可压缩流动：相关的预备知识

7.1 引言

空气动力学

A. 连续流动
Continuum Flow

B. 低密度和自由分子流
Low-density and free-molecule flows

C.粘性流动
Viscous Flow

D. 无粘流动
Inviscid Flow

E. 不可压缩流动 Incompressible Flow

F. 可压缩流动 Compressible Flow

G. 亚音速流
Subsonic

H.跨音速流
Transonic

I.超音速流
Supersonic

J.高超音速流
Hypersonic

可压缩流动的基本特征：

- ＊The pivotal aspect of high -speed flow is that the density is a variable——密度是变量.

- ＊Another pivotal aspect of high-speed compressible flow is energy. A high-speed flow is a high energy flow.——是一个高能量的流动.

- ＊Energy transformation and temperature changes are important considerations.——必须考虑能量转换与温度变化.

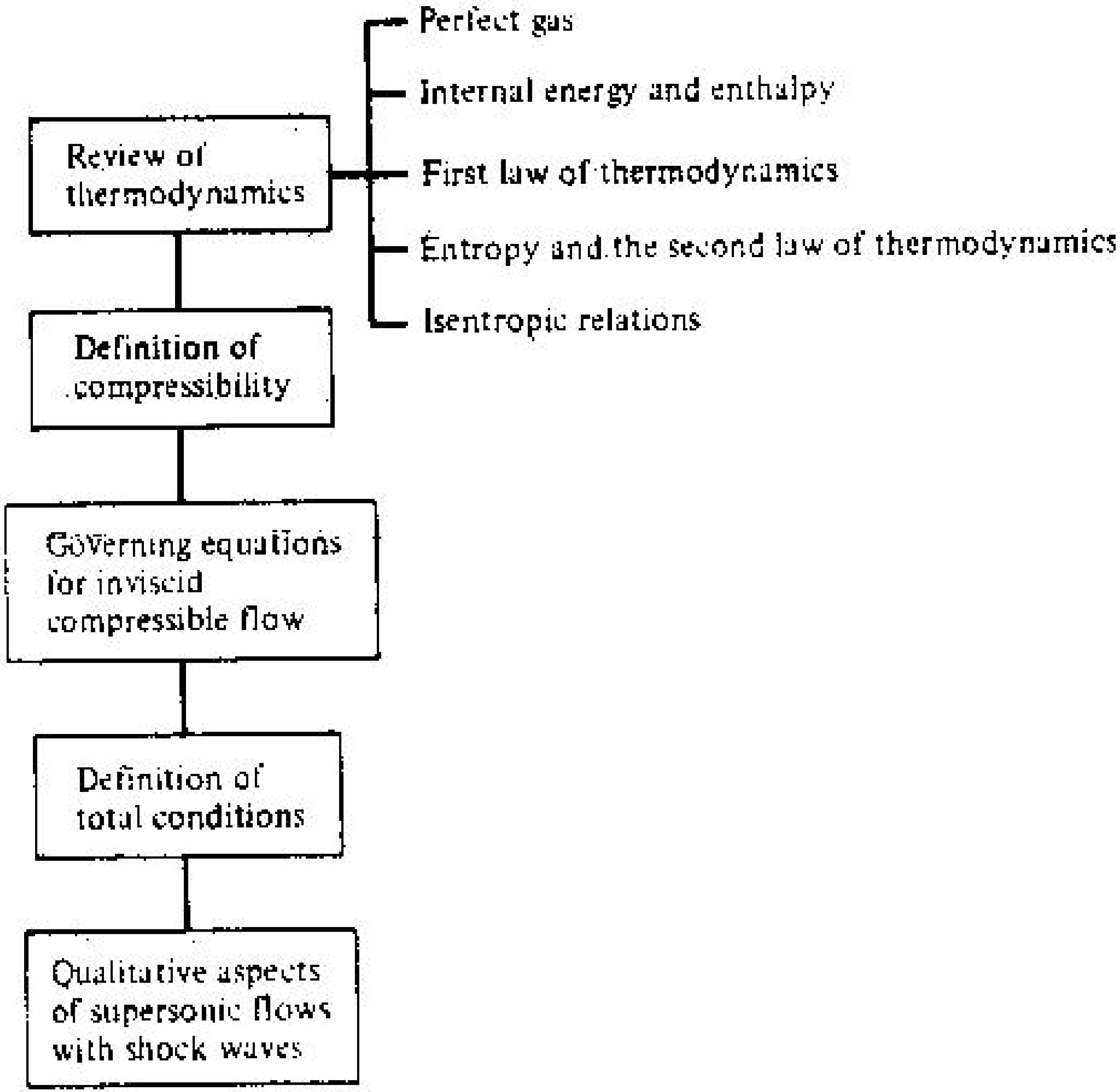


FIGURE 7.1
Road map for Chap. 7.

第七章路线图

- 热力学复习
  - 完全气体
  - 内能和焓
  - 热力学第一定律
  - 熵及热力学第二定律
  - 等熵关系式
- 压缩性定义
- 无粘可压缩流动的控制方程
- 总条件的定义
- 有激波的超音速流动的定性了解

7 .2 A BRIEF REVIEW OF THERMODYNAMICS

(热力学简要复习）

7.2.1 Perfect gas (完全气体)

定义：

A gas in which the intermolecular forces are neglected is defined as a *perfect gas*. (忽略分子间作用力的气体定义为完全气体）

完全气体满足*状态方程：*　*（equation of state)*

$$\boxed{p = \rho RT} \tag{7.1}$$

