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1 OVERVIEW

Default risk is the uncertainty surrounding a firm's ability to service its debts and obligations. Prior to default, there is no way to discriminate unambiguously between firms that will default and those that will not. At best we can only make probabilistic assessments of the likelihood of default. As a result, firms generally pay a spread over the default-free rate of interest that is proportional to their default probability to compensate lenders for this uncertainty.

Default is a deceptively rare event. The typical firm has a default probability of around 2% in any year. However, there is considerable variation in default probabilities across firms. For example, the odds of a firm with a AAA rating defaulting are only about 2 in 10,000 per annum. A single A-rated firm has odds of around 10 in 10,000 per annum, five times higher than a AAA. At the bottom of the rating scale, a CCC-rated firm's odds of defaulting are 4 in 100 (4%), 200 times the odds of a AAA-rated firm.

The loss suffered by a lender or counterparty in the event of default is usually significant and is determined largely by the details of the particular contract or obligation. For example, typical loss rates in the event of default for senior secured bonds, subordinated bonds and zero coupon bonds are 49%, 68%, and 81%, respectively.

Cross default clauses in debt contracts usually ensure that the default probabilities for each of the classes of debt for a firm are the same. That is, the default probability of the firm determines the default probability for all of the firm's debt or counterparty obligations. However, the loss in the event of default for each of the classes of obligations can vary widely depending on their nature (security, collateral, seniority, etc.).

Although in general a poor investment strategy, it is possible to be rewarded for taking on large concentrations of risk in equities because these concentrations at times produce large returns. However, overwhelming evidence of the ineffectiveness of this stock-picking strategy has been available since the early 1970s and, as a result, the majority of equity investments are managed in diversified portfolios. Unlike equities, debt has no upside potential and thus the case for managing default risk in well-diversified portfolios is even more compelling. The limited upside potential of debt spreads means that there are no possible circumstances under which an investor or counterparty can be rewarded for taking on concentrations of default risk. Like other rare events with high costs, default risk can only be effectively managed in a portfolio.

In addition to knowing the default probability and loss given default, the portfolio management of default risk requires the measurement of default correlations. Correlations measure the degree to which the default risks of the various borrowers and counterparties in the portfolio are related. The elements of credit risk can therefore be grouped as follows:

Standalone Risk

Default probability—the probability that the counterparty or borrower will fail to service obligations.

Loss given default—the extent of the loss incurred in the event the borrower or counterparty defaults.

Migration risk—the probability and value impact of changes in default probability.

Portfolio Risk

Default correlations—the degree to which the default risks of the borrowers and counterparties in the portfolio are related.

Exposure—the size, or proportion, of the portfolio exposed to the default risk of each counterparty and borrower.

While each of these items is critical to the management of credit portfolios, none are more important or more difficult to determine, than the default probability. The remainder of this paper will focus on the determination of default probability using information from a firm's financial statements and the market price of its equity.

2 MEASURING DEFAULT PROBABILITY: THE PROBLEM

There are three main elements that determine the default probability of a firm:

- **Value of Assets:** the market value of the firm's assets. This is a measure of the present value of the future free cash flows produced by the firm's assets discounted back at the appropriate discount rate. This measures the firm's prospects and incorporates relevant information about the firm's industry and the economy.

- **Asset Risk:** the uncertainty or risk of the asset value. This is a measure of the firm's business and industry risk. The value of the firm's assets is an estimate and is thus uncertain. As a result, the value of the firm's assets should always be understood in the context of the firm's business or asset risk.

- **Leverage:** the extent of the firm's contractual liabilities. Whereas the relevant measure of the firm's assets is always their market value, the book value of liabilities relative to the market value of assets is the pertinent measure of the firm's leverage, since that is the amount the firm must repay.

For example, Figure 1 illustrates the evolution of the asset value and book liabilities of Winstar Communications, a New York Telephone company that filed for Chapter 11 bankruptcy protection in April 2001.

The default risk of the firm increases as the value of the assets approaches the book value of the liabilities, until finally the firm defaults when the market value of the assets is insufficient to repay the liabilities.

