“Teach A Level Maths”

Vol. 2: A2 Core Modules

29: Volumes of Revolution

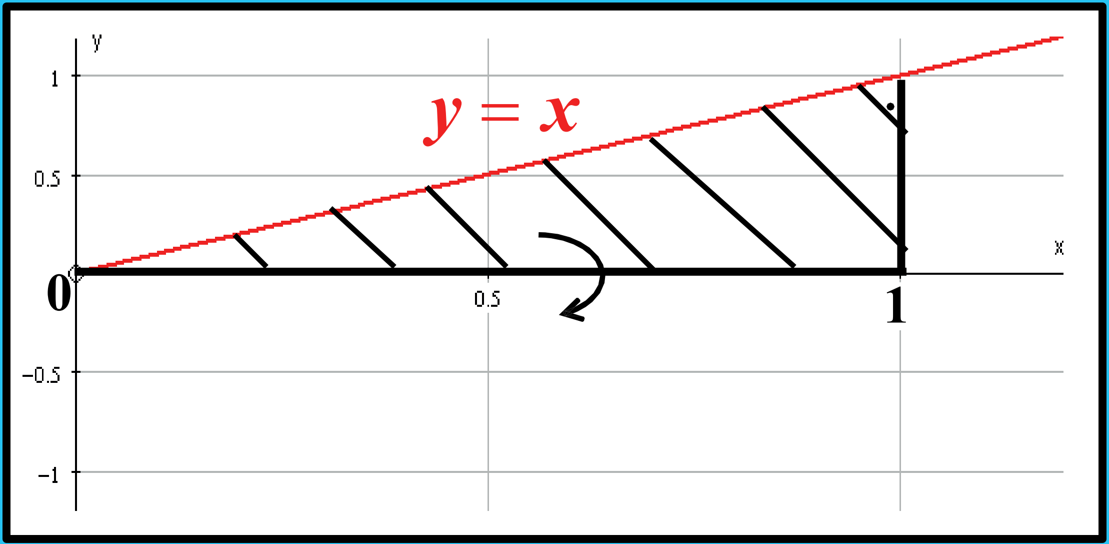
Module C3	**Module C4**
AQA	Edexcel
OCR	MEI/OCR

We'll first look at the area between the lines

$y = x$, . . .

$x = 1$, . . .

and the x-axis.



Can you see what shape you will get if you rotate the area through $360°$ about the x-axis?

Ans: A cone (lying on its side)

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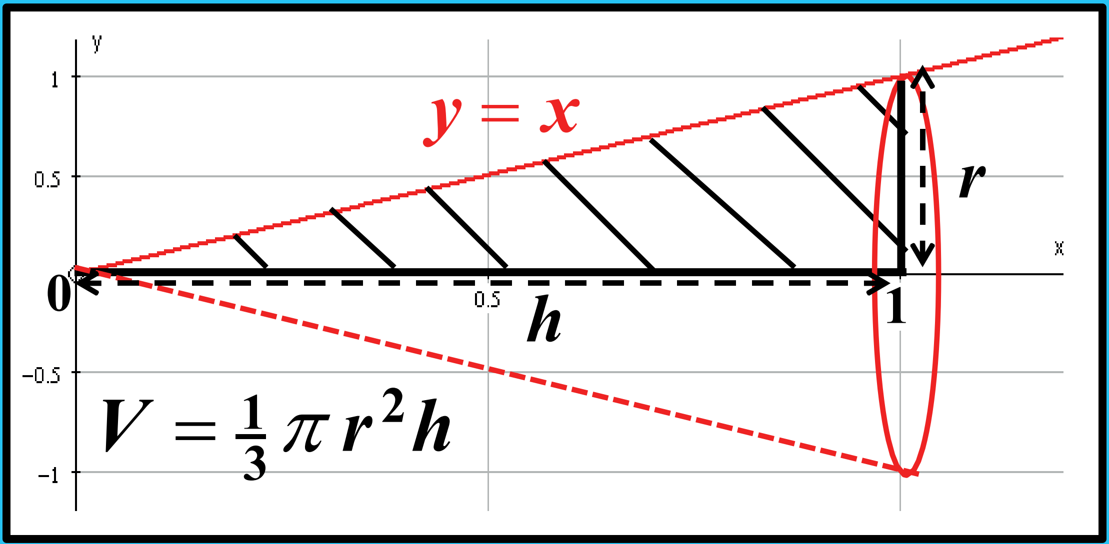
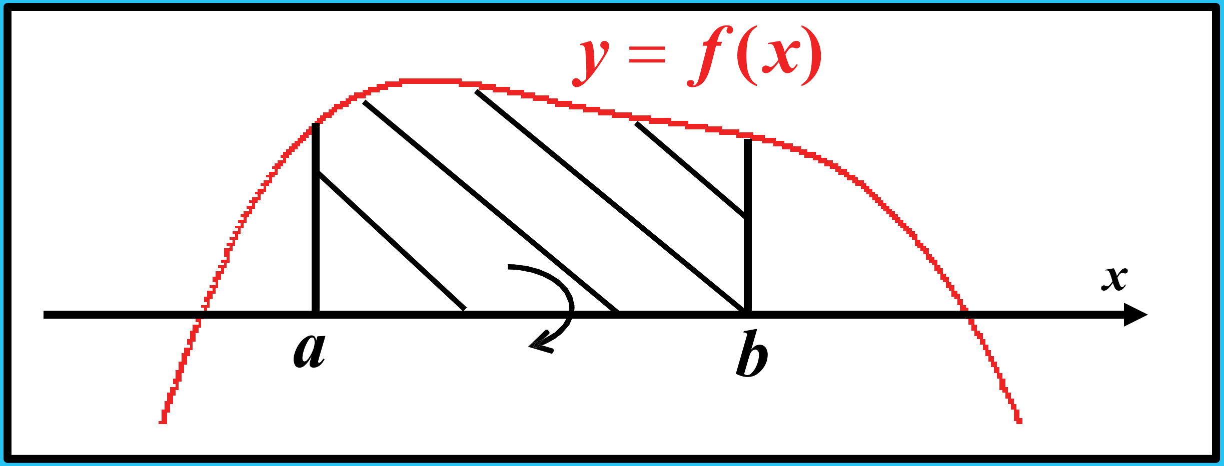