$$\boxed{pv = RT} \tag{7.2}$$

R为气体常数（specific gas constant) R=287J/(kg·K)

7.2.2 Internal Energy and Enthalpy（内能和焓）

The energy of a given molecule is the sum of its *translational, rotational, vibrational, and electronic* energies. （一个给定分子的能量是其*平动动能*、*转动动能*、*振动动能和电子能*的总和。）

对于由大量分子组成的给定体积的气体，所有分子所具有的能量的总和称为气体的内能。

单位质量气体的内能称为气体的比内能。（The internal energy per unit mass of gas is defined as the specific internal energy.)

用e表示.

与比内能e相联系的另一个量为比焓h，定义为：

$$\boxed{h = e + pv = e + \frac{p}{\rho}} \tag{7.3}$$

对于完全气体，*e*和*h*都只是温度的函数：

$$e = e(T) \tag{7.4a}$$

$$h = h(T) \tag{7.4b}$$

$$de = c_v dT \qquad (7.5a)$$

$$dh = c_p dT \qquad (7.5b)$$

c_v ——定容比热

Specific heats at constant volume

c_p —— 定压比热

Specific heat at constant pressure

$T<1000\ K$，气体为*量热完全气体*（*calorically perfect gas*),

c_v、 c_p 为常数

$$e = c_v T \tag{7.6a}$$

$$h = c_p T \tag{7.6b}$$

注意：e和h均为热状态变量（thermodynamic state variables),

它们只依赖与气体的状态而与过程无关. （they depend only on the state of the gas and are independent of any process)

$$e = c_v T \qquad (7.6a)$$

$$h = c_p T \qquad (7.6b)$$

(7.6b)-(7.6a):