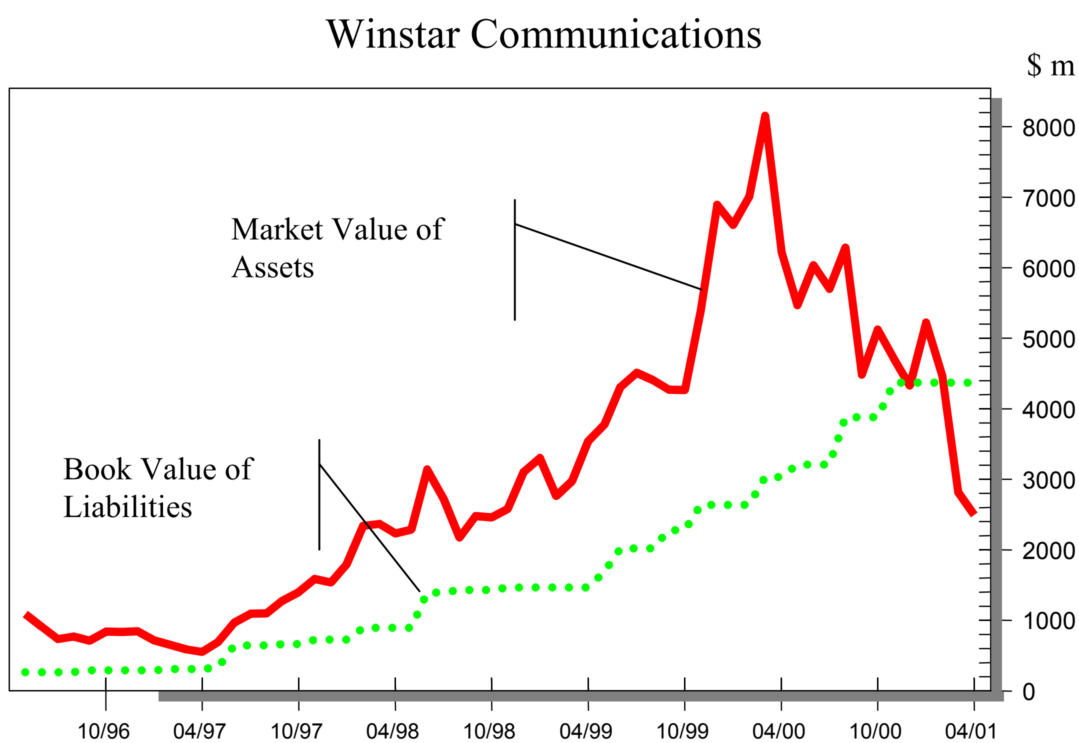

FIGURE 1 Evolution of asset value and book liabilities for Winstar Communications

In our study of defaults, we have found that in general firms do not default when their asset value reaches the book value of their total liabilities. While some firms certainly default at this point, many continue to trade and service their debts. The long-term nature of some of their liabilities provides these firms with some breathing space. We have found that the default point, the asset value at which the firm will default, generally lies somewhere between total liabilities and current, or short-term, liabilities.

The relevant net worth of the firm is therefore the market value of the firm's assets minus the firm's default point.

$$\begin{bmatrix}\text{Market Value}\\ \text{of Assets}\end{bmatrix}-\begin{bmatrix}\text{Default}\\ \text{Point}\end{bmatrix} \qquad (1)$$

A firm will default when its market net worth reaches zero.

Like the firm's asset value, the market measure of net worth must be considered in the context of the firm's business risk. For example, firms in the food and beverage industries can afford higher levels of leverage (lower market net worth) than high technology businesses because their businesses, and consequently their asset values, are more stable and less uncertain.

For example, Figure 2 shows the evolution of asset values and default points for Compaq Computer and Anheuser-Busch. Figure 3 shows the corresponding evolution of the annual default probabilities. The default probabilities shown in this figure are the one-year default rates, the probability that the firm will default in the ensuing year, and are displayed on a logarithmic scale.

