Outline

- Review(FS,CTFT,DTFT)
- Relation between periodicity and continuity
- Comparison between CTFT and DTFT(properties)
- Discrete FT(DFT)
- Self-exercise

Review

Fourier Series:

Continuous time

$$x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t} \qquad \omega_0=2\pi/T$$

$$a_k=\frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt=\frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$$

Discrete time

$$x[n]=\sum_{k=\langle N\rangle}a_k e^{jk\omega_0 n}=\sum_{k=\langle N\rangle}a_k e^{jk(2\pi/N)n}$$

$$a_k=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk\omega_0 n}=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk(2\pi/N)n}$$

Only for periodic signals!

Review

Fourier Transform

Continuous-Time Fourier Transform(CTFT)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

Discrete-Time Fourier Transform(DTFT)

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

Both periodic and non-periodic signals can have FT!!

THE FOURIER TRANSFORM FOR PERIODIC SIGNALS

To obtain the general result for periodic signals, let us first consider the Fourier transform of the complex exponential $x(t) = e^{j\omega_0 t}$

From the analysis equation, $X(j\omega) = \int_{-\infty}^{+\infty} e^{j\omega_0 t} e^{-j\omega t} dt = \frac{1}{j(\omega_0 - \omega)} e^{j(\omega_0 - \omega)t} \Big|_{-\infty}^{+\infty}$

However, this integral does not converge.

Consider the Fourier transform pair $\delta(t)$ and 1.

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} 1 \cdot e^{j\omega t} d\omega$$

$$2\pi\delta(t) = \int_{-\infty}^{+\infty} 1 \cdot e^{j\omega t} d\omega$$

$$2\pi\delta(-\omega) = \int_{-\infty}^{+\infty} e^{-ja\omega} da$$

This equation says that the Fourier transform of unit *dc* is $2\pi\delta(-\omega)$.

$$2\pi\delta(\omega_0 - \omega) = \int_{-\infty}^{+\infty} e^{-jt(\omega-\omega_0)} dt = \int_{-\infty}^{+\infty} e^{j\omega_0 t} e^{-j\omega t} dt$$

$e^{j\omega_0 t}$ ⟹ $2\pi\delta(\omega_0 - \omega)$ Remember it!

For an arbitrary periodic signal $x(t)$, First representing $x(t)$ with the Fourier series as

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

calculating the Fourier transform on both sides of this equation,

Relation between periodicity and continuity

FS

Time Domain:	Frequency Domain:
CT: Continuous	Aperiodic
Periodic	Discrete
DT: Discrete	Periodic
Periodic	Discrete

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$$

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\omega_0 n} = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk(2\pi/N)n}$$