信号与系统习题解答

1.1

(1) f(t)=ε(t)

解$\because P=\lim_{\tau\to\infty}\frac{1}{2\tau}\int_{-\tau}^{\tau}|f(t)|^2dt$

$=\lim_{\tau\to\infty}\frac{1}{2\tau}\int_{0}^{\tau}dt=\frac{1}{2}$

$E_{总}=\lim_{\tau\to\infty}\int_{-\tau}^{\tau}|f(t)|^2\ dt=\lim_{\tau\to\infty}\int_{0}^{\tau}dt=\infty$

$\therefore f(t)=\varepsilon(t)$为功率信号。

(2) f(t)=ε(t)-ε(t-1)

解$\because f(t)$是矩形脉冲信号，故为能量信号。

(3)$f(t)=6t\varepsilon(t)$

解：书中已作证明斜坡信号为非公非能信号。

(4)$f(t)=5e^{j(\omega_0 t+\varphi)}$

解$\because |f(t)|=5$

$\therefore \mathrm{P}=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}|f(t)|^2dt$

$=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}25dt=25$

$E_{总}=\lim_{T\to\infty}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}|f(t)|^2dt=\lim_{T\to\infty}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}25dt=\infty$

$\therefore f(t)$为功率信号

(5) $f(t)=e^{-t}\sin 2t\varepsilon(t)$

解：$\mathrm{E}_{总}=\lim_{\tau\to\infty}\int_{-\tau}^{\tau}|f(t)|^2dt=\lim_{\tau\to\infty}\int_{-\tau}^{\tau}(e^{-t}\sin 2t)^2dt$

$=\lim_{\tau\to\infty}\int_{0}^{\tau}\frac{e^{-2t}(e^{j2t}-e^{-j2t})}{(2j)^2}dt=(-\frac{1}{4})\lim_{\tau\to\infty}\int_{0}^{\tau}e^{-2t}(e^{j4t}+e^{-j4t}-2)dt$

$=(-\frac{1}{4})\lim_{\tau\to\infty}\int_{0}^{\tau}[e^{-(2-j4)t}+e^{-(2+j4)t}]dt$

$$=(-\frac{1}{4})\lim_{\tau\to\infty}[-\frac{e^{-(2-j4)t}}{2-j4}-\frac{e^{-(2+j4)t}}{2+j4}]|_0^{\tau}$$

$$=(-\frac{1}{4})[\frac{1}{2-j4}+\frac{1}{2+j4}-1]$$

$$=(-\frac{1}{4})[\frac{2+j4+2-j4}{4+16}-1]=\frac{1}{5}$$

$$P=\lim_{\tau\to\infty}\frac{E_{总}}{2\tau}=0$$

$\therefore f(t)=e^{-t}\sin 2t\varepsilon(t)$为能量信号

(6) $f(t)=\frac{1}{1+t}\varepsilon(t)$

解：$E_{总}=\lim_{\tau\to\infty}\int_{-\tau}^{\tau}f^2(t)dt=\lim_{\tau\to\infty}\int_{-\tau}^{\tau}\frac{1}{(1+t)^2}dt$

$$=\lim_{\tau\to\infty}(-\frac{1}{1+\tau})+1=1$$

$$P=\lim_{\tau\to\infty}\frac{E_{总}}{2\tau}=0$$

$\therefore f(t)$为能量信号

1.2 判断下列信号是否为周期信号，如果是周期信号，试确定其周期。

（1）$f(t)=3\cos(2t)+2\cos(\pi t)$

解 $\because \frac{\omega_1}{\omega_2}=\frac{T_2}{T_1}=\frac{2}{\pi}$ 是无理数

$\therefore$改组合正弦信号$f(t)$是非周期信号

(2)显然$f(t)=|\cos(2t)|$为周期信号

(3) $f(t)=3e^{j(2t+45^{\circ})}$为周期信号

(4) $f(t)=\cos(\frac{\pi}{2}t)+\cos(\frac{\pi}{3}t)+\cos(\frac{\pi}{6}t)$

$$\frac{\omega_1}{\omega_2}=\frac{\frac{\pi}{2}}{\frac{\pi}{3}}=\frac{3}{2}$$

$$T_1=2\pi/\frac{\pi}{2}=4s$$

$$T_2=2\pi/\frac{\pi}{3}=6s$$

$$T^{'}=mT_1=12s$$

$$T=5\times 12=60s$$

$\therefore f(t)$为周期信号，周期为60s.

$(3)\,f(t)=3e^{-t}\sin(3t+\pi)=3e^{-t}\,\mathrm{Im}[e^{j(3t+\pi)}]=3e^{-t}\cos(3t+\frac{\pi}{2})$

$(4)\,f(t)=je^{(j100t-2)}=e^{j\frac{\pi}{2}}e^{(j100t-2)}=e^{-2}e^{j(100t+\frac{\pi}{2})}$

$\mathrm{Re}[f(t)]=e^{-2}\cos(100t+\frac{\pi}{2})$

$(5)\,f(t)=[\![\sin(t-\frac{\pi}{6})]\!]^2$为周期信号，周期为$\pi(s)$.

