

28-April-2014



OptiStruct

随机振动分析

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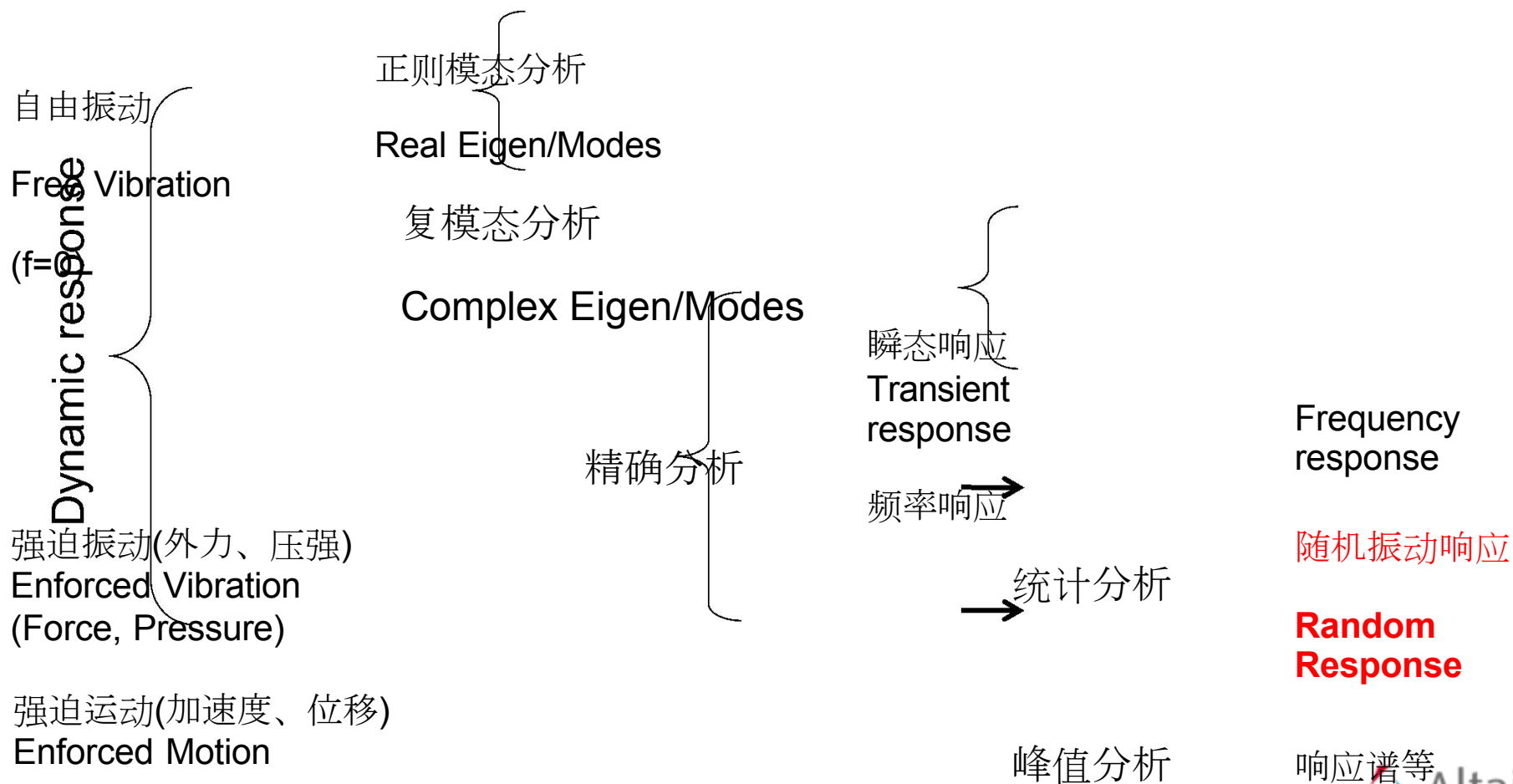
support@altair.com.cn (support email)

4006-196-186 (support hotline)

振动分类—按载荷形式和响应类型

Classification of vibration

线性系统Linear system

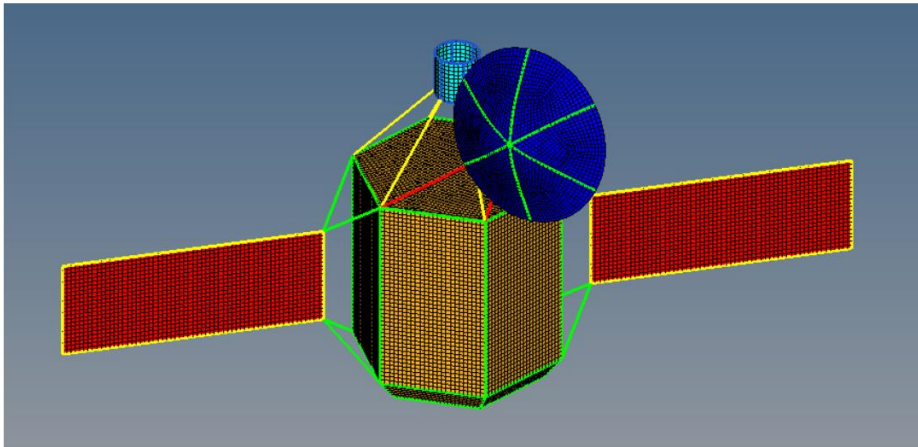
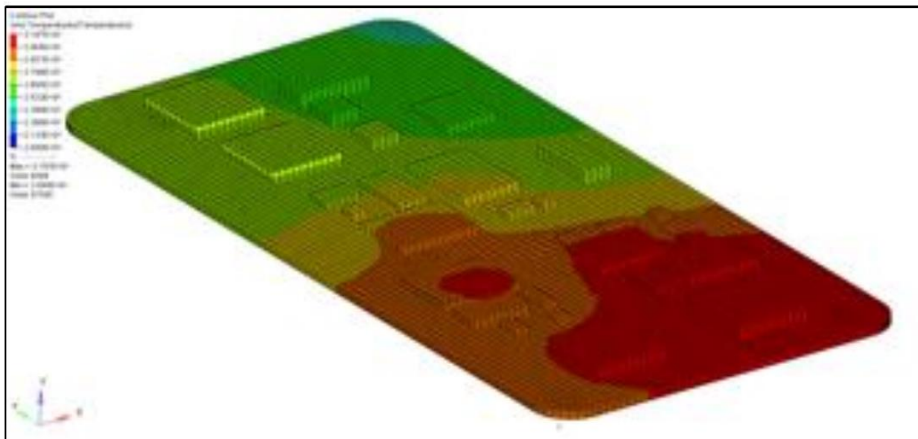