$$c_p - c_v = R \qquad (7.7)$$

定义比热比 $\boxed{\gamma = \dfrac{c_p}{c_v}}$

$$1 - \frac{c_v}{c_p} = \frac{R}{c_p} \qquad (7.8)$$

$$1 - \frac{1}{\gamma} = \frac{R}{c_p}$$

$$\boxed{c_p = \frac{\gamma R}{\gamma - 1}} \qquad (7.9)$$

$$\boxed{c_v = \frac{R}{\gamma - 1}} \qquad (7.10)$$

7.2.3 First Law of Thermodynamics（热力学第一定律）

系统－system　　环境-surroundings　　边界-boundary

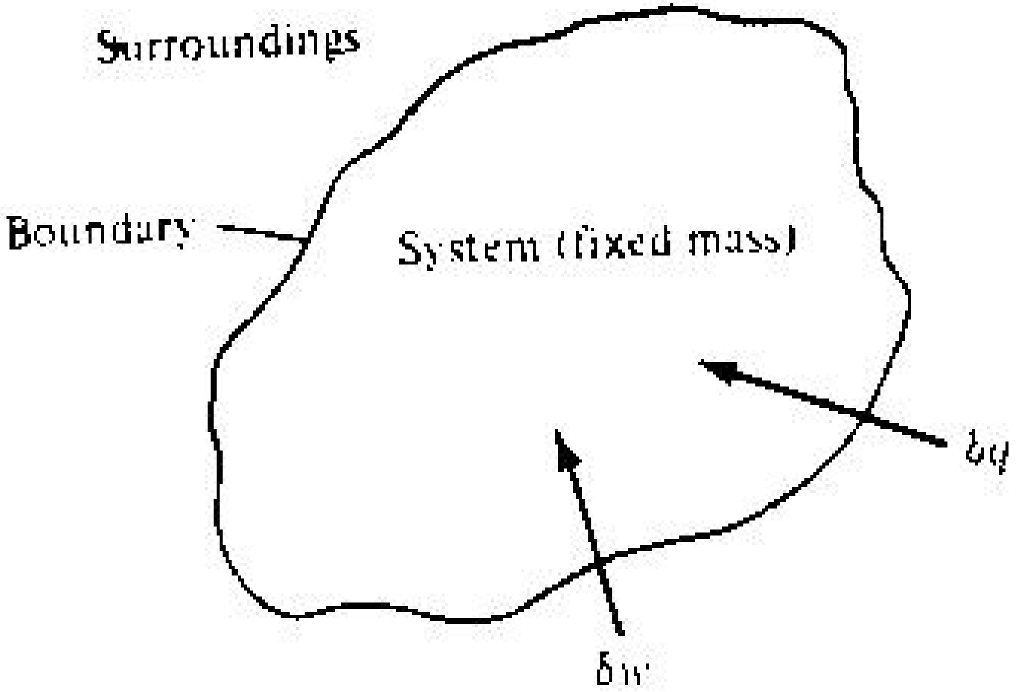


FIGURE 7.2
Thermodynamic system.

$\boxed{\delta q}$ ：外界通过边界加于系统的热增量

$\boxed{\delta w}$ ：外界对系统的做的功

Assume this system is stationary: (假定系统为静止的)

$$\boxed{\delta q + \delta w = de} \qquad \text{（7.11）}$$

要使系统产生内能增量de，有无数多种对系统做功和给系统加热的方式（过程）． --For a given *de*, there are in general an infinite number of different ways (processes) by which heat can be added and work done on the system.

我们主要考虑三个常见过程：

1． Adiabatic process　(绝热过程）

A process during which there is no heat transfer.

在过程中没有热传导。

2． Reversible process (可逆过程）

No dissipative phenomena occur. 没有耗散现象发生.

3． Isentropic process（等熵过程）

绝热、可逆。

对于可逆过程，有： $\boxed{\delta w = -pdv}$

$$\boxed{\delta q - pdv = de} \qquad (7.12)$$

热力学第一定律小结:

热力学第一定律就是能量守恒原理的一种表达，说明能量既不能产生也不能消失，只能从一种形式转换为另一种形式。

系统与外界的能量交换可以通过加热(heat)、做功（work)、质量流（mass flow)实现。对于我们研究的封闭系统（closed system) 我们不考虑质量流。

Heat is defined as the form of energy that is transferred between a system and its surroundings by virtue of a temperature difference. (热量是一种能量，它是由于系统与外界的温度差而引起。）

heat transfer：Temperature is the driving force for heat transfer. The larger the temperature difference, the higher is the rate of heat transfer.(温度差是热传导的驱动力)

热传递的形式：Conduction(传导）, Convection（对流）, Radiation（辐射）.

