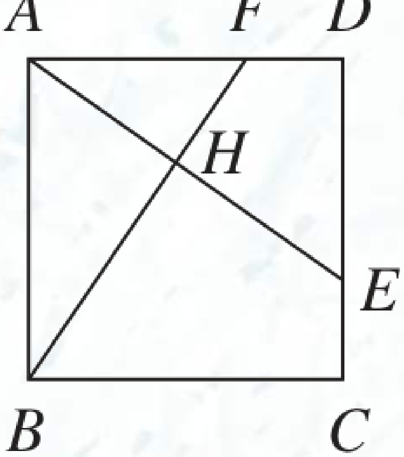
【证明】∵ 四边形$ABCD$是正方形，

∴ $AD=CD=AB$，$\angle BAF=\angle D=90°$．

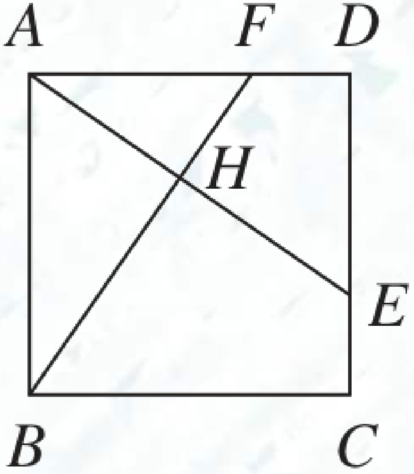
又∵ $CE=DF$，∴ $DE=AF$.

在$\triangle ADE$和$\triangle BAF$中，$\begin{cases} AD=BA, \\ \angle D=\angle BAF, \\ DE=AF, \end{cases}$

∴ $\triangle ADE\cong\triangle BAF$(SAS). ∴ $AE=BF$.



（2）若$AB=4$，$CE=1$，求AH的长.



【解】$\because AD = AB = 4$，$DF = CE = 1$，

$\therefore AF = 4 - 1 = 3.$

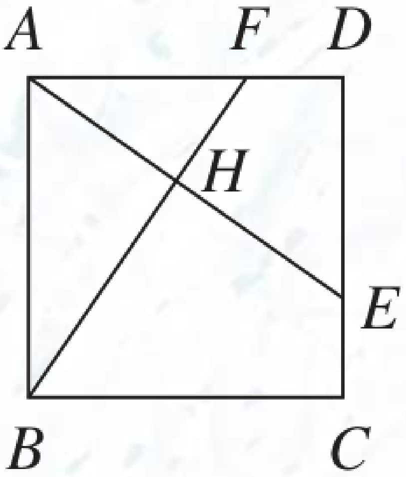
$\therefore BF = \sqrt{AB^2 + AF^2} = \sqrt{4^2 + 3^2} = 5.$

$\because \triangle ADE \cong \triangle BAF$，$\therefore \angle DAE = \angle ABH.$

又$\because \angle ABH + \angle AFB = 90°$，

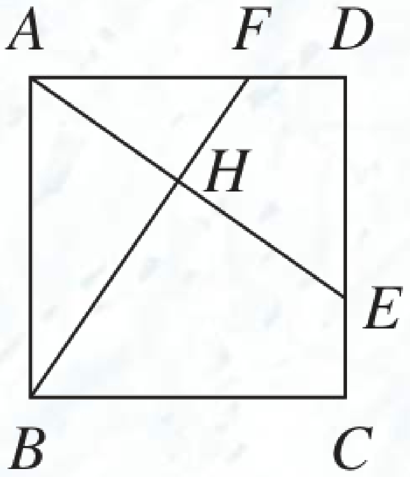
$\therefore \angle DAE + \angle AFB = 90°$.

$\therefore \angle AHF = 90°$ $\therefore AH \perp BF.$

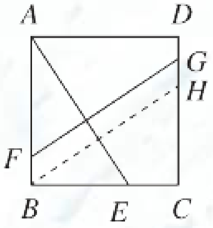


∴ $\triangle ABF$的面积为$\frac{1}{2}BF \bullet AH=\frac{1}{2}AB \bullet AF$.

∴ $AH=\frac{AB \cdot AF}{BF}=\frac{4\times 3}{5}=\frac{12}{5}$.



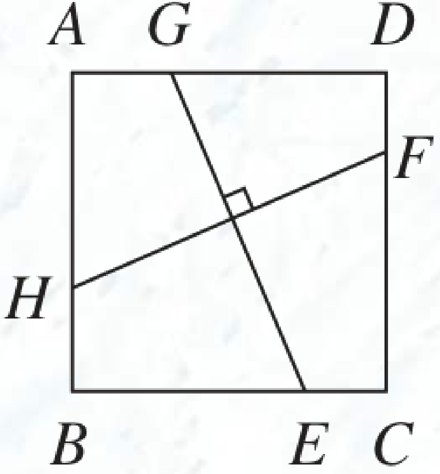
类型2 端点不为顶点

图示	条件	结论
		

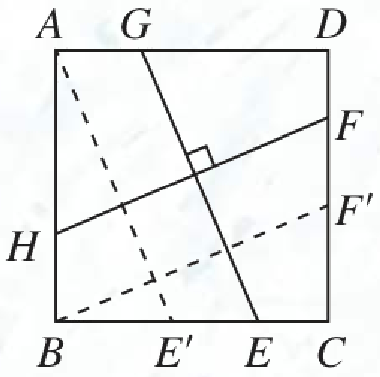
正方

3.[2024·济宁模拟] 如图，四边形$ABCD$是正方形.若点E，F，G，H分别在BC，CD，DA，AB上，且$EG \perp HF$.

求证：$EG = HF$.



【证明】如图，过点A作$AE'//GE$交BC于点E'，过点B作$BF'//HF$交CD于点F'.



∵ 四边形$ABCD$是正方形，∴ $AD//BC$，$AB//DC$.

∴ 四边形$AE'EG$与四边形$HBF'F$是平行四边形.

∴ $GE = AE'$，$HF = BF'$.