(Acceleration, Displacement)

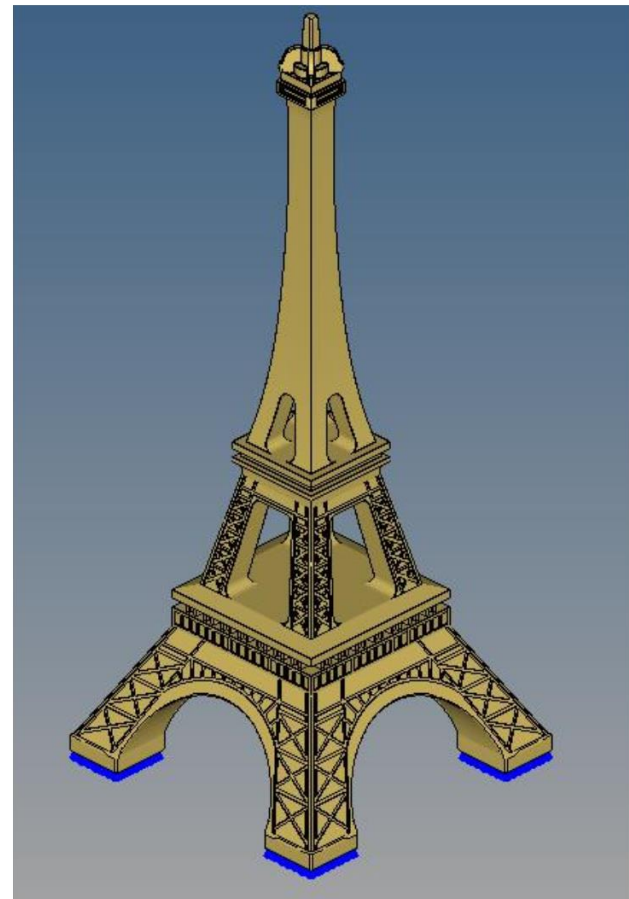
SRS, DDAM

强迫振动？强迫运动激励？

- 强迫运动激励首先一定是固定在某些“基础”之上的。



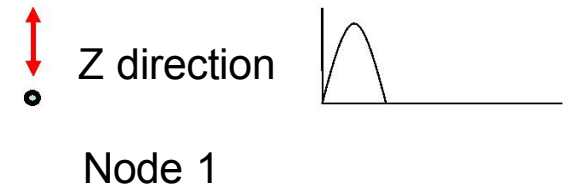
Lunch load



地震波

强迫运动 – 虚拟约束法SPCD

- 定义节点1的位移时间历程的自动强迫运动
- HyperMesh中定义两个约束：
 - SPCD 定义节点1的强迫运动位移
 - SPC定义节点约束
 - SPC必须约束住加载自由度
 - SPC是否需要约束其它自由度？



create update

nodes

size = 5 0 0 . 0 0 0

☒ label constraints

constant value

☐ dof1	=	0 . 0 0 0
☐ dof2	=	0 . 0 0 0
☒ dof3	=	1 . 0 0 0
☐ dof4	=	0 . 0 0 0
☐ dof5	=	0 . 0 0 0
☐ dof6	=	0 . 0 0 0

load types = S P C D

create create/edit reject review return

create update

nodes

size = 5 0 0 . 0 0 0

☒ label constraints

constant value

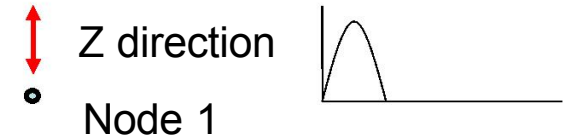
☒ dof1	=	0 . 0 0 0
☒ dof2	=	0 . 0 0 0
☒ dof3	=	0 . 0 0 0
☒ dof4	=	0 . 0 0 0
☒ dof5	=	0 . 0 0 0
☒ dof6	=	0 . 0 0 0

load types = S P C

create create/edit reject review return

强迫运动 – 虚拟约束法SPCD

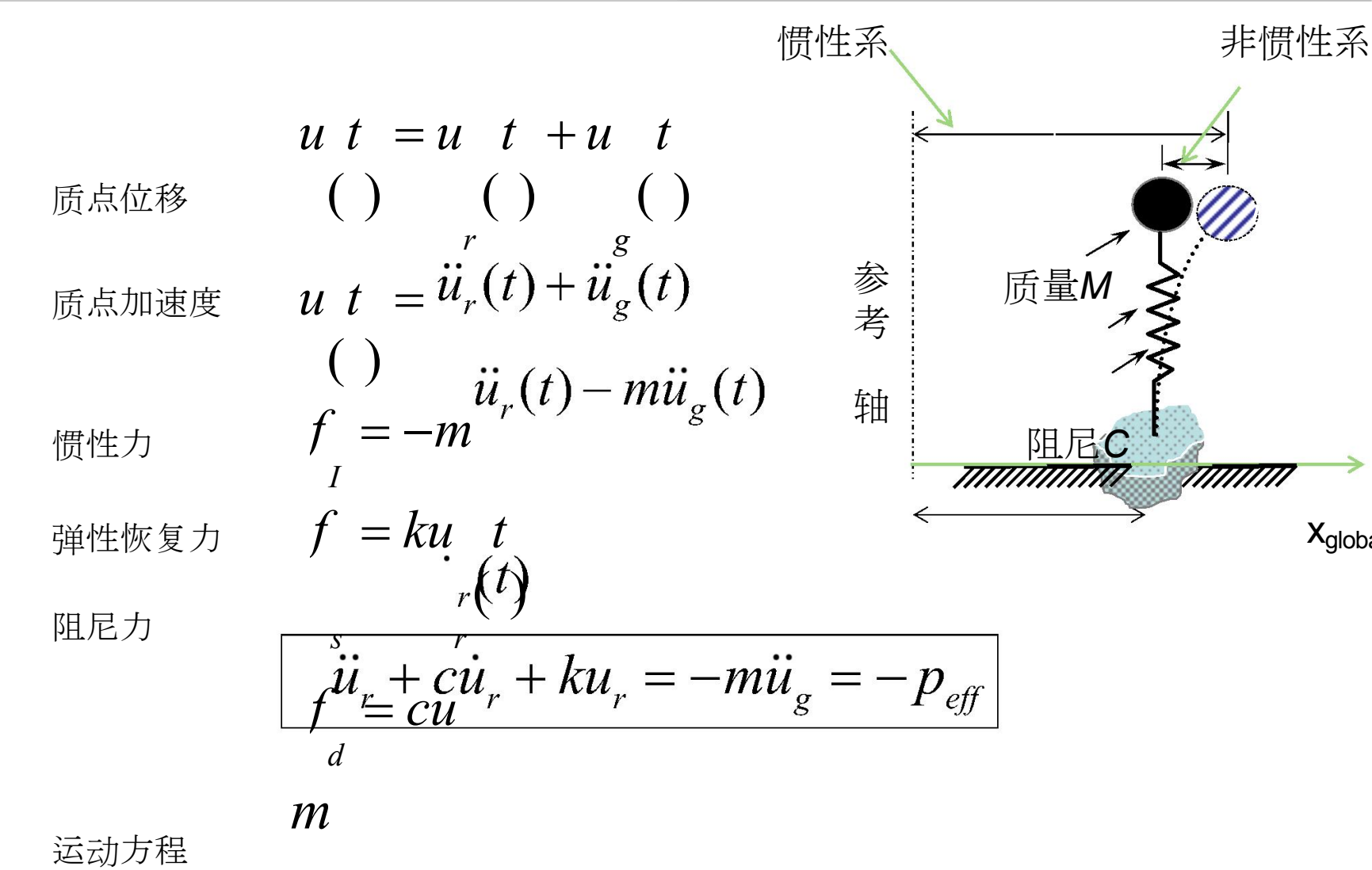
- SPCD 定义节点1的强迫运动量
- SPC 定义节点1的约束，这是一个虚拟约束