$(6)\,f(k)=(\frac{8}{7}\pi-\frac{\pi}{8})$

$\frac{2\pi}{\Omega}=\frac{2\pi}{\frac{8}{7}\pi}=\frac{7}{4}$

$\therefore f(k)$为周期序列，$N=7$.

1.3.

$(1)\,f(t)=-6=6e^{j\pi}$

$(2)\,f(t)=2\sqrt{2}e^{j\frac{\pi}{4}\cos(2t+2\pi)}=2\sqrt{2}\cos(2t+\frac{\pi}{4})$

1.4 （波形略）

1.5 设$f(t)=0\ \ (t<3)$，是确定下列个信号的零值时间区间。

(1) $f(1-t)=0\quad(-2<t)$

(2) $f(1-t)+f(2-t)=0\quad(-2<t)$

(3) $f(2t)=0\quad\left(t<\frac{3}{2}\right)$

(4) $f(1-t)+f(2-t)=0\quad(-1<t)$

(5) $f\left(\frac{t}{2}\right)=0\quad(t<6)$

1.6 试绘出题图 1-6 所示各连续信号波形的表达式。

(a) $f_1(t)=2\varepsilon(t+1)-\varepsilon(t-1)-\varepsilon(t-2)$

(b) $f_2(t)=2\Delta_4(t-1)$

(c) $f_3(t)=5\sin\pi t[\varepsilon(t)-\varepsilon(t-1)]$

(d)

$f_4(t)=2\varepsilon(-t)+4t^2[\varepsilon(t)-\varepsilon(t-1)]+2(t-1)\varepsilon(t-1)-2(t-2)\varepsilon(t-2)$

1.7试证明$\delta(t)=\lim_{\alpha\to 0}\frac{\alpha}{\pi(\alpha^2+t^2)}$.

$$\lim_{\alpha\to 0}\frac{\alpha}{\pi(\alpha^2+t^2)}=\begin{cases}\infty & t=0\\ 0 & t\neq 0\end{cases}$$

$$\delta(t)\lim_{\alpha\to 0}\frac{\alpha}{(\alpha^2+t^2)}$$

1.8 (1) $f(t)=\sin(\omega t-\varphi)\delta(t)=\sin(-\varphi)\delta(t)$

(2) $f(t)=\sin(\pi t)\delta(t+\frac{1}{4})=\sin(-\frac{\pi}{4})\delta(t+\frac{1}{4})=-0.707\delta(t+\frac{1}{4})$

(3) $f(t)=\sin(\omega t-\varphi)\delta'(t)=\sin(-\varphi)\delta'(t)-\omega\cos(-\varphi)\delta(t)$

(4) $f(t)=\sin(\pi t)\delta'(t+\frac{1}{4})=\sin(-\frac{\pi}{4})\delta'(t+\frac{1}{4})+\pi\cos(-\frac{\pi}{4})\delta(t+\frac{1}{4})$

1.9 (1) $\int_{-\infty}^{\infty}\sin(t)\delta(t-\frac{\pi}{4})dt=\sin\frac{\pi}{4}=0.707$

(2) $\int_{-\infty}^{\infty}\frac{\sin 5t}{t}\delta(t)dt=\int_{-\infty}^{\infty}5Sa(5t)\delta(t)dt=5$

(3) $\int_{-\infty}^{\infty}e^{-2t}[\delta(t)+\delta'(t)]dt=1+2e^{-2t}\Big|_{t=0}=3$

(4) $\int_{-\infty}^{\infty}(t^2+t+1)\delta(\frac{t}{2})dt=\int_{-\infty}^{\infty}(t^2+t+1)|2|\delta(t)dt=2$

(5) $\int_{0}^{3}(t^2+2)\delta(t-5)dt=0$

(6) $\int_{0}^{10}(t^2+2)\delta(t-5)dt=\int_{0}^{10}(5^2+2)\delta(t-5)dt=27$

(7) $\int_{0}^{t}\sin\tau\delta(\tau-5)d\tau=\sin 5\varepsilon(t-5)$

(8) $\int_{-\infty}^{t}(\tau^2+\tau+1)\delta(\frac{\tau}{2})d\tau=\int_{-\infty}^{t}(\tau^2+\tau+1)2\delta(\tau)d\tau=2\varepsilon(t)$

(9) $\int_{-10}^{10}(2t^2+t-5)\delta'(t+\frac{1}{4})dt=-[2t^2+t-5]'\Big|_{t=-\frac{1}{4}}=-[4t+1]\Big|_{t=-\frac{1}{4}}=0$

(10) $\int_{-\infty}^{t}(1-\tau)\delta'(\tau)d\tau=\int_{-\infty}^{t}[\delta'(\tau)+\delta(\tau)]d\tau=\delta(t)+\varepsilon(t)$

1.13:　　$f_1(k)=k\varepsilon(k)$, $f_2(k)=(\alpha)^{k-1}\varepsilon(k-1)$.

(1) $f_1(k)+f_2(k)=k\varepsilon(k)+(a)^{k-1}\varepsilon(k-1)$.

(2) $f_1(k)-f_2(k)=k\varepsilon(k)-(a)^{k-1}\varepsilon(k-1)$.

(3) $f_1(k)\times f_2(k)=k(a)^{k-1}\varepsilon(k-1)$.