Work: Work is the energy transfer associated with a force acting through a distance. （功是力与力的作用距离相联系的能量形式）

加热和做功的共同特征：

1、都通过系统边界作用于系统。

2、系统拥有能量，而不是热和功。

3、热和功都和过程相联系，而不是状态。

4、它们都是路径的函数。（即它们的大小依赖于过程和状态）。

*可逆过程的理解：

可逆过程的定义：对于一个给定的系统和它的外界，如果它们完全可以由终止状态回到他们的初始状态，这样的过程就是可逆过程。（If a process can be reversed so that the system of particles and its surroundings are restored so that the system in all respects to their initial conditions, the exchange process is said to be reversible.)

热力学研究的是平衡的系统。处于平衡状态的系统内没有力的不平衡，没有温度的梯度，没有质量的扩散。

可逆过程举例：

1、无摩擦的摆锤　2、气体的准平衡膨胀与压缩

* 不可逆过程：

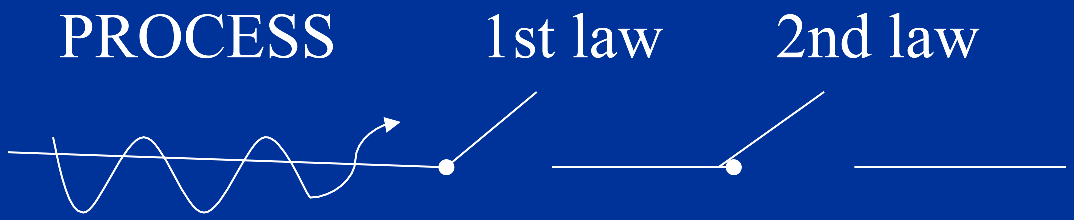
举例: 不可逆现象

7.2.4 Entropy and the Second Law of Thermodynamics

(熵及热力学第二定律)

热力学第一定律解决了能量在一个过程中的守恒问题。热力学第二定律则要解决过程会向哪个方向进行的问题，表明能量既有“质”（quality)也有“量”（quantity)。

A process cannot take place unless it satisfies both the first and second law. （一个过程只能向同时满足热力学第一定律和第二定律的方向进行。）



PROCESS

1st law

2nd law

The second law leads to the definition of a new property called ***entropy***. （热力学第二定律引入了一个新性质，*熵*）

Let us define a new state variable, the ***entropy s***, as follows:

$$\boxed{ds = \frac{\delta q_{rev}}{T}}$$

（7.13）

δq_{rev}

where δq_{rev} is an incremental amount of heat added *reversibly* to the system and *T* is the system temperature. （为可逆地加于系统的热增量， *T* 为系统的温度。）

The quanti δq_{rev} is just an artifice(人为的量） which can always be assigned to relate the initial and end points of an irrevers δq process, where the ***actual amount of heat added is*** ***（实际的加热量）***

Hence:

$$ds = \frac{\delta q}{T} + ds_{irrev} \tag{7.14}$$

ds_{irrev} is the generation of entropy due to the irreversible, dissipative phenomena

These dissipative phenomena *always increase the e* $ds_{irrev} \geq 0$ *s:*

(7.15)

The equals sign denotes a *reversible process*

（等号表示

Therefore:

$$ds \geq \frac{\delta q}{T}$$

(7.16)

$$\delta q = 0$$

If the process is adiabatic,

$$ds \geq 0$$

then

This is the *second law of thermodynamics* that tells in *what direction* a process will take place:

（热力学第二定律指明过程进行的方向）

The entropy of the system and its surroundings always increases or, at best, stays the same.