对于**SPCD**方法，虚约束自由度同时又是运动加载自由度！

TLOAD1	4	5		ACCE	1
LSEQ	1	5	6		
SPCD	6	1	3	386.1	
DLOAD	2	1.	1.	4	
\$ Displacement Constraints of Load Set : acceleration					
SPC1	1	123	1		
\$ Referenced Dynamic Load Tables					
\$ Dynamic Load Table : field					
TABLED1	1				
	0.	0.	.1	0.	.11
	.5	0.	1.	0.	1.
				ENDT	.49

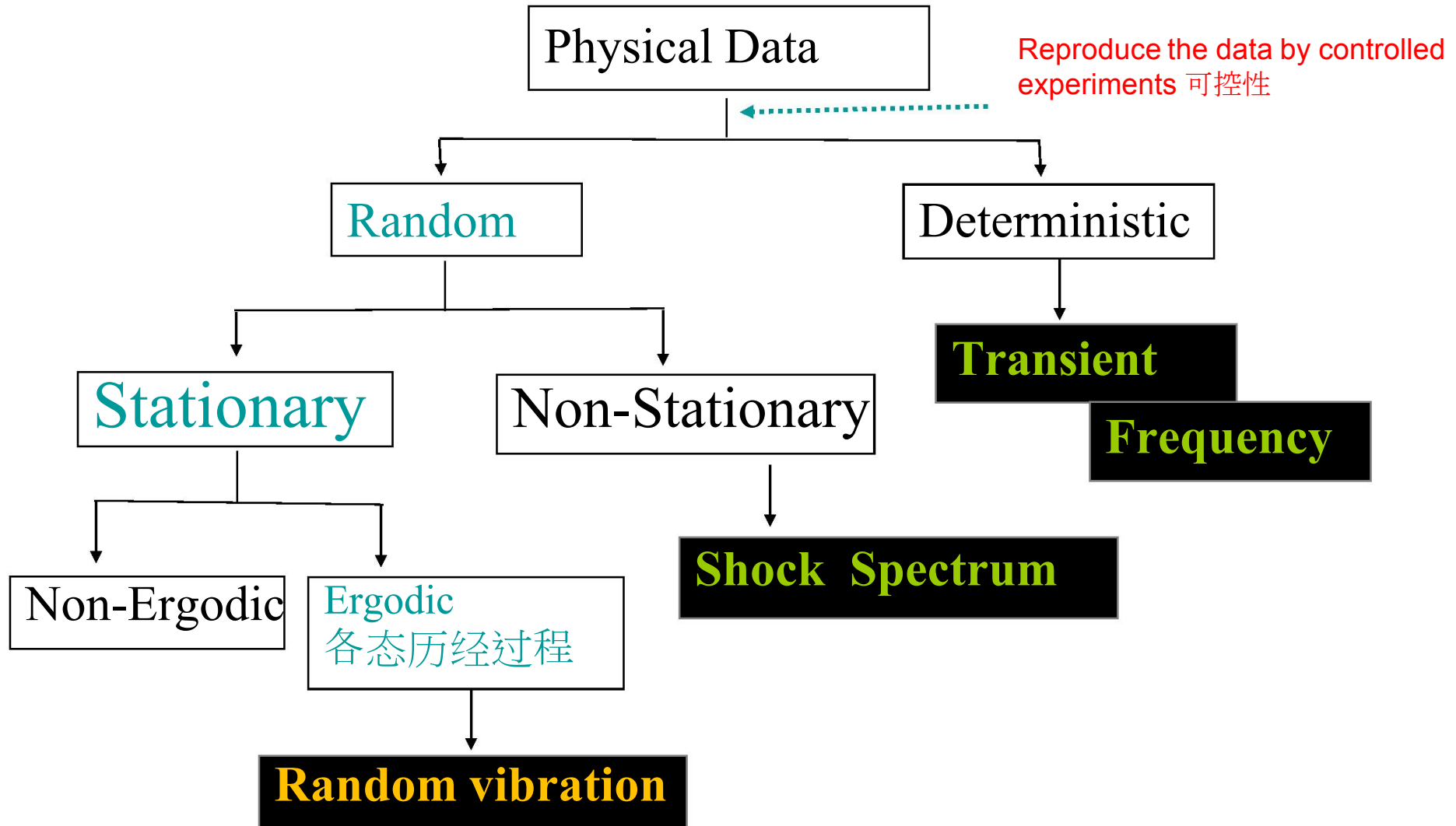
强迫运动的绝对响应和相对响应



PARAM,ENFMOTN

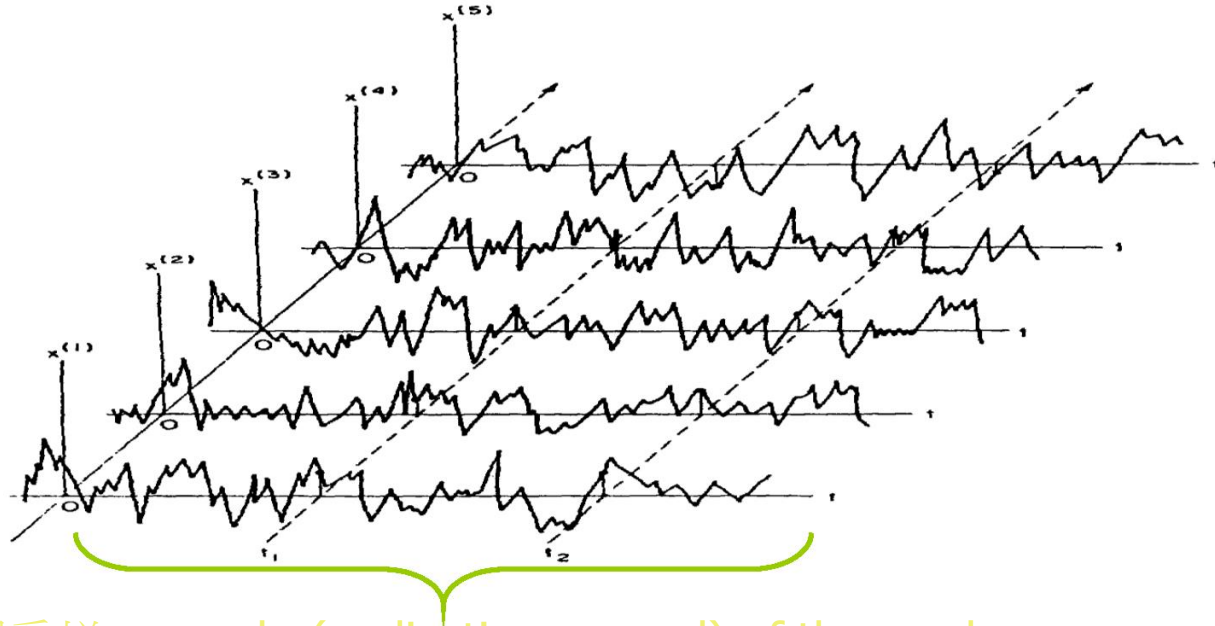
- ENFMOTN参数控制绝对位移和相对位移的输出
