Summary

Traffic circle, as a special form of intersections in communication systems, has attracted more and more attentions. Various of traffic flow-control methods appear in our sights. Some involve yield or stop signs to give priority to certain flows; Some place common traffic lights; Others may have different designs. Hence, the optimality of the control methods has come into question.

Our objective is to set up a model to determine which flow-control strategy should come into use under different conditions. We propose three different strategies: Free Strategy, Sign Strategy and Traffic Light Strategy. The condition factors which may have influences on our choice vary in different forms, such as circle size, entrance flow, time of day, flow distribution and the number of approaches.

We construct a simple but useful Delay Model, based on a discrete split method that breaks a traffic circle into several sections. The basic unit is called a "processor" with a certain time-delay function. With this fundamental model, more detailed ones come up for the three different strategies. They measure the quality of three strategies in terms of average passing time between each pair of entrances.

Then we use computer programs to simulate vehicle flow in, around and out of a given circle in order to explore the effectiveness of each strategy. The simulation results tell us which strategy to choose under certain conditions. When traffic lights come into use, we use both theoretical equations and computer-based genetic algorithm to achieve the optimized value of the traffic lights' parameters.

The results of our model come up with a very important priciple of method-choosing: the dominant factor that would influence our choice is the intensity of traffic flow. Traffic flow intensity refers to the quotient of total vehicle flow over the area of the circulatory roadway. Our conclusion is to choose Free or Sign Strategy under light traffic flow intensity while Traffic Light Strategy under heavy traffic flow intensity. Other factors would have little influence on our choice. But those factors all together determine the parameters of traffic lights. A Case Study shows the detailed steps of our method and successfully achieves the goal of method-choosing for a real traffic circle in Washington D.C..

We validate our model in both sensitivity and robustness using theoretical analysis and repeated program tests. The results show that our model performs well. We also discuss the strengths and weaknesses of our model and propose some related future work. Finally, the Technical Summary is presented at the end of paper.

Contents

Traffic Circle Control Methods: Choosing Be Easy

1 Introduction

Generally, there are two styles of juctions all over the world, crossroads and traffic circles. Among them, the traffic circle, with its effect to slow down the cars and hence to decrease the number of accidents, was widely used nowadays. However, with the development of the economics and the gradually improvement of people's living standards, the number of cars are increasing consequently. Thus, the pressure of the juctions are also increasing larger and larger, which results in the fact that many traffic circles could not meet the continuously growing demanding so that the congestion in the traffic circles are becoming much more severe.[1]

In the past, to solve the problem, some stop signs were positioned either on each incoming road to decrease the velocity the vehicles flow into the circle, or in the circle at each incoming road to prevent the vehicles already in the traf- fic circle from intertweaving with the vehicles from the entry road. However, though these methods could to some extent improve the traffic condition of the circle, the improvement would be quite limited. Thus, more practical methods are in great need.

This paper simulates different patterns of control strategies, such as positioning some yield or stop signs or traffic lights on each entry road, and calculates the average time delay of all vehicles to determine the optimal method of improving the traffic condition of the circle.

1.1 Restatement of the Problem

Our objective of this paper is to use a model to determine how best to control the traffic flow in, around and out of the circle. To reach that goal, we should weaken and even eliminate the possibility of congestions in the circle. And for a vehicle already in the circle, there are two main factors leading to congestion and hence the time waste of travelling, its intertweaving with vehicles from the entry road and its being obstacled by other vehicles in front of it. Ultimately, congestion disrupts the smooth flow of vehicles and prolongs the time traveling in the circle.

To minimize the effects of congestion, some stop signs or traffic lights should be positioned in some place, such as on the incoming road or in the circle at the incoming road. Moreover, to better solve the problem, the phase of the traffic lights should be coordinated so that the total waiting time in the circle could be minimized. While it is not difficult to develop solutions that specify the process of one vehicle's in and out, such solutions for all vehicles are difficult to implement. Appropriate treatment of the problem calls for careful and balanced analyses that weigh the practicality of implementation, performance and variability.