For this cone, $r = 1,\ h = 1 \quad \Rightarrow \quad V = \frac{1}{3}\pi$

The formula for the volume found by rotating <u>any</u> area about the x-axis is

$$V = \pi \int_a^b y^2 \, dx$$



where $y = f(x)$ is the curve forming the upper edge of the area being rotated.

a and b are the x-coordinates at the left- and right-hand edges of the area.

We leave the answers in terms of π

$$V = \pi \int_a^b y^2 \, dx$$

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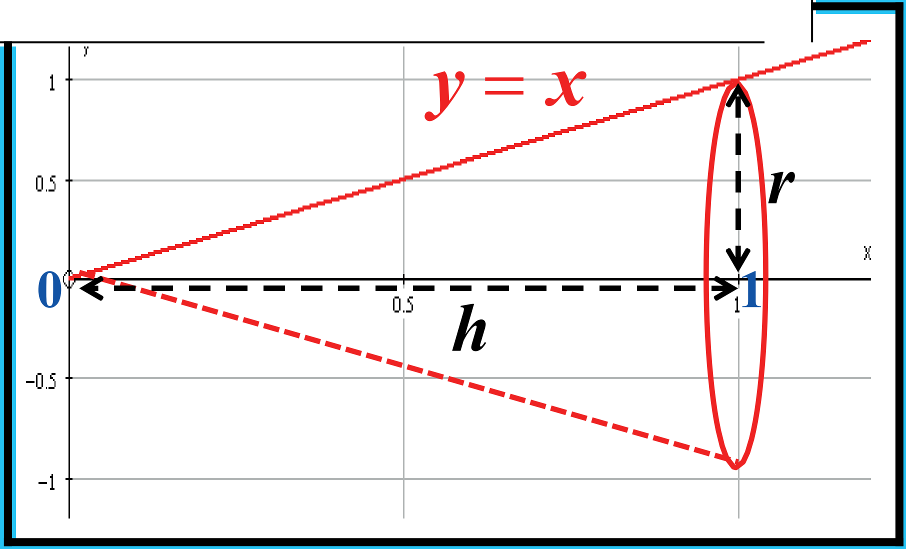
We must substitute for y using $y = f(x)$ before we integrate.

$$= \pi \left[\frac{x^3}{3} \right]_0^1$$

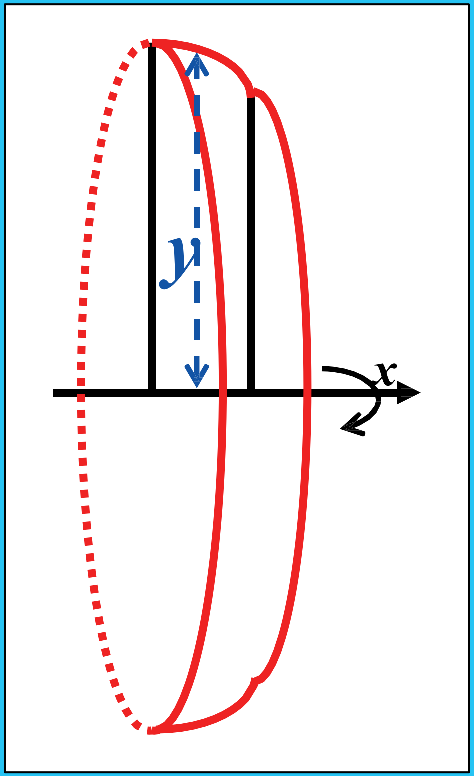
$$= \pi \left(\frac{1}{3} - 0 \right)$$

$$= \frac{1}{3} \pi$$



I'll outline the proof of the formula for you.

The formula can be proved by splitting the area into narrow strips . . . which are rotated about the x-axis.



Each tiny piece is approximately a cylinder (think of a penny on its side).

Each piece, or element, has a volume $\approx \pi r^2 h = \pi y^2$

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