- PARAM,ENFMOTN,ABS
 - ABS的缺省值表示分析结果是模型的绝对运动
- PARAM,ENFMOTN,REL
 - 假如值被定义为 **REL**, 那么运动相对于强迫运动的基础。在模态振动分析中, 这等效于用大质量法和从分析中排除刚体运动。

Classification of Physical Data



Stationary and Ergodic

稳态数据和各态历经过程

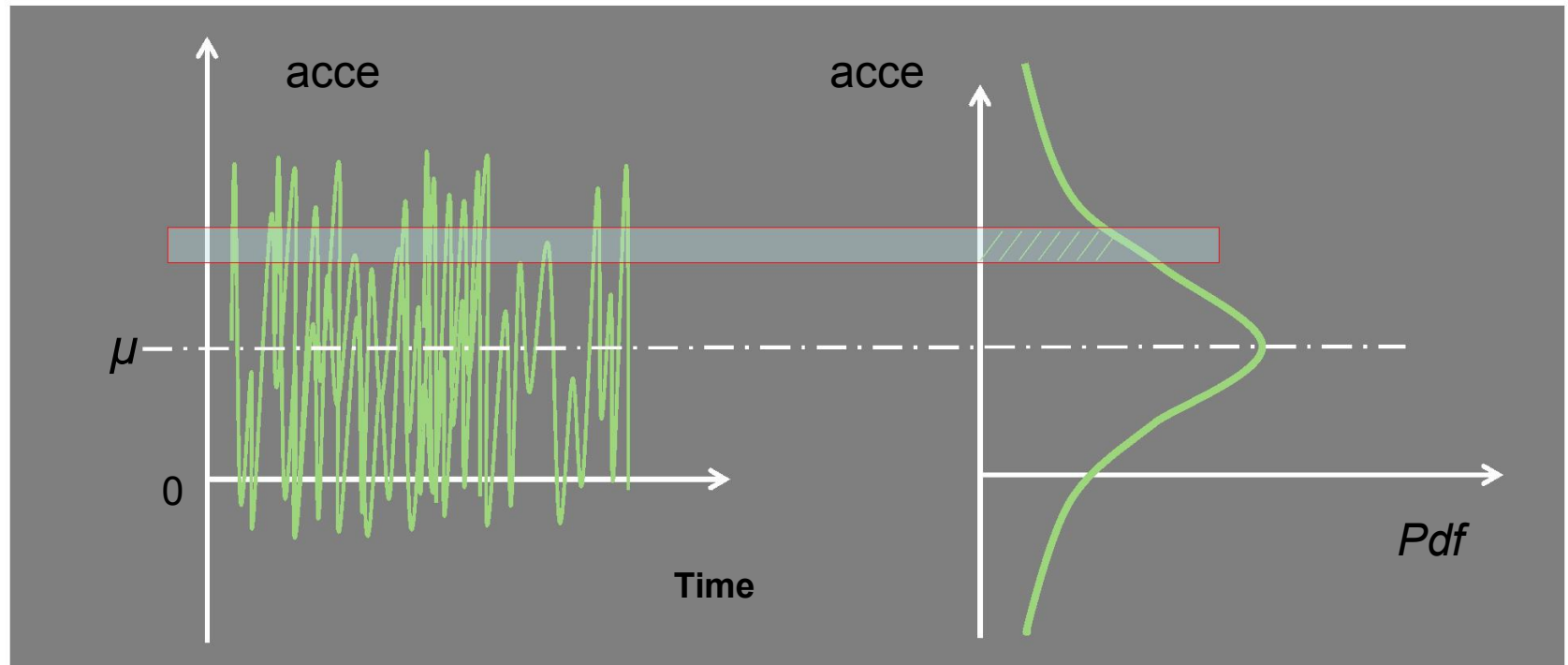


数据采集 a sample (realization, record) of the random process

- **Stationary:** For every t , PDF (Probability Density Function), μ_x and σ_x^2 of every record are constant. 对每个采样数据, 概率密度函数, μ_x 和 σ_x^2 是恒定的。