(4) $f_1(k-1)+f_2(k+1)=(k-1)\varepsilon(k-1)+a^k\varepsilon(k)$.

(5) $f_1(k-1)\times f_2(k+1)=(k-1)a^k\varepsilon(k-2)$.

1.18. (1)偶、偶谐　　　　　（2）偶、奇谐

（3）偶、偶谐奇谐（非唯一）　　　　（4）奇、奇谐

(5)奇偶谐　　　　　（6）奇、奇谐 偶谐

1.19 解：（1）

$$U_C=U_S-2I-I'$$
$$U_C=RI_2+2I_2'=I_2+2I_2'$$
$$I_2=I-I_C$$
$$I_C=C\frac{du_c}{dt}=I_2'+2I_2''$$
$$U_C=(I-U_S'+2I'+I'')+2(I'-U_S''+2I''+I''')$$
$$U_S-2I-I'=I-U_S'+2I'+I''+2I'-2U_S''+4I''+2I'''$$

整理得：

$$2I'''+5I''+5I'+3I=2U_S''+U_S'+U_S$$

（2）

$$U_C=\frac{1}{2}\int_{-\infty}^{t}U(\tau)d\tau+U$$

$$U_C'=\frac{1}{2}U+U'$$

$$I_C=CU_C'=U_C'=\frac{1}{2}U+U'$$

$$I=I_C+I_2=\frac{1}{2}U+U'+\frac{1}{2}\int_{-\infty}^{t}U(\tau)d\tau$$

$$2I+I'+\int_{-\infty}^{t}U(\tau)d\tau+U=U_s$$

整理得：

$$2U''' + 5U'' + 5U' + 3U = 2U_S'$$

1.20 解：由题意 y(k)=y(k-1)+ α y(k-1)- β y(k-1)+f(k)
∴y(k)-(1+ α - β)y(k-1)=f(k)

1.21 解：由题意 y(1)=f(1)+ β y(1)

Y(2)=f(2)+y(1)+ β y(1)

第 k 个月的全部本利为 y(k)，第 k-1 个月初的全部本利为 y(k-1)，则第 k 个月初存入银行的款数为

Y(k)-(1-β)y(k-1)=f(k)

1.22 解：由题意 $y(k)=\frac{2}{3}y(k-1)$

$\therefore y(k)-\frac{2}{3}y(k-1)=0$

1.23 解：由题意

(1) $y_x = e^{-t}x(0)$　　$y_f=\int_0^t \sin\tau f(\tau)d\tau$

$x_1(0)+x_2(0) \dashrightarrow e^{-t}[x_1(0)+x_2(0)] = e^{-t}x_1(0)+e^{-t}x_2(0)=y_{x1}+y_{x2}$

满足零输入线性

$f_1+f_2 \dashrightarrow \int_0^t \sin\tau[f_1(\tau)+f_2(\tau)]d\tau = \int_0^t \sin\tau f_1(\tau)d\tau + \int_0^t \sin\tau f_2(\tau)d\tau = y_{f1}+y_{f2}$ 满足零状态线性

∴为线性系统

（2）$y(t)=\sin[x(0)t]+f^2(t)$

$x_1(0)+x_2(0) \dashrightarrow \sin\{[x_1(0)+x_2(0)]t\} \neq \sin[x_1(0)t]+\sin[x_2(0)t]$ 不满足零输入线性

(3) $y(t)=f(t)x(0)+\int_0^t f(t)d\tau$　不满足分解性，所以是非线性系统；

(4) $y(t)=x(0)\lg f(t)$　是非线性系统；

(5) $y(t)=\lg x(0)+f'(t)$　不满足零线性输入，所以是非线性系统；

(6) $y(t)=\sqrt{x(t_0)}+\int_{t_0}^t f(\tau)d\tau$ 不满足零输入线性

$\int_{t_0}^{t}[f_1+f_2]d\tau = y_1+y_2$ 满足零状态线性，故为非线性系统；

（7） $y(k)=\frac{1}{2^k}x(0)+f(k)f(k-2)$

$$x_1(0)+x_2(0)\to\frac{1}{2^k}[x_1(0)+x_2(0)]=\frac{1}{2^k}x_1(0)+\frac{1}{2^k}x_2(0)=y_{x_1}+y_{x_2}$$

满足零输入线性

$$y_1(k)+y_2(k)\to[y_1(k)+y_2(k)][y_1(k-2)+y_2(k-2)]\neq y_{f_1}(k)+y_{f_2}(k)$$

不满足零状态线性，因而是非线性系统；

（8）

$y(k)=kx(0)+\sum_{n=0}^{k}f(n)$ $\quad x_1(0)+x_2(0)\to k\,x_1(0)+k\,x_2(0)=y_{x_1}(k)+y_{x_2}(k)$

$f_1(k)+f_2(k)\to\sum_{n=0}^{k}[f_1(n)+f_2(n)]=\sum_{n=0}^{k}f_1(n)+\sum_{n=0}^{k}f_2(n)$ 因而为线性系统；