1.2 Survey of Previous Research

1.2.1 About the Capacity of Roundabout

To achieve the goal mentioned above, we should first build a model to estimate the traffic capacity of the roundabout. After scanning some ebooks on the Internet and refering some books in the library, we find that there are various models to determine the capacity of the roundabout. However, these models are all based on one of the following three theories:

- **Interweave Theory**
  With Wardrop Equation being the representitive, Interweave Theory's guidance idea is that bottle-neck interweave capacity determine capacity of the whole roundabout.

- **Gap Acceptance Theory**
  Gap Acceptance Theory is a theory that using the maximum flow into the circle to reflect the capacity of the whole roundabout.

- **Regression Theory**
  This theory, through linear regression of all the parameters that could affect the capacity of the whole circle, reflects the flow into the circle, and hence the capacity of the roundabout.

Though all the three theories are widely used in the models analyzing the capacity of the whole roundabout, we just take some reference to them instead of using them directly because they are either much too complex for our model or based on a large amount of real statistic data observed from a given roundabout.

1.2.2 About the Control of Traffic Lights

There are various studies about the methods to control traffic lights, such as "The Urban Traffic Lamp Control System with Self-Organization" [5], which gives a control strategy of traffic lights based on their self-organization. However, all the references about the traffic lights control we have found are too complex. Thus, we would develop a relatively simple control strategy making use of our model.

2 Assumptions

The brief statement of this problem hides many layers of complication. To account for the physical limitations and uncertainties involved in the traffic circle systems, we make several assumptions as follows in order to restrict the model:

- The traffic circle system is an isolated system.
- No cyclists and pedestrians are in this system.
- There is no limitation on any exits.
- All vehicles are of the same size and free speed.
- There is no long-time congestion caused by some sudden accidents.
- No vehicles would travel in the circle for more than one loop.
- Traffic lights only consist of red and green lights while no yellow light exists.

3 Symbols

	The Variables	
Notation	**Description**	**Units**
D	the time delay of the processor	s
T_D	passing time in the circle system	s
T_L	period of each traffic light	s
T_g	the time for green light	s
T_r	the time for red light	s
ΔT_g	the phase difference of every two traffic light	s
$T_{i,j}$	average passing time for the ith and jth entry road	s
L	the length of a processor	m
$\bar{v}$	the average speed of vehicles in the processor	m/s
$\bar{v}i$	the average speed of vehicles in circulatory roadway	m/s v
f	the free-moving speed of vehicles in the processor	m/s
v_w	the average speed of vehicles in weaving part	m/s
Q_e	flow of vehicles from the entry road	pcu/h
Q_c	flow of vehicles from the circulatory roadway	pcu/h
Q_i	the traffic flow of the ith entry road	pcu/h
N	the current number of vehicles in the processor	
N_c	the capacity of the number of vehicles in the processor	
C_w	coefficient for weaving	
γ	the split of each traffic light	

4 Control Strategy

From analysis of the problem, we think out of various strategies to control the traffic flow in, around and out of the circle. Among them, we select three of them, which, in our point of view, would be more realistic:

- **Free Strategy**
  Free Strategy, a strategy that there is no sign or traffic light located in the traffic circle, means that all the vehicles, wether in the traffic circle or on the incoming roads, have the same priority to travel through the roundabout.

- **Sign Strategy**
  Sign Strategy is the strategy that some signs, such as stop signs or yield signs, would be positioned on the incoming roads. If a traffic circle adopts this strategy, there is a related rule that drivers of vehicles from the entry road should first judge whether there are any vehicles on the circulatory roadway and unless there are no vehicles they could not move into the circle.

- **Traffic Light Strategy**
  Traffic Light Strategy would position some traffic lights on each incoming road to force the vehicles to stop to alleviate the traffic pressure in the circle. The traffic lights would change its color, that is green or red, periodically; That means the green light would stay on for certain seconds and the red light would take its place, then after another certain seconds, the green light would stay on again.