- **Ergodic:** If the random process is stationary , and the characters of any single record are the same as the ensemble of records.对所有的采样数据，任意时间段内都是稳态的

各态历经随机信号, $N(\mu, \sigma)$ Ergodic Random signal



$$F(x) = \frac{1}{\sigma} \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2} = RMS$$

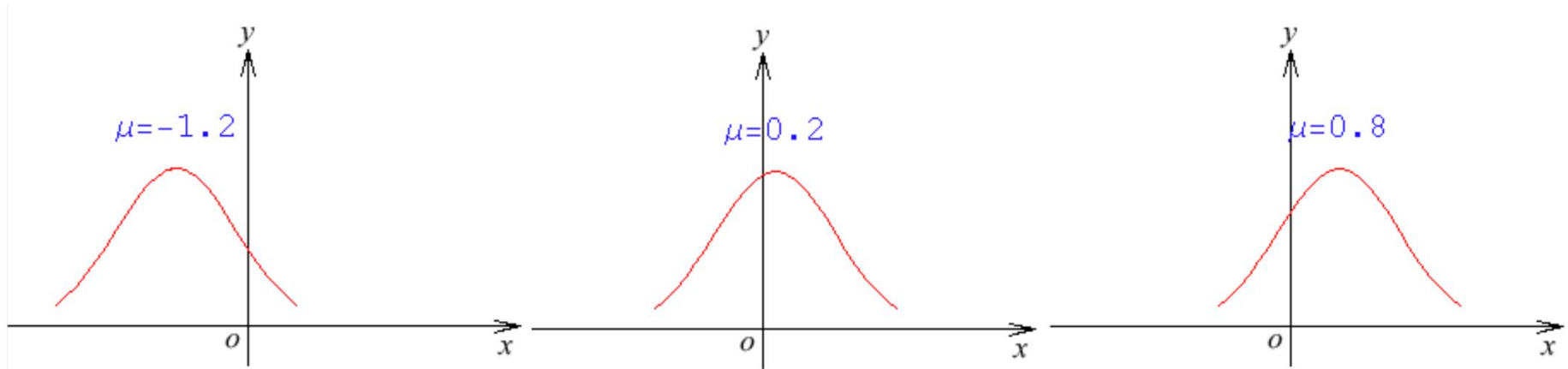
$$= \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sigma} e^{-\frac{x^2}{2\sigma^2}} dx$$

工程上常见的随机信号，可以用高斯分布，即正态分布来表示。
 均方根(标准差**Standard deviation**)：每个样本点到平均值的平均距离。
 μ_x , σ_x 只能反映载荷的幅值或者强度变化，但是完全不能反映频谱特性

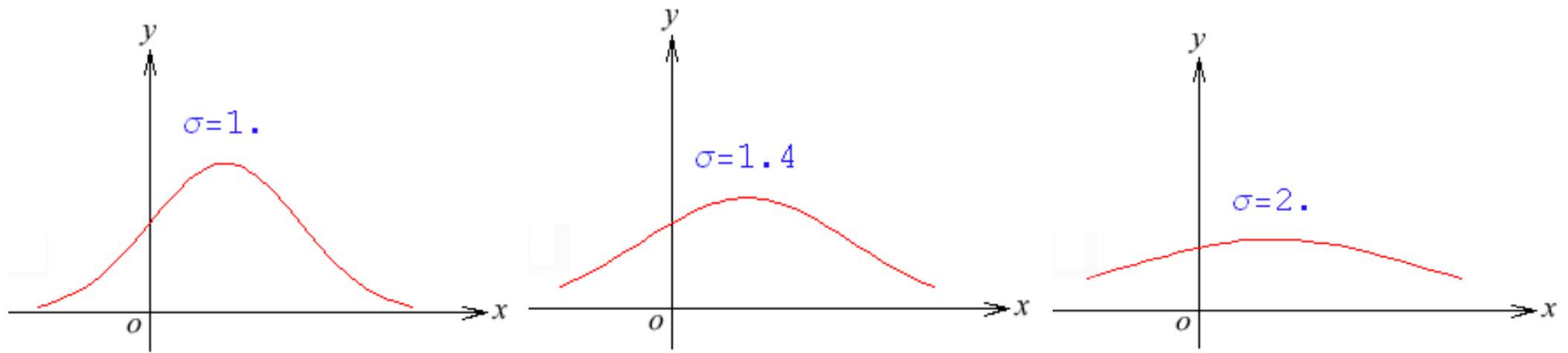
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正态分布的特征