1.24 （1） $y(t)=\int_{-\infty}^{t}f(\tau)d\tau$ 为线性系统；

$f(t-t_d)\to\int_{-\infty}^{t}f(\tau-t_d)d\tau \underline{\underline{x=\tau-t_d}} \int_{-\infty}^{x}f(x)dx$ 因而是时不变系统；

$(2)y(t)=\int_{0}^{t}f(\tau)d\tau$ 线性

$f(t-t_d)\longrightarrow\int_{0}^{t}f(\tau-t_d)d\tau \underline{\underline{x=\tau-t_d}} \int_{-t_d}^{t-t_d}f(x)f(x)dx$ 时变

$(3)y(t)=|f(t)|$

$f_1+f_2=|f_1+f_2|\neq|f_1|+|f_2|$ 非线性

$f(t-t_d)\longrightarrow|f(t-t_d)|=y(t-t_d)$ 非时变

$(4)y(t)=e^{f(t)}$ 非线性非时变

$(5)y'+2y=f'-2f$ 非线性非时变

$(6)y'+\sin y=f'$ 线性时变

$(7)[y'(t)]^2+2y(t)=f(t)$ 非线性非时变

$(8)\, y'(t)+2y(t)=tf(t)$　　线性时变

$(9)\, y(k)+(k-1)y(k-1)=f(k)$　　线性时变

$(10)\, y(k)+y(k-1)y(k-2)=f(k)$　　非线性非时变

1.25 $(1)\because \delta(t)=\dfrac{d\varepsilon(t)}{dt}$　　$\therefore y_{f_2}(t)=\dfrac{dy_{f_1}(t)}{dt}=\dfrac{d}{dt}[e^{-2t}\varepsilon(t)]=\delta(t)-2e^{-2t}\varepsilon(t)$

$(2)\because R(t)=\int_0^t \varepsilon(\tau)d\tau$

$\therefore y_{f_3}(t)=\int_0^t y_{f_1}(\tau)d\tau=\int_0^t e^{-2t}d\tau\varepsilon(t)=-\dfrac{1}{2}e^{-2t}\,|_0^t\, \varepsilon(t)=\dfrac{1}{2}(1-e^{-2t})\varepsilon(t)$

1.26 解：由题意

$y_{x_1}=2e^{-t}+3e^{-3t}$，$y_{x_2}=4e^{-t}-2e^{-3t}$，$y_f=2+e^{-t}+2e^{-3t}$

$y(t)=2y_{x_1}+5y_{x_2}+3y_f$

$=4e^{-t}+6e^{-3t}+20e^{-t}-10e^{-3t}+6+3e^{-t}+6e^{-3t}$

$=6+27e^{-t}+2e^{-3t}$

1.27 解：由题意

（1）　$2y(t)=3y_1-y_2$　，

（2）　$\left.\begin{array}{l} y_1(t)=y_{x_1}+y_{x_2}+y_f \\ y_2(t)=y_{x_1}+y_{x_2}+3y_f \end{array}\right\} 3y_1-y_2=2y_{x_1}+2y_{x_2}=10e^{-2t}-8e^{-3t}$，

$y_2-y_1=2y_f=2e^{-2t}-2e^{-3t}$，

$\therefore y_f(t)=e^{-2t}-e^{-3t}=y(t)$　。

1.28 解：$y_1(k)=y_x(k)+y_f(k)=\varepsilon(k)$

$y_2(k)=y_x(k)-y_f(k)=\left[2\left(\dfrac{1}{2}\right)^k-1\right]\varepsilon(k)$

$y_1+y_2=2y_x(k)=2\left(\dfrac{1}{2}\right)^k\varepsilon(k)$，　　$\therefore y_x(k)=\left(\dfrac{1}{2}\right)^k$

$$y_1 - y_2 = 2y_f(k) = 2\varepsilon(k) - 2\left(\frac{1}{2}\right)^k \varepsilon(k)$$

$$\therefore y_f(k) = \varepsilon(k) - \left(\frac{1}{2}\right)^k \varepsilon(k) \quad 。$$

$$\therefore y(k) = 2y_x(k) + 4y_f(k) = 2\left(\frac{1}{2}\right)^k + 4\varepsilon(k) - 4\left(\frac{1}{2}\right)^k \varepsilon(k)$$

$$= 4\varepsilon(k) - 2\left(\frac{1}{2}\right)^k \varepsilon(k)$$

1.29（1） $f(t) = 0(t \le 0)$有$y' + 2y = -3$　非因果非线性非时变

(2) $y'(t) = 2tf^2(t) + f(t+5)$　当$t \le 0$　$f(t) = 0$

有$y'(t) = f(5)$　非线性非因果时变

(3) $y_f(t) = |f(t)|$　非线性非时变因果

(4)　$y_f(t) = f(t)\cos()$　线性时变因果

(5)　$y_f(t) = f(-t)$　线性非时变非因果

(6)　$y_f(K) = f(K-2)f(K)$　线性时变因果

(7)　$y_f(K) = \sum_{n=0}^{K} f(n)$　线性时变因果

$$f(K-K_0) \to \sum_{n=0}^{K} f(n-K_0) \xrightarrow{m=n-K_0} \sum_{-K_0}^{K-K_0} f(m) \ne y(K-K_0)$$

