COMP 210
Discrete Structures

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Today's contents:

- Set theory
- Functions

Definition

A *set* is an unordered collection of elements.

Examples:

{0,3,5} is the set containing "0" and "3" and "5."
{0, 0, 3, 3,5} = {0, 3, 5} since repetition is irrelevant.
{0, 3, 5} = {5, 0, 3} since sets are unordered.
{1, 2, 3, …} is a way we denote an infinite set (in this case, the natural numbers).
∅ = {} is the empty set, or the set containing no elements.

Note: $\emptyset \neq \{\emptyset\}$

A set includes an element "empty set"

Methods to define a set

- Explicitly: {Tom, John, Andrew}
- Implicitly: {1,3,5,…}, or {2,3,5,7,11,13,17,…}
- Set builder: { x : x is even }, { x | x is negative }. In general { x : P(x) is true }, where P(x) is some description of the set.

Example:

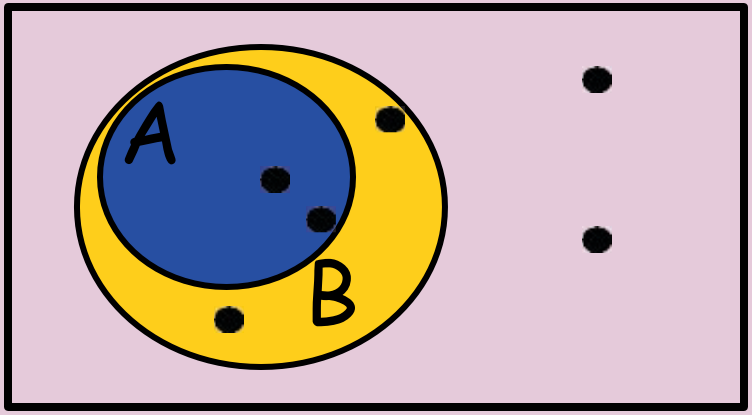
Let P(x,y) denote "xy is divisible by 3."

Definitions

$x \in A$ means "x is an element of set A."
$x \notin A$ means "x is not an element of set A."

$A \subseteq B$ means "A is a subset of B."

or, "B contains A."
or, "every element of A is also in B."
or, $\forall x ((x \in A) \rightarrow (x \in B))$.



Venn Diagram

Universal Domain

Definitions

$A \subseteq B$ means "A is a subset of B."
$A \supseteq B$ means "A is a superset of B."

$A = B$ if and only if A and B have exactly the same elements.

If and only if

iff, $A \subseteq B$ and $B \subseteq A$
iff, $A \subseteq B$ and $A \supseteq B$
iff, $\forall x ((x \in A) \leftrightarrow (x \in B))$.

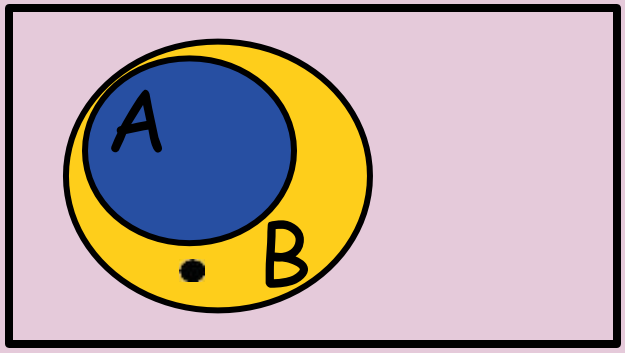
So to show equality of sets A and B, show:

- $A \subseteq B$
- $B \subseteq A$

Definitions

$A \subset B$ means "A is a proper subset of B."

- $A \subseteq B$, and $A \neq B$.



$A \not\subset A$

Examples

- $\{0,3,5\} \subseteq \{0,1,2,3,4,5\}$
- $\{0,3,5\} \subset \{0,1,2,3,4,5\}$

Is $\varnothing \subseteq \{0,3,5\}$?

Yes! $\forall x\,(x \in \varnothing) \rightarrow (x \in \{0,3,5\})$
holds, because $(x \in \varnothing)$ is false.

Is $\varnothing \in \{0,3,5\}$? No!

Is $\varnothing \subseteq \{\varnothing, 0,3,5\}$? Yes!

Is $\varnothing \in \{\varnothing, 0,3,5\}$? Yes!