characters of normal distribution



平均值的影响

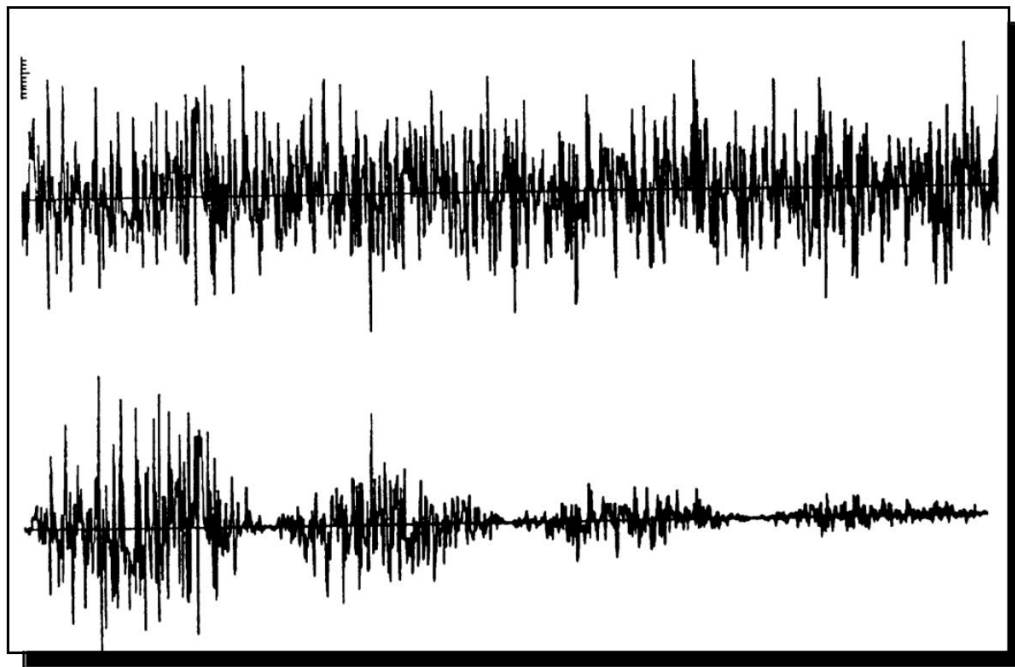


标准差的影响

随机振动响应的两个基本假设

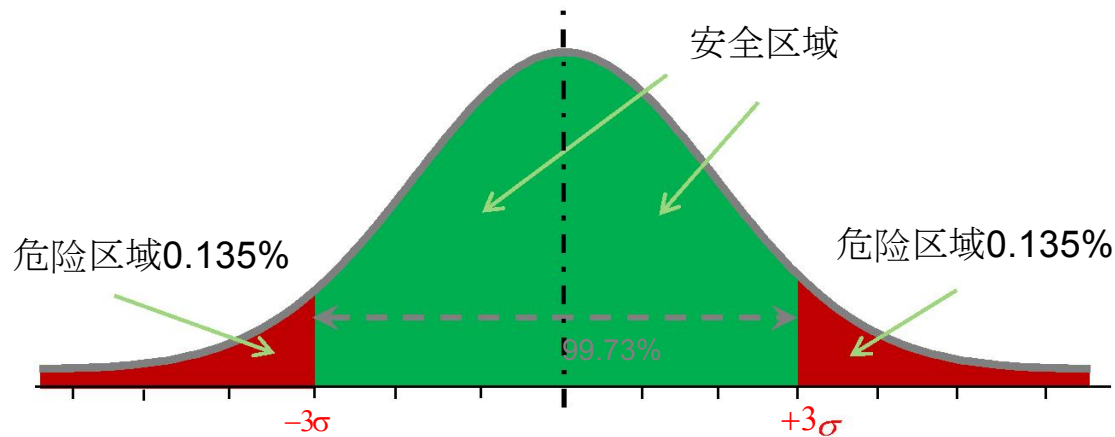
● 随机振动分析方法有两个基本假设：

- 载荷和响应 满足正态分布假设
- 载荷和响应 满足零平均值假设



正态分布的置信水平

1σ~3σ



$$F(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$\lambda\sigma$	$P(x \leq \lambda\sigma)$	$P(x > \lambda\sigma)$	Per Million/每百万
1σ	68.27%	31.7%	317,300
2σ	95.45%	4.55%	45,500
2.5σ	98.76%	1.24%	12,400

$$\pi \sigma^2$$

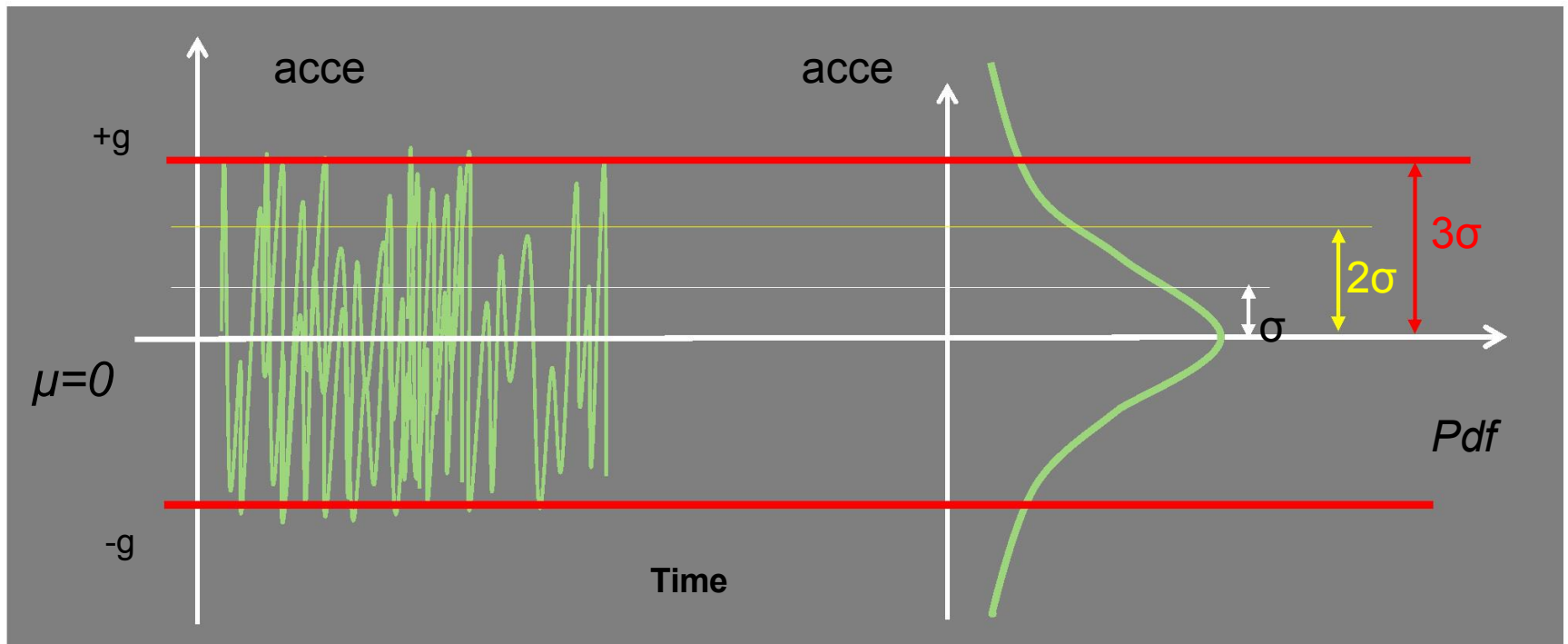
$$\sigma_{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^n (a_i - \mu)^2}$$

随机振动采用双边置信水平

3σ	99.73%	0.27%	2,700
3.5σ	99.953%	0.047%	470
4σ	99.9937%	0.0063%	63
5σ	99.999943%	0.000057%	0.57
6σ	99.9999998%	0.0000002%	0.002