(8)　$y_f(K) = f(1-k)$　线性非时变非因果

$$f(K) = 0(K<0) \to y_f(K) = f(1) \ne 0$$

1.30（1）　$y''' + 6y'' + 12y' + 8y = 5f'' + f$

（2）　y(k+3)-y(k+2)+y(k+1)=f(k+1)+αf(k)

（3）y(k)-y(k-2)=3f(k-1)-f(k-2)

1.31

(1) $y''' + 3y'' + y' = \alpha f'' + f' + 3f$

(2) y(k+2)-2y(k+1)+3y(k)=4f(k+2)-5f(k+1)+6f(k)
(3) y(k+2)-2y(k+1)+4y(k)= α f(k+1)+f(k) 或 y(k)-2y(k-1)+4y(k-2)= α f(k-1)+f(k-2)

1.32
解：有题图可得，

$y_1' = \beta_0 f + \alpha_1 y_1 + \alpha_0 y$

$y_1 = y' - \beta_1 f$

所以，$y'' - \beta_1 f' = \beta_0 f + \alpha_1 y' - \alpha_1 \beta_1 f + \alpha_0 y$

整理得，$y'' - \alpha_1 y' - \alpha_0 y = \beta_1 f' + (\beta_0 - \alpha_1 \beta_1) f$

与给定微分方程可得，

$a_1 = -\alpha_1, a_0 = -\alpha_0, b_1 = \beta_1, b_0 = \beta_0 - \alpha_1 \beta_1$

1、（1） y″+5y′+6y=0　 $y'(0_-)=-1, y(0_-)=1$

解：特征方程$\lambda^2+5\lambda+6=0$

特征根：$\lambda_1=-2, \lambda_2=-3.\quad \therefore y_h(t)=C_1e^{-2t}+C_2e^{-3t}$

代入初始状态有：$\begin{cases} y_h(0_-)=C_1+C_2=1 \\ y_h'(0_-)=-2C_1-3C_2=-1 \end{cases}$ 解之：$C_1=2, C_2=-1$

$\therefore y_h(t)=2e^{-2t}-e^{-3t}$

(2)　$y''+y'=0\qquad y'(0_-)=0, y(0_-)=2$

解：$\lambda^2+1=0\quad \lambda_{1,2}=\pm j$

$\therefore y_h(t)=C_1\cos t+C_2\sin t$　代入初始状态得：$C_1=2, C_2=0$

$\therefore y_h(t)=2\cos t\quad t\geq 0$

2、(1) y″(t)+3y′(t)+2y(t)=f(t), $y'(0_-)=1, y(0_-)=0, f(t)=\varepsilon(t)$

对微分方程两端关于t从0_-到0_+作积分有

$$\int_{0_-}^{0_+} y''(t)dt+3\int_{0_-}^{0_+} y'(t)dt+2\int_{0_-}^{0_+} y(t)dt=\int_{0_-}^{0_+}\varepsilon(t)dt$$

$y'(0_+)-y'(0_-)=0$, $y(0_+)-y(0_-)=0$

得$y'(0_+)=y'(0_-)=1, y(0_+)=y(0_-)=0$

(2)　y″+6y′+8y=f'　$y'(0_-)=1, y(0_-)=0, f=\varepsilon(t)$

$$\therefore \int_{0_-}^{0_+} y''dt+6\int_{0_-}^{0_+} y'dt+8\int_{0_-}^{0_+} ydt=\int_{0_-}^{0_+}\delta(t)dt$$

得：　　　$y'(0_+)-y'(0_-)=1$，$y(0_+)-y(0_-)=0$

$$\therefore \begin{cases} y'(0_+)=1+y'(0_-)=2 \\ y(0_+)=y(0_-)=0 \end{cases}$$

3）$y''+4y'+3y=f'+f$，$y'(0_-)=1, y(0_-)=0, f=\varepsilon(t)$

上式可写为　$y''+4y'+3y=\delta(t)+\varepsilon(t)$

$t=0$时微分方程左端只有y''含冲激，其余均为有限值，故有

$$\int_{0_-}^{0_+} y''dt+4\int_{0_-}^{0_+} y'dt+3\int_{0_-}^{0_+} ydt=\int_{0_-}^{0_+}\delta(t)dt+\int_{0_-}^{0_+}\varepsilon(t)dt$$

得 $y'(0_+)-y'(0_-)=1, y(0_+)-y(0_-)=0$

$$\therefore \begin{cases} y'(0_+)=1+y'(0_-)=2 \\ y(0_+)=y(0_-)=0 \end{cases}$$

4）$y''+4y'+5y=f', y'(0_-)=2, y(0_-)=1, f(t)=e^{-2t}\varepsilon(t)$

$f'(t)=\delta(t)-2e^{-2t}\varepsilon(t)$

原方程可写为 $y''+4y'+5y=\delta(t)-2e^{-2t}\varepsilon(t)$

$$\int_{0_-}^{0_+}y''dt+4\int_{0_-}^{0_+}y'dt+5\int_{0_-}^{0_+}ydt=\int_{0_-}^{0_+}\delta(t)dt-2\int_{0_-}^{0_+}e^{-2t}\varepsilon(t)dt$$