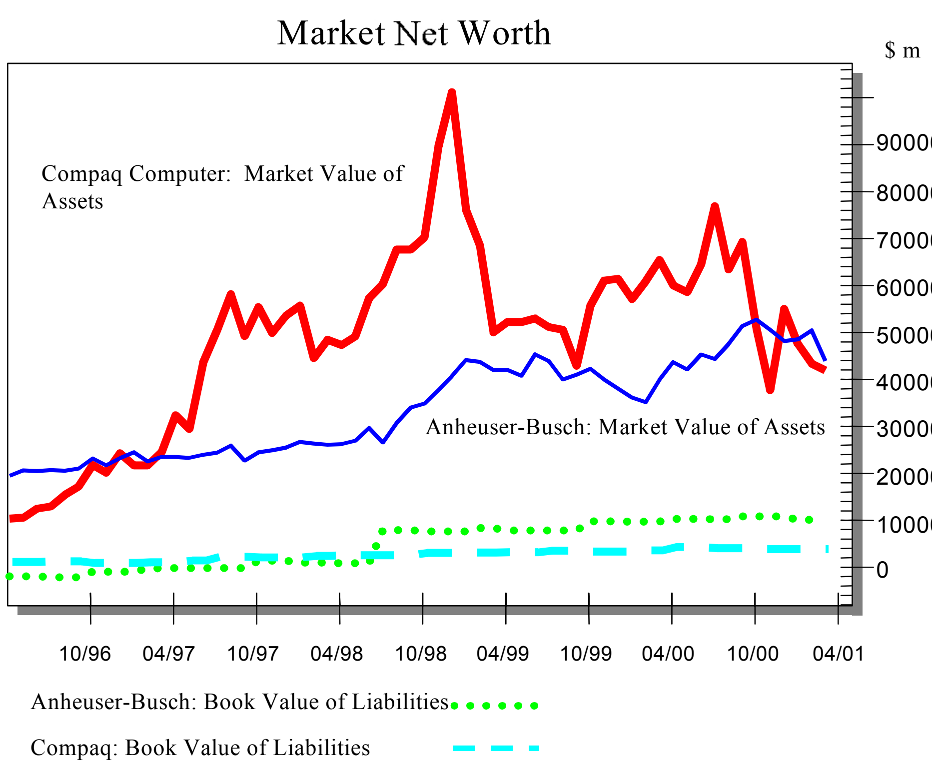

FIGURE 2 Evolution of asset values and default points for Compaq and Anheuser-Busch