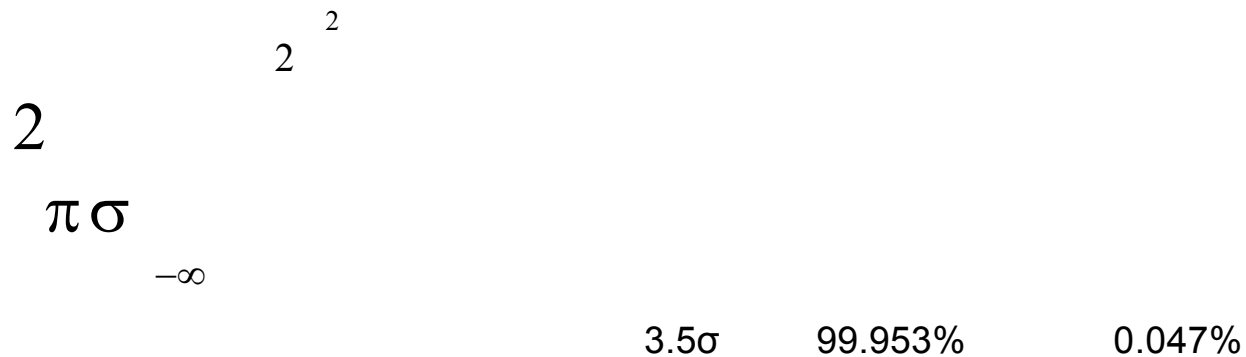
随机振动结果的常用置信水平

Common confidence level for random vibration



$$F(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{x^2}{2\sigma^2}} dx$$

$\lambda\sigma$	安全率	失效率
2.5 σ	98.76%	1.24%
3 σ	99.73%	3‰



随机振动结果通常使用 3σ 双边置信，即 $\pm 3\sigma$

PSD定义公式

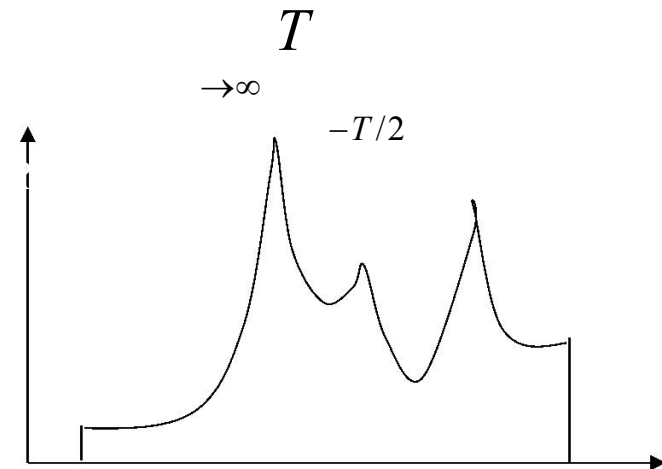
PSD Function definition

自相关函数：
(Auto)correlation

$$R_{xx}(\tau) = E \left[\frac{x(t)x(t+\tau)}{T} \right] = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau) dt$$

自功率谱密度函数定义
(Auto)PSD

$$= \int_{-\infty}^{\infty} \sqrt{\frac{1}{T}} \left[\text{Fourier Transform of } R_{xx}(\tau) \right] d\omega$$



$$\begin{aligned}
 & S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{i\omega\tau} d\tau \\
 & R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{-i\omega\tau} d\omega \\
 & \text{PSD} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |A(t)|^2 dt \\
 & \text{RMS} = \sqrt{\frac{1}{T} \int_{-T/2}^{T/2} |A(t)|^2 dt}
 \end{aligned}$$

