

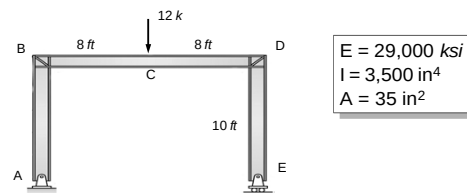
Virtual Work for Frames

- Applying the virtual work equations to a frame structure is as simple as separating the frame into a series of "beams" and summing the virtual work for each section.
- In addition, when evaluating the deformation of a frame structure, you may have to consider both bending and axial internal force components.

Virtual Work for Frames

Compute the **deflection** at point C on the frame shown below.

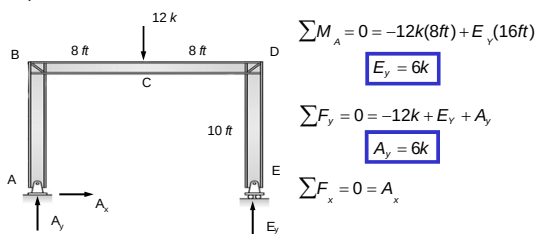
Include only the effects of bending in your virtual work equation (no axial work).



Virtual Work for Frames

The first step is to find the equation for moment in each section of the frame due to the **real loads**.

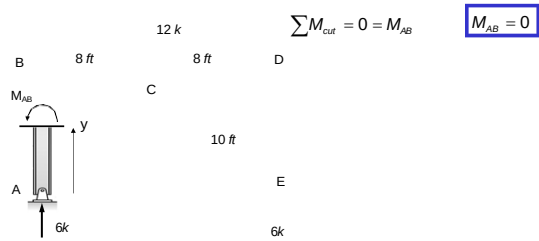
To do develop the moment expression we need the reaction at points A and E.



Virtual Work for Frames

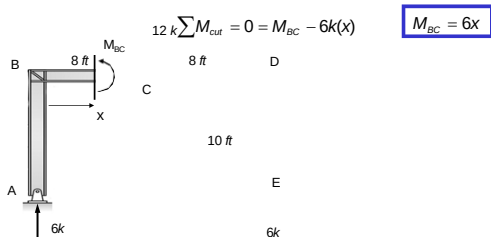
The next step is to find the equation for moment in each section of the frame.

Consider section AB



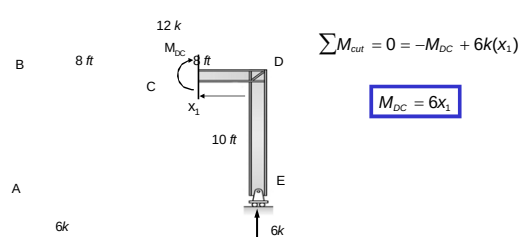
Virtual Work for Frames

Consider section BC



Virtual Work for Frames

Consider section DC



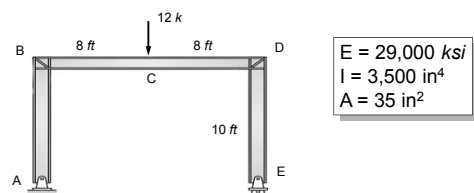
Virtual Work for Frames

- Applying the virtual work equations to a frame structure is as simple as separating the frame into a series of "beams" and summing the virtual work for each section.
- In addition, when evaluating the deformation of a frame structure, you may have to consider both bending and axial internal force components.

Virtual Work for Frames

Compute the **deflection** at point C on the frame shown below.

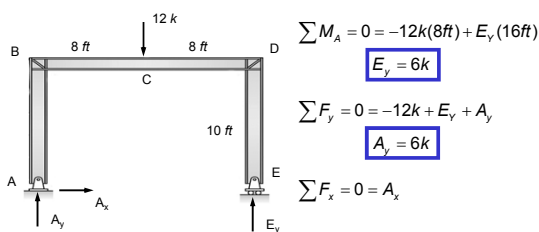
Include only the effects of bending in your virtual work equation (no axial work).



Virtual Work for Frames

The first step is to find the equation for moment in each section of the frame due to the **real loads**.

To do develop the moment expression we need the reaction at points A and E.

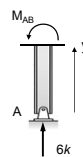


Virtual Work for Frames

The next step is to find the equation for moment in each section of the frame.