Default Probability

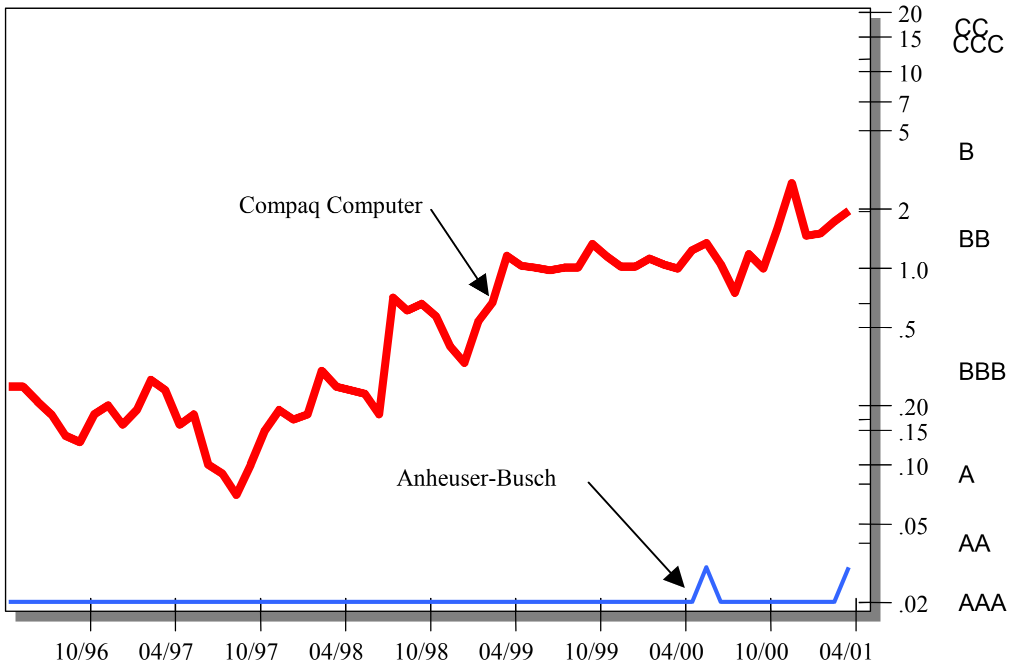

FIGURE 3 Corresponding evolution of the annual default probabilities

The effect of the relative business risks of the two firms is clear from a comparison of the two figures. For instance, as of April 2001, the relative market values, default points, asset risks and resulting default probabilities for Compaq and Anheuser-Busch were:

TABLE 1 Market Value of Assets & Default Point

	Anheuser-Busch	**Compaq Computer**
Market Value of Assets	44.1	42.3
Default Point	5.3	12.2
Market Net Worth ($b)	38.8	30.1
Asset Volatility	21%	39%
Default Probability (per annum)	.03%	1.97%

The asset risk is measured by the asset volatility, the standard deviation of the annual percentage change in the asset value. For example, Anheuser-Busch's business risk is 21%, which means that a one standard deviation move in their asset value will add (or remove) $9 bn from its asset value of $44.1 bn. In contrast, a one standard deviation move in the asset value of Compaq Computer will add or remove $16.5 bn from its asset value of $ 42.3 bn. The difference in their default probabilities is thus driven by the difference in the risks of their businesses, not their respective asset values or leverages.

As you would expect, asset volatility is related to the size and nature of the firm's business. For example, Figure 4 shows the asset volatility for several industries and asset sizes.