$$\therefore y'(0_+)-y'(0_-)=1, y(0_+)-y(0_-)=0$$

$$\begin{cases}y'(0_+)=y'(0_-)=3\\ y(0_+)=y(0_-)=1\end{cases}$$

$3.(1)\ y''+4y'+3y=f, y'(0_-)=y(0_-)=1, f(t)=\varepsilon(t)$

解：①求 $y_x(t)$　　$y_x''+4y_x'+3y_x=0$　　$\lambda^2+4\lambda+3=0$

$\lambda_1=-1, \lambda_2=-3$

$y(t)=c_1e^{-t}+c_2e^{-3t}$

$$\begin{cases}y_x(0_-)=C_1+C_2=1\\ y_x'(0_-)=-C_1-3C_2=1\end{cases}$$

解之:　$C_1=2$　　$C_2=-1$

$\therefore y_x(t)=2e^{-t}-e^{-3t}$　　$t\ge 0$

②求 $y_f(t)$　　$y_f(t)=C_{f1}e^{-t}+C_{f2}e^{-3t}+y_p(t)$

设 $y_f(t)=P_0$ 带如原微分方程有 $3P=1$ 即 $P_0=\frac{1}{3}$

故：$y_f(t)=C_{f1}e^{-t}+C_{f2}e^{-3t}+\frac{1}{3})$

对原微分方程两端从 0_- 到 0_+ 关于 t 积分有

$$\int_{0_+}^{0_-}y_f''dt+4\int_{0_+}^{0_-}y_f'dt+3\int_{0_+}^{0_-}y_fdt=\int_{0_+}^{0_-}\varepsilon(t)dt$$

$$\begin{cases}y_f'(0_+)-y_f'(0_-)=0\\ y_f(0_+)-y_f(0_-)=0\end{cases}\qquad\begin{cases}y_f'(0_+)=0\\ y_f'(0_+)=0\end{cases}$$

有：$$\begin{cases}y_f'(0_+)=-C_{f1}-3C_{f2}=0\\ y_f'(0_+)=C_{f1}+C_{f2}+\frac{1}{3}=0\end{cases}$$

解之：$C_{f1}=-\frac{1}{2}\quad C_{f2}=-\frac{1}{6}$

$\therefore\ y_f(t)=(-\frac{1}{2}e^{-t}+\frac{1}{6}e^{-3t}+\frac{1}{3})\varepsilon(t)$

③求全响应 $y(t)$。

$$y(t)=y_x(t)+y_f(t)=2e^{-t}-e^{-3t}-\frac{1}{2}e^{-t}+\frac{1}{6}e^{-3t}+\frac{1}{3}$$
$$=\frac{3}{2}e^{-t}-\frac{5}{6}e^{-3t}+\frac{1}{3}\qquad t\geq 0$$

（2）$y''+4y'+4y=f'+3f$ ，$y'(0\_)=2, y(0\_)=1, f=e^{-t}\varepsilon(t)$

解：① $\lambda^2+4\lambda+4=0\qquad \lambda_{1,2}=-2$。

$y_x(t)=(C_{x0}+C_{x2})e^{-2t}$

$$\begin{cases}y'_x(0_-)=-2C_{x_0}+C_{x_1}=2\\ y_x(0_-)=C_{x_0}=1\end{cases}\qquad 得\ C_{x_0}=1, C_{x_1}=4$$

$\therefore\ y_x(t)=(1+4t)e^{-2t}\qquad t\geq 0$

(2)求$y_f(t)$

$y_f(t)=(C_{f_0}+C_{f_1}t)e^{-2t}+y_{fp}(t)$

设$y_{fp}(t)=p_1e^{-2t}$并代入原微分方程，有

$(p_1e^{-2t})''+4(p_1e^{-2t})'+4(p_1e^{-2t})=(e^{-t})'+3e^{-t}$

得 $p_1-4p_1+4p_1=-1+3\qquad$ 即$p_1=2$

故 $y_f(t)=(C_{f_0}+C_{f_1}t)e^{-2t}+2e^{-t}$

由$\int_{0_-}^{0_+}y_f''dt+4\int_{0_-}^{0_+}y_f'dt+4\int_{0_-}^{0_+}y_f dt=\int_{0_-}^{0_+}[\delta(t)-e^{-t}\varepsilon(t)]dt+3\int_{0_-}^{0_+}e^{-t}\varepsilon(t)dt$

有$\begin{cases}y'_f(0_+)-y'_f(0_-)=1\\ y_f(0_+)-y_f(0_-)=0\end{cases}\qquad \begin{cases}y'_f(0_+)=1+y'_f(0_-)=1\\ y_f(0_+)=0\end{cases}$

$\therefore\begin{cases}y'_f(0_+)=-2C_{f_0}+C_{f1}-2=1\\ y_f(0_+)=C_{f_0}+2=0\end{cases}\qquad$ 解之：$C_{f_0}=-2, C_{f1}=-1$