5 Model Overview

Research into traffic flow control in a traffic circle has taken several approaches. However, analytic approaches to the problem, as mentioned above, could not be used in our model, due to their complexity or high reliance on large amount of statistics data. Thus, we would rely to some extent on the simulation of computers. For simulation allows for the complexity of the problem to be distributed, and hence presents a simpler formulation. We here present a simulative model which can be considered as a vehicle-based approach.

Our preliminary idea for the roundabout is just to treat it as a simple circle, with destinations (the outlets) at certain distances proportional to the real distances along the circle. And each vehicle moves along the circle until reaching its destinations. Moreover, every vehicle has a speed and may just be constrained by other vehicles in front of it. However, to regard the roundabout as a simple circle would be too complex for the computers to simulate the process of the vehicles, this idea is so difficult for us to realize that it needs to be improved.

We then build a basic model, Delay Model, to scheme the roundabout to simlify our simulation, as the circle is too complex for computers to simulate the process of the vehicles' in, around and out. The whole circle is broken into several discrete units and each of the units would contain the roads between two adjacent inlets while the inlets and outlets would be respectively inputs and outputs of the units. We call these units 'processors' since they determine the rate that an individual vehicle moves through the system. With this model, we could simulate the vehicles' process in a roundabout without any sign or traffic light.

The next extended model, Delay Model With Signs, take into account the influence of stop signs or yield signs. As we could easily get this extended model from our Delay Model by just alter some parameters, we would not describe this model as specific as our basic model. We also consider the influence of traffic lights based on the Delay Model and build another extended model, Delay Model With Traffic Lights. The function of the traffic lights would be to minimize the interweave between vehicles.

6 Details of the Model

In this section, we would first define the criterion for our model and then give a specific description of our two models, beginning with Delay Model, our basic model, and then continuing with its extensions.

6.1 Criterion For Our Model

Beforing describing our models, we should first define the criterion for our model. That is, we should define a factor that could represent degree of goodness of our method to control the traffic flow in, around and out of the roundabouts. Based on analysis of the problem, there would be various factors that could reflect the goodness of our method. Among them, we choose the average passing time of every part of the traffic circle as our criterion. That is, we would calculate the average time of a large number of vehicles that travels the same part of the roundabout, which means they take the same incoming road and have the same destination, and would take it as our criterion for our models.

6.2 Delay Model

The basic model is the foundation of our experimental results. We model the topology of the circle using a compartmental model. We divide the roundabout into compartments called 'processors'. Each processor represents the roads between two adjacent incoming roads. The processor's input would be the combination of the traffic flow that comes from last processor and the traffic flow that is about to come into the circle from the incoming road while its output would be the traffic flow that would divide into the vehicles that travel to the next processor and the vehicles that comes out of the circles through the outcoming road. By connecting several processors seriesly, the number of which would be determined by the number of incoming roads, we can model a variety of roundabout topologies. The topology given by Figure 1 would be an example, which gives the topology of a traffic circle that contains incoming roads.