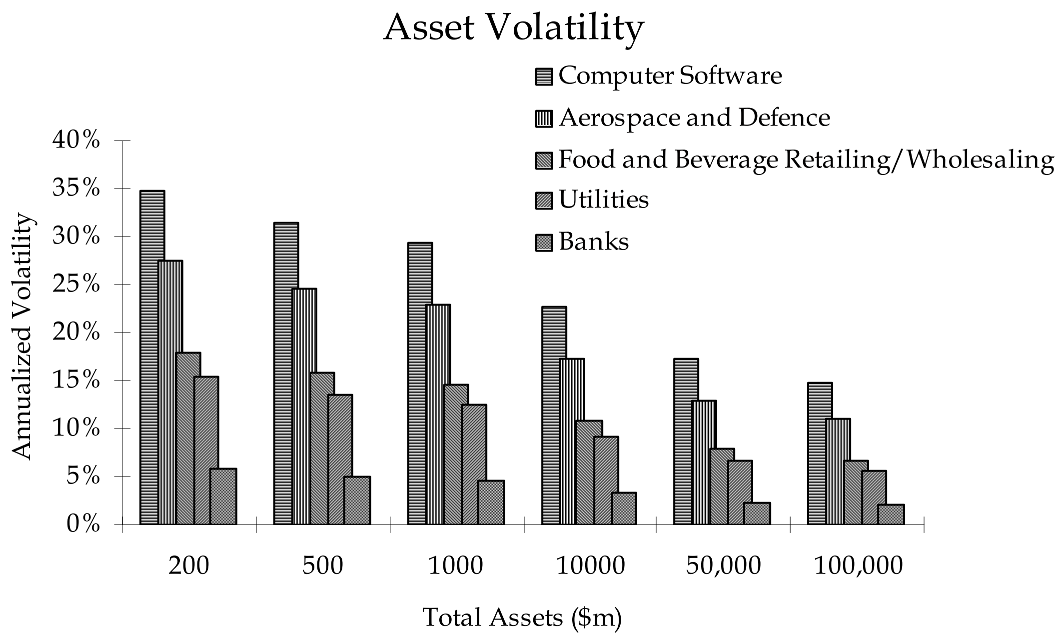

FIGURE 4 Asset volatility for several industries and asset sizes

Asset volatility is related to, but different from, equity volatility. A firm's leverage has the effect of magnifying its underlying asset volatility. As a result, industries with low asset volatility (for example, banking) tend to take on larger amounts of leverage while industries with high asset volatility (for example, computer software) tend to take on less. As a consequence of these compensatory differences in leverage, equity volatility is far less differentiated by industry and asset size than is asset volatility.

Asset value, business risk and leverage can be combined into a single measure of default risk which compares the market net worth to the size of a one standard deviation move in the asset value. We refer to this ratio as the distance-to-default and it is calculated as:

$$\begin{bmatrix}\text{Distance}\\ \text{to Default}\end{bmatrix} = \frac{\begin{bmatrix}\text{Market Value}\\ \text{of Assets}\end{bmatrix} - \begin{bmatrix}\text{Default}\\ \text{Point}\end{bmatrix}}{\begin{bmatrix}\text{Market Value}\\ \text{of Assets}\end{bmatrix}\begin{bmatrix}\text{Asset}\\ \text{Volatility}\end{bmatrix}} \qquad (2)$$

For example, in April 2001 Anheuser-Busch was approximately 4.2 standard deviations away from default while, in contrast, Compaq Computer was only 1.8 standard deviations away from default. That is, it would take a 4.2 standard deviation move in the asset value of Anheuser-Busch before it will default while only a 1.8 standard deviation move is required in Compaq's asset value to result in its default.

The distance-to-default measure combines three key credit issues: the value of the firm's assets, its business and industry risk, and its leverage. Moreover, the distance-to-default also incorporates, via the asset value and volatility, the effects of industry, geography and firm size.

The default probability can be computed directly from the distance-to-default if the probability distribution of the assets is known, or, equivalently, if the default rate for a given level of distance-to-default is known.

3 MEASURING DEFAULT PROBABILITY: A PRACTICAL APPROACH

There are three basic types of information available that are relevant to the default probability of a publicly traded firm: financial statements, market prices of the firm's debt and equity, and subjective appraisals of the firm's prospects and risk. Prices, by their nature, are inherently forward looking. Investors form debt and equity prices as they anticipate the firm's future. In determining the market prices, investors use, amongst many other things, subjective appraisals of the firm's prospects and risk, financial statements, and other market prices. This information is combined using their own analysis and synthesis and results in their willingness to buy and sell the debt and equity securities of the firm. Market prices are the result of the combined willingness of many investors to buy and sell and thus prices embody the synthesized views and forecasts of many investors.

We have found that the most effective default measurement, therefore, derives from models that utilize both market prices and financial statements. There is no assertion here that markets are perfectly efficient in this synthesis. We assert only that, in general, it is difficult to do a better job than they are doing. That is, in general, it is very difficult to consistently beat the market. Consequently, where available, we want to utilize market prices in the determination of default risk because prices add considerably to the predictive power of the estimates.

Oldrich Vasicek and Stephen Kealhofer have extended the Black-Scholes-Merton framework to produce a model of default probability known as the Vasicek-Kealhofer (VK) model. This model assumes the firm's equity is a perpetual option with the default point acting as the absorbing barrier for the firm's asset value. When the asset value hits the default point, the firm is assumed to default. Multiple classes of liabilities are modeled: short-term liabilities, long-term liabilities, convertible debt, preferred equity, and common equity. When the firm's asset value becomes very large, the convertible securities are assumed to convert and dilute the existing equity. In addition, cash payouts such as dividends are explicitly used in the VK model. A default database is used to derive an empirical distribution relating the distance-to-default to a default probability. In this way, the relationship between asset value and liabilities can be captured without resorting to a substantially more complex model characterizing a firm's liability process.