（系统和其环境的熵总是增加的或不变的）

In summary, the concept of entropy in combination with the second law allows us to predict the *direction* that nature takes.

（熵与热力学第二定律相结合，使我们能预计过程进行的*方向*）

Practical calculation of entropy(熵的实际计算)

If head is added *reversibly,* then:

$$\delta q = Tds$$

and

$$\boxed{Tds = de + pdv}$$

(7.18)

From the definition of enthalpy

$$h=e+pv$$

$$\rightarrow \quad dh=de+pdv+vdp$$

(7.19)

and, by combining these equations:

$$\boxed{Tds = dh - vdp} \qquad (7.20)$$

For a perfect gas, $de = c_v dT$

and $dh = c_p dT$

Substituting in precedent relations, one obtain

$\rightarrow$ $$ds = c_v \frac{dT}{T} + \frac{p\,dv}{T}$$

(7.21) $$ds = c_p \frac{dT}{T} - \frac{v\,dp}{T}$$

With the equation of stat $pv = RT$

or $$\frac{v}{T} = \frac{R}{p}$$

we get a convenient form for integration

$$\rightarrow \qquad \boxed{ds = c_p \frac{dT}{T} - R\frac{dp}{p}}$$

Consider a thermodynamic process with initial state 1 and end state 2, the integration leads to:

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \int_{p_1}^{p_2} R \frac{dp}{p}$$

For a *calorically perfect gas*, both *R and c_p* are constant; hence:

$$\boxed{s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1}}$$

(7.25)

In a similar fashion, we obtain:

$$\boxed{s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}}$$

(7.26)

Note that *s* is a function of two thermodynamic state variables; （熵是两个热力学状态参数的函数）e.g.

$$s = s(v, T)$$

or

$$s = s(p, T)$$

7.2.5 Isentropic relations(等熵关系式)

For an ***adiabatic proce*** $\delta q = 0$

And for an ***isentropic proce*** $ds_{irrev} = 0$

Hence:

$$0 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

or

$$\ln\frac{p_2}{p_1} = \frac{c_p}{R}\ln\frac{T_2}{T_1}$$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{c_{p/R}}} \qquad (7.27)$$

$$\boxed{\frac{c_p}{R} = \frac{\gamma}{\gamma - 1}}$$

$$\boxed{\rightarrow \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}} \qquad (7.28)$$

In a similar fashion

$$0 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$\ln\frac{T_2}{T_1} = -\frac{R}{c_v}\ln\frac{v_2}{v_1}$$

$$\frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{-c_v/R} \qquad (7.29)$$

$$\frac{c_v}{R} = \frac{1}{\gamma - 1}$$

$$\boxed{\frac{v_2}{v_1} = \left(\frac{T_2}{T_1}\right)^{-1/(\gamma-1)}} \qquad (7.30)$$

$$\boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)}} \qquad (7.31)$$

$$\boxed{\frac{p_2}{p_1}=\left(\frac{\rho_2}{\rho_1}\right)^{\gamma}=\left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}} \qquad (7.32)$$

Equation (7.32) is very important ; it relates pressure, density , and temperature for an isentropic process.

方程（7.32)非常重要，它将等熵过程中的压强、密度、温度联系起来

Why is these equation so important?

An isentropic process seems so restrictive, requiring both *adiabatic* and *reversible conditions?*

- Because a large number of practical compressible flows can be assumed to be isentropic（绝大多数实际流动问题可以被假设为等熵的）：
- valid for a *calorically perfect gas* ,(量热完全气体)
- and the flow outside the boundary layer which is

Concept of barotropic: 正压流的概念

满足密度是压力的唯一函数的流动,称为正压流.