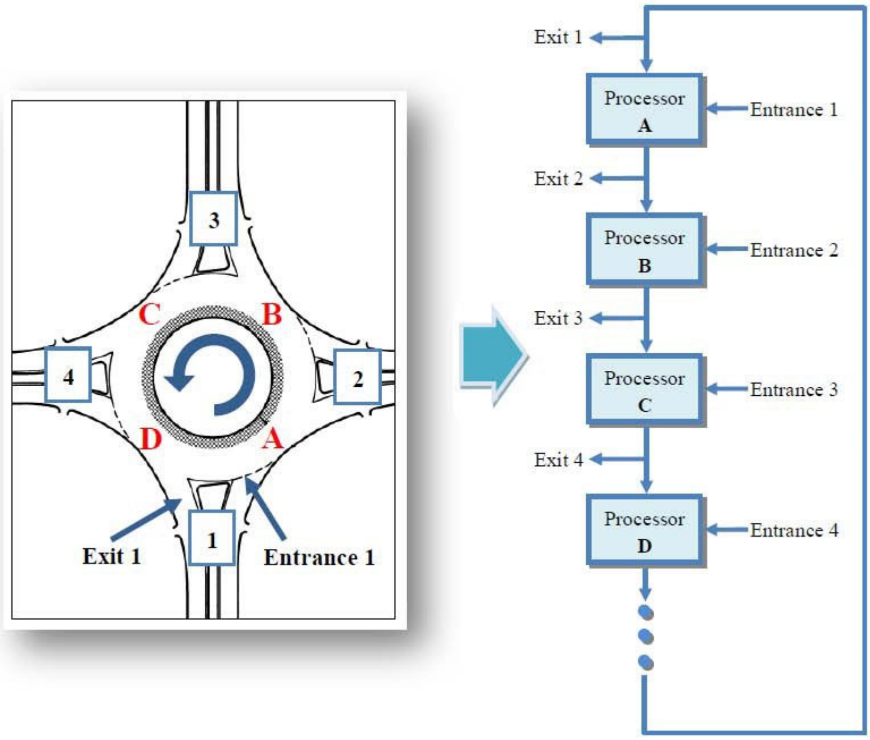

Fig 1: Model of Traffic Circle

Before giving the specific description of the processor and its working mechanism, we should first define the states of vehicles. During the simulation of the model, all the vehicles may be in one of the following three states:

- **Into the Processor.**
  Vehicles in this state are just going to enter the traffic circle on an incoming road.

- **Out to the Next Processor.**
  Vehicles in this state are just leaving one processor and going to enter the next processor, which means that these vehicles do not reach their destination, the outlet that they would get out of the circle.

- **Out of the Circle.**
  Vehicles in this state are just leaving one processor and going to get out of

the roundabout, which means they have already arrived at their destination.

With all the work above done, we now would give the detailed description of the processor.

Since processors have already broken the traffic circle into several pieces, we would consequently analyze the processor separately to reflect the whole process of the traffic circle. As the criterion for our model would be the average passing time on each part of the roundabout, we should focus our study on the average time delay of each processor.

As mentioned above, each processor would represent the part between two adjacent incoming roads. Thus, we conclude that there are mainly two factors that could influence the average time delay of the processor, which is the time delay caused by relatively large intensity of vehicles in the circle and the time delay caused by the weaving of the two traffic flow at each incoming road. Meantime, we would transfer the average time delay of the processor to the average speed of vehicles in the processor, as to build the relationship between the time delay and these two factors are much more difficult for us than to build the relationship between the average speed of vehicles in the processor and these two factors. We make use of the following equation to transfer the time delay to the average speed:

$$D = \frac{L}{\overline{v}} \tag{1}$$

where D represents the time delay of the processor, L represents the length of processor, and $\overline{v}$ represents the average speed of vehicles in the processor.

Then, we would study these two factors separately and then we would combine the two factors and give the final formulation of the average time delay of our model.

Vehicle Intensity

First, we should determine the time delay due to the relatively large intensity of the vehicles. As is known to all, the speed would decrease with the increasement of the intensity, and hence we estimate that the speed would be on negative growth as the intensity increases. We find that in 1961 Underwood proposed an exponential model of average speed versus intensity as shown in the following equation, which to some extent prove the correctness of our estimation:[2]

$$\overline{v}_i = v_f e^{-\frac{K}{K_c}} \tag{2}$$

where $\overline{v}_i$ means the average speed of vehicles in the processor except for the weaving part, v_f means the speed of vehicles when they could move freely in the processor, K means the current intensity of the vehicles, and K_c means the capacity of intensity, the intensity of vehicles that could stop the vehicles from moving forward.

Though this equation could easily provide us the relationship between the average speed of vehicles and the intensity of them, we would not directly take advantages of it for the reason that we could not simulate the intensity directly in the computer. Thus, we would find some other variable that could replace the variable of intensity. Finally, we find the variable of the number of the vehicles in the processor to reflect the intensity because the intensity would be the unit-area number of vehicles and hence using the number of vehicles would not affect the quotient of K and K_c. Meanwhile, we could easily get the number of vehicles in the processor for we could just let the computer to count it. As a result, we give the following equation:

$$\overline{v}_i = v_f e^{-\frac{N}{N_c}} \tag{3}$$

where N and N_c represent the current number of vehicles in the processor and the capacity of the number, the number of vehicles that could prevent vehicles from moving forward. Utilizing this equation, we could easily model the relationship of average speed of vehicles versus the number of them.

