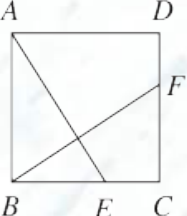




阶段拔尖专训3 正方形中的“十字架”模型

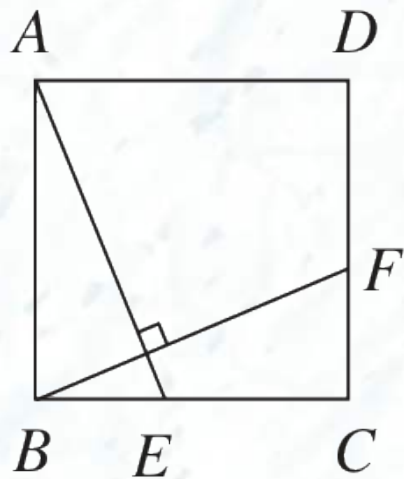


类型1 端点为顶点

图示	条件	结论
 <p>A square with vertices labeled A (top-left), B (bottom-left), C (bottom-right), and D (top-right). Diagonals AC and BD intersect at point E. A point F is located on the side CD.</p>		No Image

正方形

1.如图, 四边形 $ABCD$ 是正方形.若 E, F 分别是 BC, CD 上的点, 且 $AE \perp BF$.求证:
 $\triangle ABE \cong \triangle BCF$.



【证明】∵ 四边形 $ABCD$ 是正方形,

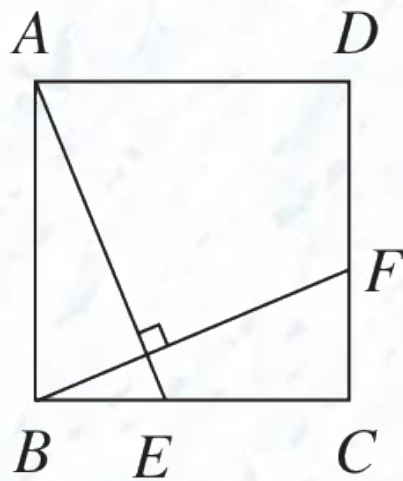
∴ $AB = BC$, $\angle ABC = \angle BCD = 90^\circ$.

∵ $AE \perp BF$, ∴ $\angle AEB + \angle CBF = 90^\circ$.

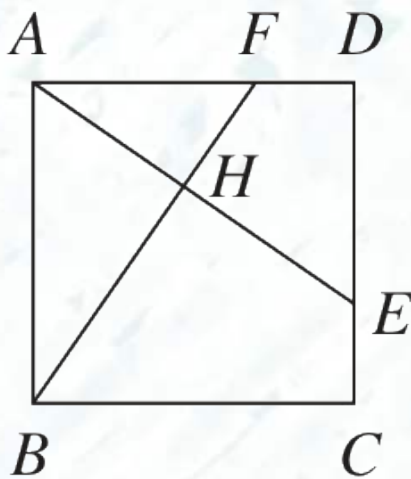
又∵ $\angle BFC + \angle CBF = 90^\circ$,

∴ $\angle AEB = \angle BFC$.

∴ $\triangle ABE \cong \triangle BCF$ (AAS).



2.[2024·泰安模拟] 如图, 正方形 $ABCD$ 中, 点 E, F 分别为边 CD, AD 上的点, $CE = DF$, AE, BF 交于点 H .



(1) 求证: $AE = BF$;

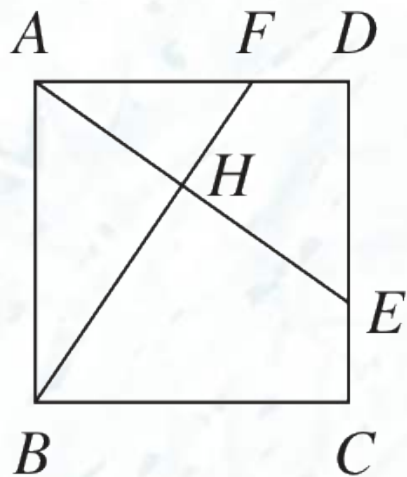
【证明】 \because 四边形 $ABCD$ 是正方形,

$\therefore AD = CD = AB, \angle BAF = \angle D = 90^\circ$.

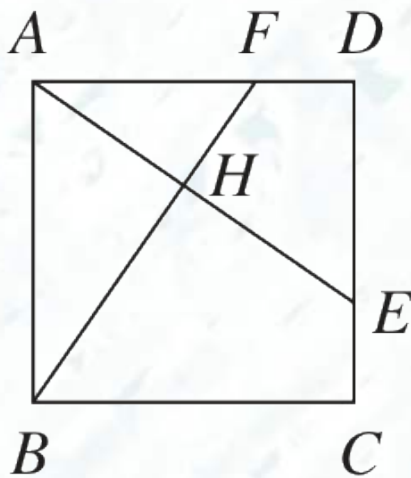
又 $\because CE = DF, \therefore DE = AF$.

