

Outline

- Review(FS,CTFT,DTFT)
- Relation between periodicity and continuity
- Comparison between CTFT and DTFT(properties)
- Discrete FT(DFT)
- Self-exercise

Review

Fourier Series:

Continuous time

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \quad \omega_0 = 2\pi / T$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt$$

Discrete time

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

Only for **periodic** signals!

Review

Fourier Transform

Continuous-Time Fourier Transform(CTFT)

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

Discrete-Time Fourier Transform(DTFT)

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$

Both periodic and non-periodic signals can have FT!!

THE FOURIER TRANSFORM FOR PERIODIC SIGNALS

To obtain the general result for periodic signals, let us first consider the Fourier transform of the complex exponential $x(t) = e^{j\omega_0 t}$

From the analysis equation,
$$X(j\omega) = \int_{-\infty}^{+\infty} e^{j\omega_0 t} e^{-j\omega t} dt = \frac{1}{j(\omega_0 - \omega)} e^{j(\omega_0 - \omega)t} \Bigg|_{-\infty}^{+\infty}$$

However, this integral **does not converge**.

Consider the Fourier transform pair $\delta(t)$ and 1.

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} 1 \cdot e^{j\omega t} d\omega$$

$$2\pi\delta(t) = \int_{-\infty}^{+\infty} 1 \cdot e^{j\omega t} d\omega$$

$$2\pi\delta(-\omega) = \int_{-\infty}^{+\infty} e^{-ja\omega} da$$

This equation says that the Fourier transform of unit dc is $2\pi\delta(-\omega)$.

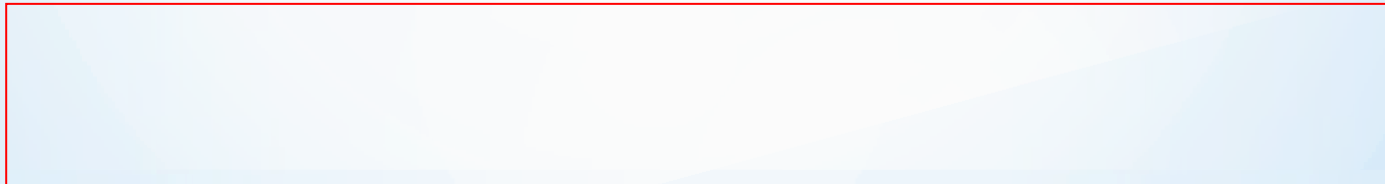
$$2\pi\delta(\omega_0 - \omega) = \int_{-\infty}^{+\infty} e^{-jt(\omega - \omega_0)} dt = \int_{-\infty}^{+\infty} e^{j\omega_0 t} e^{-j\omega t} dt$$

$$e^{j\omega_0 t} \longrightarrow 2\pi\delta(\omega_0 - \omega) \quad \text{Remember it!}$$

For an arbitrary periodic signal $x(t)$, First representing $x(t)$ with the Fourier series as

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

calculating the Fourier transform on both sides of this equation,



Relation between periodicity and continuity

FS

Time Domain:

CT: Continuous

Periodic

DT: Discrete

Periodic

Frequency Domain:

Aperiodic

Discrete

Periodic

Discrete

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

以上内容仅为本文档的试下载部分，为可阅读页数的一半内容。如要下载或阅读全文，请访问：<https://d.book118.com/198061116057006026>