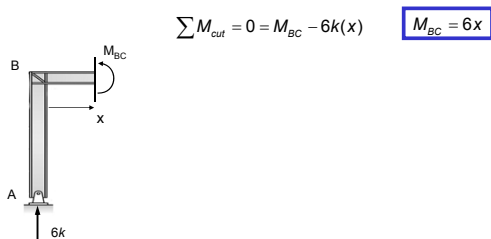
Consider section AB

$$\sum M_{cut} = 0 = M_{AB} \quad M_{AB} = 0$$



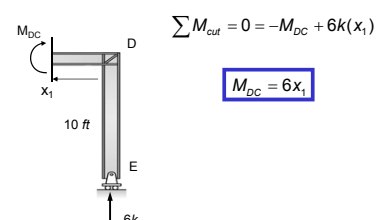
Virtual Work for Frames

Consider section BC



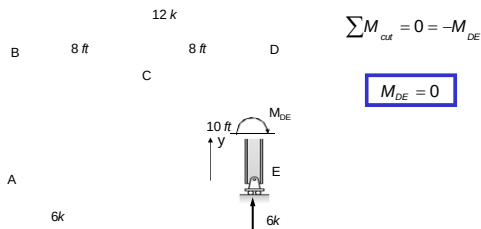
Virtual Work for Frames

Consider section DC



Virtual Work for Frames

Consider section ED



Virtual Work for Frames

In this problem, the virtual moments are the real moments divided by 12 (from superposition).

Section	M	m
AB	0	0
BC	6x	0.5x
DC	6x ₁	0.5x ₁
DE	0	0

Virtual Work for Frames

The virtual work equations are:

$$\Delta = \int_A^C \frac{M}{EI} \frac{m}{EI} dy + \int_B^C \frac{M}{EI} \frac{m}{EI} dx + \int_D^C \frac{M}{EI} \frac{m}{EI} dx + \int_D^E \frac{M}{EI} \frac{m}{EI} dy$$

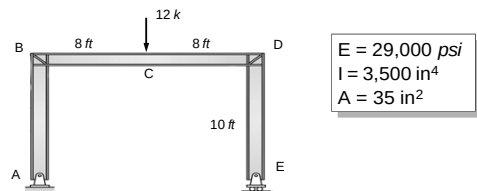
Substituting the moment expressions into the virtual work equation and integrating yields the following:

$$\Delta = \int_0^8 \frac{(6x)}{EI} \frac{1}{EI} dx + \int_0^8 \frac{(6x)}{EI} \frac{1}{EI} dx + \int_0^8 \frac{6x^2}{EI} \frac{1}{EI} dx = \frac{2x^3}{EI} \bigg|_0^8$$

$$= \frac{1,024 \text{ kft}^3}{EI} = \frac{1,024 \text{ kft}^3 (1,728 \text{ in}^3 / \text{ft}^3)}{(29,000 \text{ ksi})(3,500 \text{ in}^4)} = 0.017 \text{ in}$$

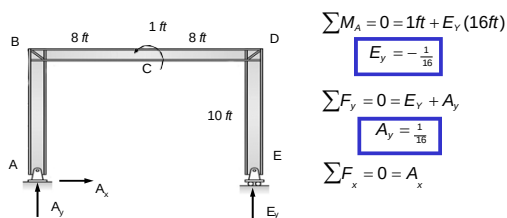
Virtual Work for Frames

Compute the **slope** at point C on the frame shown below. Include only the effects of bending in your virtual work equation (no axial work).



Virtual Work for Frames

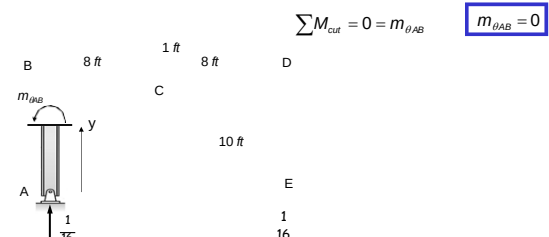
Find the moments in the frame due to a virtual couple. First, find the reaction in the frame to the virtual couple.



Virtual Work for Frames

The next step is to find the equation for moment in each section of the frame.

Consider section AB

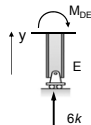


Virtual Work for Frames

Consider section ED

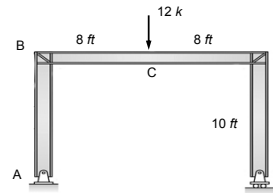
$$\sum M_{cut} = 0 = -M_{DE}$$

$$M_{DE} = 0$$



Virtual Work for Frames

In this problem, the virtual moments are the real moments divided by 12 (from superposition).



Section	M	m
AB	0	0
BC	6x	0.5x
DC	6x ₁	0.5x ₁
DE	0	0

Virtual Work for Frames

The virtual work equations are:

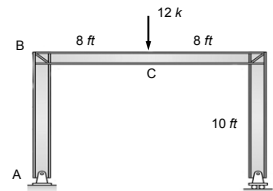
$$\Delta_C = \int_A^B \frac{M_{AB} m_{AB}}{EI} dy + \int_B^C \frac{M_{BC} m_{BC}}{EI} dx + \int_D^C \frac{M_{DC} m_{DC}}{EI} dx + \int_D^E \frac{M_{DE} m_{DE}}{EI} dy$$