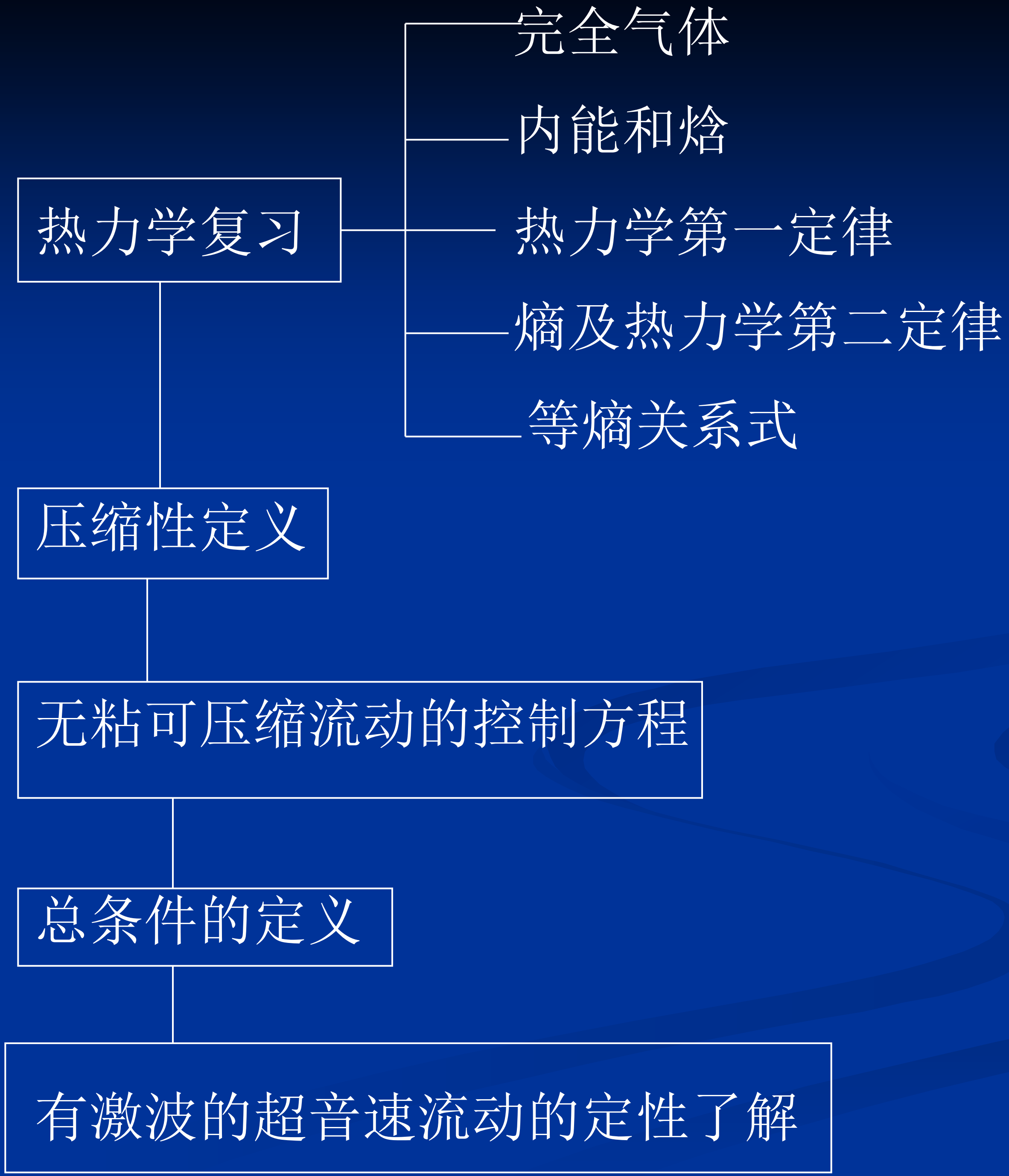
即 $\rho = \rho(p)$

$$dp = -\rho V dV \quad (3.12)$$

1.不可压流 $\rho = const.$ $p_1 + \frac{1}{2}\rho {V_1}^2 = p_2 + \frac{1}{2}\rho {V_2}^2$

2.等熵流 $\frac{p}{\rho^\gamma} = const.$ $\int_1^2 \frac{dp}{\rho} = -\int_1^2 V dV$

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{{V_1}^2}{2} = \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{{V_2}^2}{2}$$



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7.3. DEFINITION OF COMPRESSIBILITY

(压缩性定义)

All real substances are *compressible* to some greater or lesser extend.