MKMV has implemented the VK model to calculate an Expected Default Frequency™ (EDF™) credit measure which is the probability of default during the forthcoming year, or years for firms with publicly traded equity (this model can also be modified to produce EDF values for firms without publicly traded equity.) The EDF value requires equity prices and certain items from financial statements as inputs. EDF credit measures can be viewed and analyzed within the context of a software product called Credit Monitor™ (CM). CM calculates EDF values for years 1 through 5 allowing the user to see a term structure of EDF values. MKMV's EDF credit measure assumes that Default is defined as the nonpayment of any scheduled payment, interest or principal. The remainder of this section describes the procedure used by MKMV to determine a public firm's probability of default.

There are essentially three steps in the determination of the default probability of a firm:

- **Estimate asset value and volatility:** In this step the asset value and asset volatility of the firm is estimated from the market value and volatility of equity and the book value of liabilities.

- **Calculate the distance-to-default:** The distance-to-default (DD) is calculated from the asset value and asset volatility (estimated in the first step) and the book value of liabilities.

- **Calculate the default probability:** The default probability is determined directly from the distance-to-default and the default rate for given levels of distance-to-default.

3.1 Estimate Asset Value and Volatility

If the market price of equity is available, the market value and volatility of assets can be determined directly using an options pricing based approach, which recognizes equity as a call option on the underlying assets of the firm. For example, consider a simplified case where there is only one class of debt and one class of equity. See Figure 5.

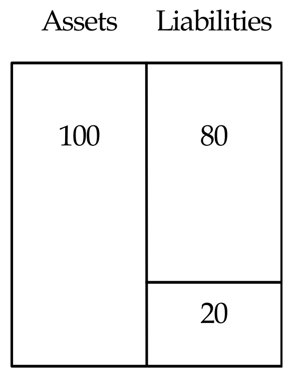

FIGURE 5

The limited liability feature of equity means that the equity holders have the right, but not the obligation, to pay off the debt holders and take over the remaining assets of the firm. That is, the holders of the other liabilities of the firm essentially own the firm until those liabilities are paid off in full by the equity holders. Thus, in the simplest case, equity is the same as a call option on the firm's assets with a strike price equal to the book value of the firm's liabilities.

The VK model uses this option nature of equity to derive the underlying asset value and asset volatility implied by the market value, volatility of equity, and the book value of liabilities. This process is similar in spirit to the procedure used by option traders in the determination of the implied volatility of an option from the observed option price.

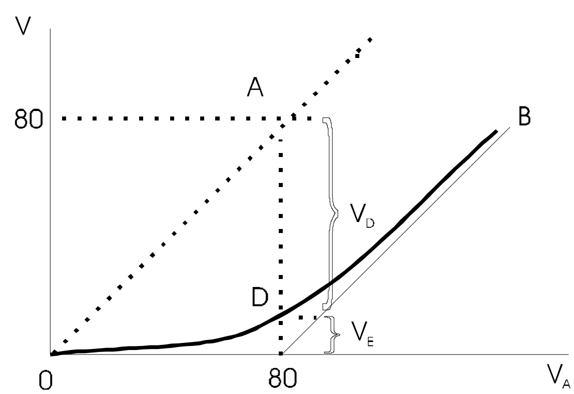

FIGURE 6

For example, assume that the firm is actually a type of levered mutual fund or unit trust. The assets of the firm are equity securities and thus can be valued at any time by observing their market prices. Further, assume that our little firm is to be wound up after five years and that we can ignore the time value of money (discounting adds little to our understanding of the relationships and serves only to complicate the picture). That is, in five years time, the assets will be sold and the proceeds divided between the debt and equity holders.

Initially, assume that we are interested in determining the market value of the equity from the market value of the assets. This is the reverse of the problem we face in practice but provides a simpler perspective to initially understand the basic option relationships. See Figure 6.

To be specific, assume that we initially invest $20 in the firm and borrow a further $80 from a bank. The proceeds, $100, are invested in equities. At the end of five years what is the value of equity? For example, if the market value of the assets at the end of year five is $60 then the value of equity will be zero. If the value of the assets is $110, then the