$\therefore\ y_f(t)=[2e^{-t}-(2+t)e^{-2t}]\varepsilon(t)$

(3)求$y(t)$

$y(t)=y_f(t)+y_x(t)=2e^{-t}+(3t-1)e^{-2t}\qquad t\geq 0$

$y''+2y'+2y=f', y'(0_-)=1, y(0_-)=0, f=\varepsilon(t)$

解:1. 求$y_x(t)$

$\lambda^2+2\lambda+2=0, \lambda_{1,2}=-1\pm j$

$\therefore y_x(t)=e^{-t}(C_{x1}\cos t+C_{x2}\sin t)$

$y'_x(t)=e^{-t}(C_{x2}\cos t-C_{x1}\sin t)-e^{-t}(C_{x1}\cos t+C_{x2}\sin t)$

代入初始状态：$y_x(0_-)=C_{x1}=0, y'_x(0_-)=C_{x2}=1$

$\therefore y_x(t)=e^{-t}\sin t \qquad t\geq 0$

2.求$y_f(t)$　　首先确定$y'_f(0_+)$与$y_f(0_+)$

$\int_{0_-}^{0_+} y_f''dt+2\int_{0_-}^{0_+} y_f'dt+2\int_{0_-}^{0_+} y_f dt=\int_{0_-}^{0_+}\delta(t)dt$

可得 $y'_f(0_+)-y'_f(0_-)=1, y_f(0_+)-y_f(0_+)=0;$

则$\begin{cases} y'_f(0_+)=1 \\ y_f(0_+)=0 \end{cases} \qquad y_f''+2y_f'+2y_f=\delta(t)$

当$t\geq 1$时，$y_f''+2y_f'+2y_f=0$

$\therefore y_f(t)=e^{-t}(A\cos t+\sin t)$

代入初始条件：$y'_f(0_+)=B=1, y_f(0_+)=A=0$

$\therefore y_f(t)=e^{-t}\sin t\varepsilon(t)$

3.求全响应$y(t)$

$y(t)=y_x+y_f=2e^{-t}\sin t \qquad t\geq 0$

2.4 （1）y(k+2)+3y(k+1)+2y(k)=0, $y_x(0)=2, y_x(1)=1$

解：特征方程 $r^2+3r+2=0$

(r+1)(r+2)=0

特征根：　$r_1=-1, r_2=-2$

y(k)=$C_{x_1}r_1^k+C_{x_2}r_2^k=C_{x_1}(-1)^k+C_{x_2}(-2)^k$

代入初始条件 $\begin{cases} C_{x_1}+C_{x_2}=2 \\ -C_{x_1}-2C_{x_2}=1 \end{cases}$ 解得 $C_{x_1}=5, C_{x_2}=-3$

$\therefore y_x(k)=5(-1)^k-3(-2)^k \qquad k\geq 0$

(2)y(k+2)+2y(k+1)+2y(k)=0.　$y_x(0)=0, y_x(1)=1.$

解：

$r^2+2r+2=0 \rightarrow r_{1,2}=-1\pm j$

$y_x(k)=C_{x_1}(-1+j)^k+C_{x_2}(-1-j)^k$

$$\begin{cases} y_x(0)=C_{x_1}+C_{x_2}=0 \\ y_x(1)=(-1+j)C_{x_1}+(-1-j)C_{x_2}=1 \end{cases}$$

$C_{x_1}=-\dfrac{j}{2}, C_{x_2}=\dfrac{j}{2}$

$\therefore y_x(k)=(-\dfrac{j}{2})(-1+j)^k+\dfrac{j}{2}(-1-j)^k$

$=(\sqrt{2})^k e^{j\frac{3\pi}{2}k}=(\sqrt{2})^k \sin\dfrac{3\pi}{4}k$

k≥0

(3)y(k+2)+2y(k+1)+y(k)=0 $y_x(0)=y_x(1)=1$

解：

$r^2+2r+1=0 \quad (r+1)^2=0 \therefore r_1=r_2=-1$

$y_x(k)=(C_{x_1}+C_{x_2}K)(-1)^k$

$$\begin{cases} y_x(0)=C_{x_1}=1 \\ y_x(1)=(C_{x_1}+C_{x_2})(-1)=1 \end{cases}$$

$$\therefore \begin{cases} C_{x_1}=1 \\ C_{x_2}=-2 \end{cases}$$

$y_x(k)=(1-2k)(-1)^k$ k≥0

(4)

$y(k)+2y(k-1)=0 \quad y_x(0)=2$

解：$\gamma-2=0 \quad \gamma=2 \quad y_x(k)=C_x(2)^k$

$y_x(0)=C_x=2$ 故 $y_x(k)=2(2)^k$ k>=0

(5)

$y(k)+2y(k-1)+4y(k-2)=0 \quad y_x(0)=0, y_x(1)=2$

解：

$\gamma^2+2\gamma+4=0$ 即 $(\gamma+1)^2+3=0$

特征根 $\gamma_{1,2}=-1\pm\sqrt{3}j$

$y_x(k)=C_{x1}(-1+\sqrt{3}j)^k+C_{x2}(-1-\sqrt{3}j)^k$

$C_{x1}=\frac{1}{j\sqrt{3}}$ $\quad C_{x2}=C_{x1}^{*}=-\frac{1}{j\sqrt{3}}$

故 $y_x(k)=\frac{1}{\sqrt{3}}[\frac{(-1+j\sqrt{3})^k}{j}-\frac{(-1-j\sqrt{3})^k}{j}]$