Substituting the moment expressions into the virtual work equation and integrating yields the following:

$$\begin{aligned} \Delta_C &= \int_0^{10} \frac{(6x)y}{2EI} dy + \int_0^8 \frac{(6x_1)x_1}{2EI} dx_1 + \int_0^8 \frac{6x_1^2}{EI} dx_1 = \frac{2x^3}{EI} \Big|_0^8 \\ &= \frac{1,024 \text{ kft}^3}{EI} = \frac{1,024 \text{ kft}^3 (1,728 \text{ in}^3 / \text{ft}^3)}{(29,000 \text{ ksi})(3,500 \text{ in}^4)} = 0.017 \text{ in} \end{aligned}$$

Virtual Work for Frames

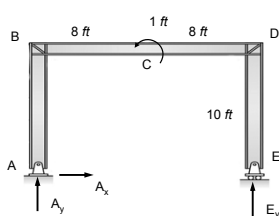
Compute the **slope** at point C on the frame shown below. Include only the effects of bending in your virtual work equation (no axial work).



$$\begin{aligned} E &= 29,000 \text{ psi} \\ I &= 3,500 \text{ in}^4 \\ A &= 35 \text{ in}^2 \end{aligned}$$

Virtual Work for Frames

Find the moments in the frame due to a virtual couple. First, find the reaction in the frame to the virtual couple.



$$\sum M_A = 0 = 1\text{ft} + E_y(16\text{ft})$$

$$E_y = -\frac{1}{16}$$

$$\sum F_y = 0 = E_y + A_y$$

$$A_y = \frac{1}{16}$$

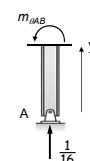
$$\sum F_x = 0 = A_x$$

Virtual Work for Frames

The next step is to find the equation for moment in each section of the frame.

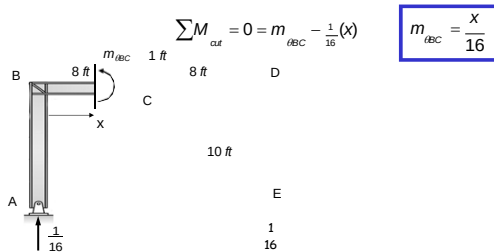
Consider section AB

$$\sum M_{cut} = 0 = m_{AB} \quad m_{AB} = 0$$



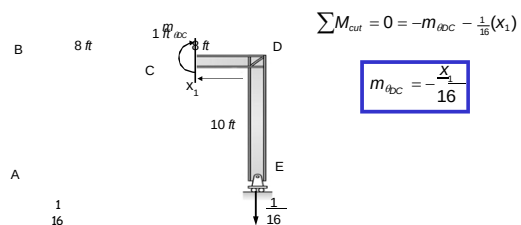
Virtual Work for Frames

Consider section BC



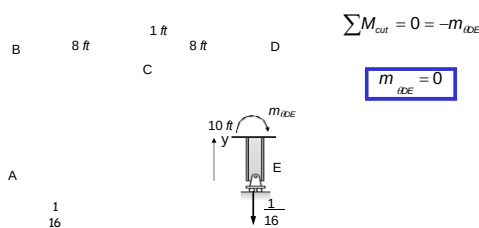
Virtual Work for Frames

Consider section DC



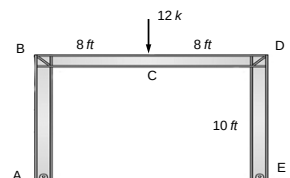
Virtual Work for Frames

Consider section ED



Virtual Work for Frames

The following table lists the moment expression due to the real loading and the moment expression due to a virtual couple at point C



Section	M	m _θ
AB	0	0
BC	6x	x/16
DC	6x ₁	-x ₁ /16
DE	0	0

Virtual Work for Frames

The virtual work equations are:

$$\theta_c = \int_A^B \frac{M}{EI} \frac{m_{AB}}{EI} dy + \int_B^C \frac{M}{EI} \frac{m_{BC}}{EI} dx + \int_C^D \frac{M}{EI} \frac{m_{DC}}{EI} dx + \int_D^E \frac{M}{EI} \frac{m_{DE}}{EI} dy$$

Substituting the moment expressions into the virtual work equation and integrating yields the following:

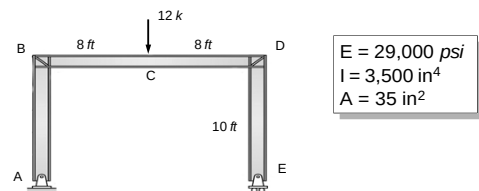
$$\theta_c = \int_0^8 \frac{(6x)x}{16EI} dx - \int_0^8 \frac{(6x_1)x_1}{16EI} dx_1 = 0$$

The slope at point C is zero

Virtual Work for Frames

Repeat the previous example and include the effects of axial work.

In order to compute the axial work, we need the axial force in the real and virtual loading systems



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