Weaving

Next, we would determine the second factor, weaving, that could result in the time delay and hence the decrease of vehicle speed in the processor. So we should build a model for the relationship between the speed of vehicles and some variable that could represent the degree of weaving. Thus, we should first define a variable that could reflect the weaving. Finally, we define P, the proportion of the number of vehicles that would intertwine with each other at each incoming road and the total number of vehicles at the same place. Then we fortunately find that Wardrop has ever proposed a equation to model the relationship of Q_m, the maximum flow of vehicles that could get through the weaving part per hour, versus some variables of the roundabouts, including P. The equation is shown as follows:[3]

$$Q_m = \frac{354(1 + \frac{e}{W})(1 - \frac{P}{3})}{1 + \frac{W}{l}} \tag{4}$$

where l means the length of the weaving part, W means the width of the weaving part, e means the arithmetic mean of entry width and circulatory roadway width, and the meaning of P and Q_m have already been defined.

However, as we can see, the Wardrop equation is too complex for our model for it would rely on so many parameters of the roundabout. Thus, we should first simplify the Wardrop equation. From analysis, we find that as e, W, l are all the parameters reflecting the roundabout and hence would be constant to a specific roundabout, so we could easily conclude that Q_m would change linearly when P changes, or rather Q_m is negatively proportional to P. Meanwhile, making use of the knowledge we have learned from physics class, we could easily find the relationship between Q_m and $\overline{v}_w$, the average speed of vehicles in weaving part, as shown in the following equation:

$$Q_m = \frac{\overline{v}_w N_l}{d} \tag{5}$$

where N_l means the number of vehicles moving side by side, and d means the sum of average length of vehicles and the average distance between two vehicles. From this equation, we find that $\overline{v}_w$ is proportional to Q_m, which leads to the conclusion that $\overline{v}_w$ would change linearly and negatively with the changes of P due to the fact that Q_m is negatively proportional to P.

Based on the fact that Q_m is negatively proportional to P, to simplify our model and combine the two factors, we take the method that we could reflect the influence of P to the final average speed $\overline{v}$ by simply mutiplying a coefficient to formula of $\overline{v}$. We define this coefficient for weaving as C_w and then we would develop the method to get this coefficient.

According to Wardrop, we could make use of Q_m to reflect P, which directly gives us the idea that we could also use Q to determine the coefficient C_w. To get that coefficient, we first define two variables, Q_e and Q_c, to represent the flow from the entry and the circulatory roadway respectively. Then we would develop the relationship between C_w and the two flow variables Q_e and Q_c. As we can see, when the sum of the two flows are constant, we could estimate the weaving would be the most severe when the two flow of vehicles are about the same size while the weaving would decrease to zero when either of the two flows is zero. And hence, the average speed would reach minimum when the two flows are roughly the same while it would reach the maximum value when there is only one flow, either from the entry or from the circulatory roadway. Thus, we could represent the coefficient C_w using quotient of the margin of the flows and the sum of them, as shown in the following equation:

$$C_w = \frac{|Q_e - Q_c|}{Q_e + Q_c} \tag{6}$$

Meanwhile the Wardrop equation tells us that the value of Q_m for $P = 1$, which means the two flows weav totally and hence Q_e equals to Q_c, would be the two thirds of the value of Q_m for $P = 0$, which means there is no weaving at all and hence either of Q_c and Q_e is zero. Thus, we should make some adjustment to equation (6). That is, we should make the value of C_w for $Q_c = Q_e$ be two thirds of the value of C_w when one of Q_c or Q_e would be zero. All we have to do is to do the linear transformation for the equation (6) and get the final equation for C_w, as shown in the following:

$$C_w = \frac{1}{3}\frac{|Q_e - Q_c|}{Q_e + Q_c} + \frac{2}{3} \tag{7}$$

Combining the two factors, we could get the final equation for the average speed in the processor, as shown in the following equation:

$$\overline{v} = \overline{v}_i C_w \tag{8}$$