=

f_1

f_2

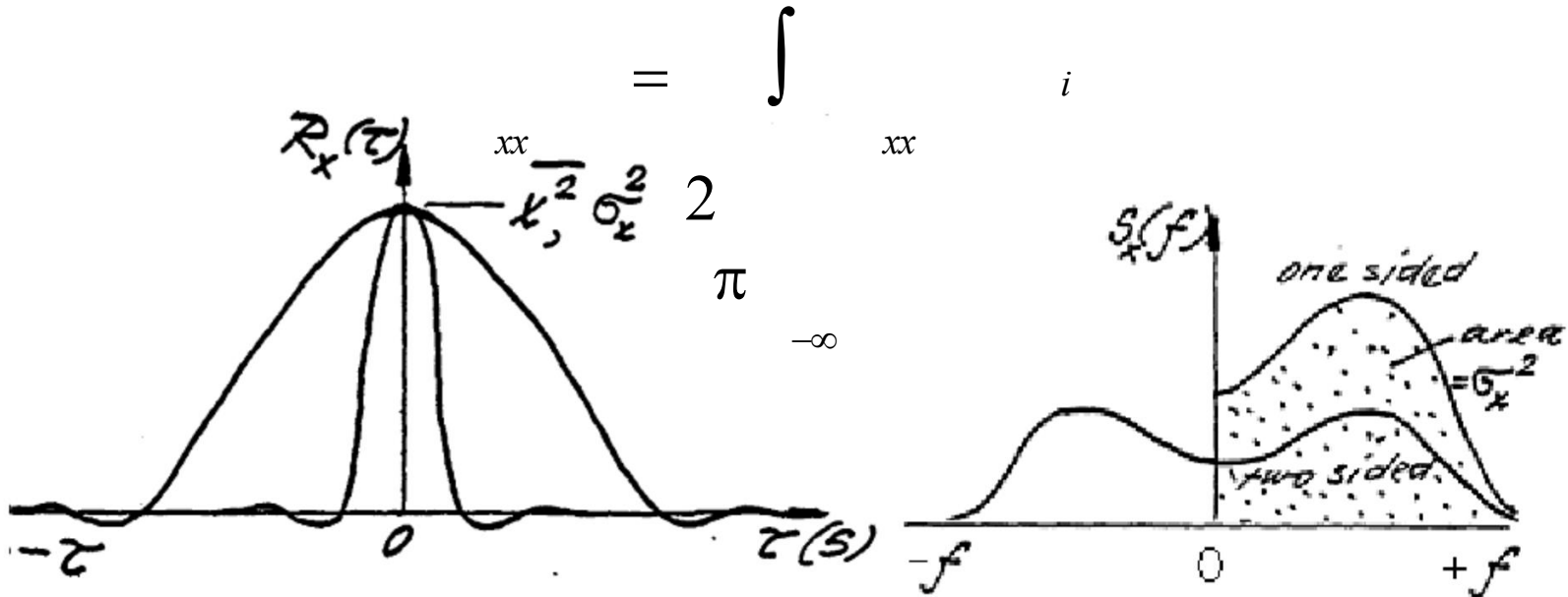
PSD计算公式

PSD Function Calculation

双边谱

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-i\omega\tau} d\tau$$

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-i\omega\tau} d\tau$$

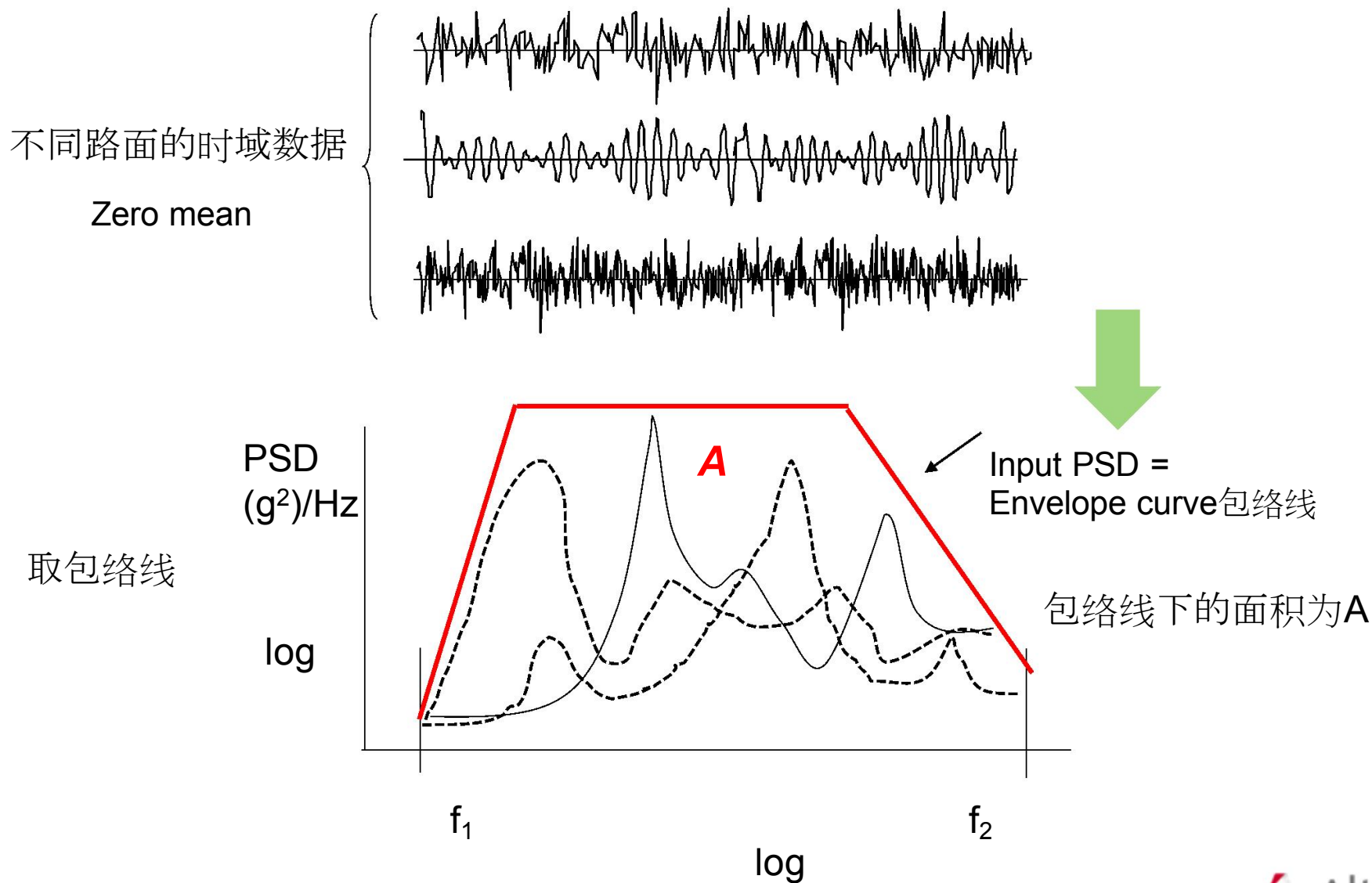


PSD使用单边谱

$$G_{xx}(\omega) = \begin{cases} 2S_{xx}(\omega) & \omega \geq 0 \\ 0 & \omega < 0 \end{cases}$$

如何生成PSD载荷

Convert time records to PSD



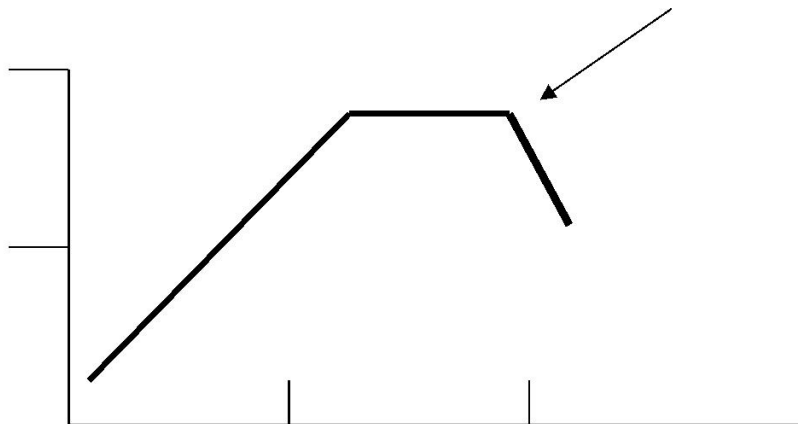
PSD输入谱的典型定义

Example 1 of random loading:

20-160 Hz	+5 dB/oct (+ slope)
160-800 Hz	0.06 g ² /Hz
800-2000 Hz	-6 dB/oct (- slope)

Overall

xxx g_{rms}



$$\begin{aligned}
 \text{二倍频} & \text{octave} & (f_2 / f_1) \\
 & = \log_2 & \\
 & \lg & (f_2 / f_1) \\
 & = \lg 2 & \\
 \text{十倍频} & \text{decade} & (f_2 / f_1) \\
 & = \lg & \\
 & & 0.30103
 \end{aligned}$$