When you squeeze or press on them, their density will change. This is particularly true of *gases*.

(所有的真实物质都是可压缩的,当我们压挤它们时,它们的密度会发生变化,对于气体尤其是这样.)

The amount by which a substance can be compressed is given by a specific property of the substance called the *compressibilty* , defined below.

物质可被压缩的大小程度称为物质的压缩性.

Consider a small element of fluid of v volume .

The pressure exerted on the sides of the element is *p*. If the ***pressure is increased*** by an dv finitesimal amount ***dp***, the ***volume will change by a negative amount***

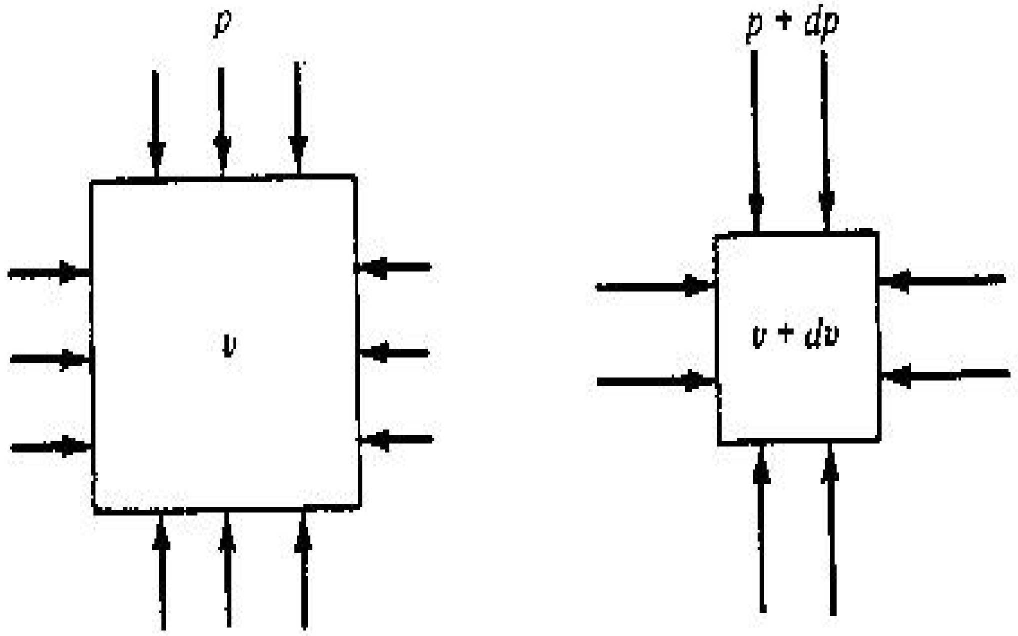

FIGURE 7.3
Definition of compressibility.

By definition, the compressibility is given by:

$$\tau = -\frac{1}{v}\frac{dv}{dp} \tag{7.33}$$

as $v = \frac{1}{\rho}$

$$\tau = \frac{1}{\rho}\frac{d\rho}{dp} \tag{7.36}$$

Physically, the compressibility is a fractional change in volume of the fluid element per unit change in pressure.(从物理上讲,压缩性就是每单位压强变化引起的流体微元单位体积内的体积变化)

If the temperature of the fluid element is τeld constant, then is identified as the ***isothermal* *compress*** ***压缩性)***

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

(7.34)

If the p place ***isentrop*** (等熵压缩性)

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s$$