在 $\triangle ADE$ 和 $\triangle BAF$ 中,
$$\begin{cases} AD = BA, \\ \angle D = \angle BAF, \\ DE = AF, \end{cases}$$

$\therefore \triangle ADE \cong \triangle BAF(SAS). \therefore AE = BF$.



(2) 若 $AB = 4$, $CE = 1$, 求 AH 的长.



【解】 $\because AD = AB = 4, DF = CE = 1,$

$$\therefore AF = 4 - 1 = 3.$$

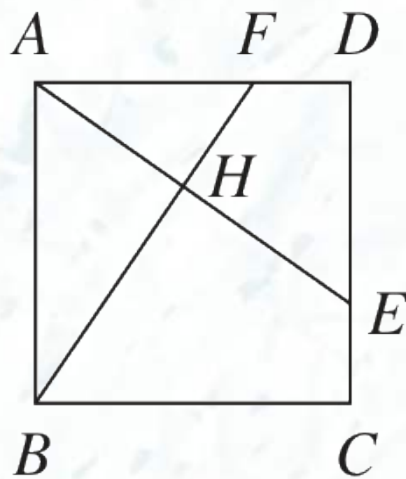
$$\therefore BF = \sqrt{AB^2 + AF^2} = \sqrt{4^2 + 3^2} = 5.$$

$\because \triangle ADE \cong \triangle BAF, \therefore \angle DAE = \angle ABH.$

又 $\because \angle ABH + \angle AFB = 90^\circ,$

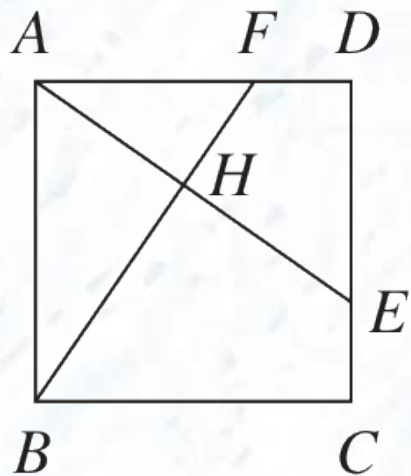
$\therefore \angle DAE + \angle AFB = 90^\circ.$

$\therefore \angle AHF = 90^\circ \therefore AH \perp BF.$

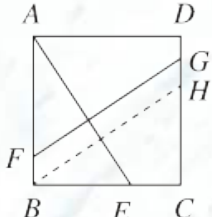


$\therefore \triangle ABF$ 的面积为 $\frac{1}{2}BF \bullet AH = \frac{1}{2}AB \bullet AF$.

$$\therefore AH = \frac{AB \cdot AF}{BF} = \frac{4 \times 3}{5} = \frac{12}{5}.$$

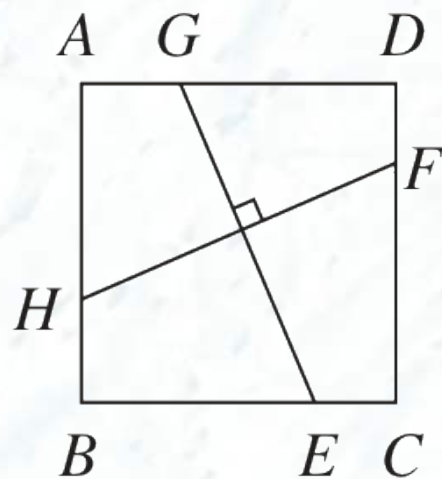


类型2 端点不为顶点

图示	条件	结论
		<p style="text-align: center;">No Image</p>

正方形

3.[2024·济宁模拟] 如图, 四边形 $ABCD$ 是正方形. 若点 E, F, G, H 分别在 BC, CD, DA, AB 上, 且 $EG \perp HF$.
求证: $EG = HF$.

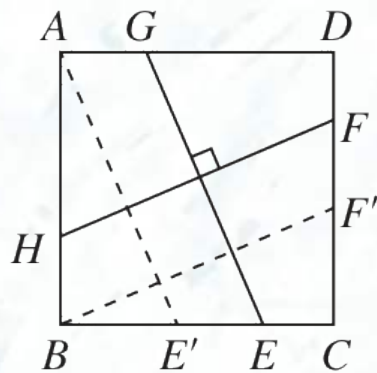


【证明】如图，过点A作 $AE' \parallel GE$ 交BC于点 E' ，过点B作 $BF' \parallel HF$ 交CD于点 F' 。

\because 四边形ABCD是正方形， $\therefore AD \parallel BC$ ，
 $AB \parallel DC$ 。

\therefore 四边形 $AE'EG$ 与四边形 $HBF'F$ 是平行四边
形。

$\therefore GE = AE'$ ， $HF = BF'$ 。



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