$=\frac{2^{k+1}}{\sqrt{3}}\frac{e^{j\frac{2\pi}{3}k}-e^{-j\frac{2\pi}{3}k}}{2j}=\frac{2}{\sqrt{3}}2^k\sin\frac{2\pi}{3}k$ k>=0

(6) $y(k)-7y(k-1)+16y(k-2)-12y(k-3)=0$

$y_x(0)=0,\quad y_x(1)=-1$，$y_x(2)=-3$

解：$\gamma^3-7\gamma^2+16\gamma-12=0$ 即 $(\gamma-3)(\gamma-2)^2=0$

$\gamma_1=3\quad \gamma_{2,3}=2$

$y_x(k)=C_{x0}(3)^k+(C_{x1}+C_{x2}k)(2)^k$

带入初始条件有

$$\begin{cases} y_x(0)=c_{x0}+c_{x1}=0 & c_{x0}=-c_{x1} \\ y_x(1)=3c_{x0}+2c_{x1}+2c_{x2}=-1 & -c_{x1}+2c_{x2}=-1 \\ y_x(2)=9c_{x0}+4c_{x1}+8c_{x2}=-3 & -5c_{x1}+8c_{x2}=-3 \end{cases}$$

解之得：$c_{x0}=1,\qquad c_{x1}=-1$，$\qquad c_{x2}=-1$

故：$\quad y_x(k)=3^k+(1+k)(2^k)\quad$ k>=0

2.5(1) $\quad y(k)+3y(k-1)+2y(k-2)=f(k),y(-1)=0,y(-2)=1$

解：$\gamma^2+3\gamma+2=0\qquad \gamma_1=-1,\gamma_2=-2$

$y_x(k)=c_{x1}(-1)^k+c_{x2}(-2)^k$

$$\begin{cases} y_x(-1)=c_{x1}(-1)^{-1}+c_{x2}(-2)^{-1}=0 \\ y_x(-2)=c_{x1}(-1)^{-2}+c_{x2}(-2)^{-2}=1 \end{cases}$$ 即：$\begin{cases} 4c_{x1}+c_{x2}=4 \\ -2c_{x1}-c_{x2}=0 \end{cases}$

解之得：$\begin{cases} c_{x1}=2 \\ c_{x2}=-4 \end{cases}$ 故：$y(k)=\left[2(-1)^k-4(-2)^k\right]k>=0$

（2）$y(k)+2y(k-1)+y(k-2)=f(k)-f(k-1)\qquad y(-1)=1, y(-2)$

解：$y_x(k)+2y_x(k-1)+y_x(k-2)=0$

$\gamma^2+2\gamma+1=0 \qquad (\gamma+1)^2=0 \qquad \gamma_{1,2}=-1$

$y_x(k)=(c_{x1}+c_{x2}k)(-1)^k$

$$\begin{cases} y_x(-1)=-c_{x1}+c_{x2}=1 \\ y_x(-2)=-c_{x1}-2c_{x2}=3 \end{cases} \qquad \begin{cases} c_{x1}=1 \\ c_{x2}=2 \end{cases}$$

故：$y_x(k)=(1+2k)(-1)^k\ k>=0$

（3） $y(k)+y(k-2)=f(k-2), y(-1)=-2, y(-2)=-1$

解：$\gamma^2+1=0; \quad \gamma_{1,2}=\pm j$

$$y_x(k)=A\cos\frac{\pi}{2}k+B\sin\frac{\pi}{2}k$$

$y(-1)=-B=-2$

$y(-2)=A=-1$

$$y_x(k)=(-\cos\frac{\pi}{2}k+2\sin\frac{\pi}{2}k)$$
$$=\sqrt{5}\cos(\frac{\pi}{2}k-63.4^\circ)k\geq 0$$

2.6 (1) $y(k)-2y(k-1)=f(k), y(-1)=-1, f(k)=2\varepsilon(k)$

解：$\gamma-2=0, \gamma=2 \qquad y(k)=C(2)^k+y_p(k)=C2^k-2$

$y_p(k)=p_0, p_0-2p_0=2, p_0=-2$

令$k=0, y(0)-2y(-1)=2, y(0)=0$

$y(0)=C-2, C=2$

所以 $y(k)=2(2)^k=2, k\geq 0$

其中$y_x(k)=C_x(2)^k$
$=-2(2)^k, k\geq 0$ $\qquad \frac{C_x}{2}=-1, C_x=-2$

$$y_f(k)=C_f(2)^k+y_{fp}(k)$$
$$=y(k)-y_x=2(2)^k-2-[-2(2)^k]$$
$$=[4(2)^k-2]\varepsilon(k)$$

(2) $y(k)+3y(k-1)+2y(k-2)=f(k)$

$y(-1)=1, y(-2)=0, f(k)=\varepsilon(k)$

解：$\gamma^2+3\gamma+2=0 \Rightarrow \gamma_1=-1, \gamma_2=-2$

$y_x(k)=C_{x_1}(-1)^k+C_{x_2}(-2)^k$

$$\begin{cases} y_x(-1)=y(-1)=-C_{x_1}-\frac{1}{2}C_{x_2} \\ y_x(-2)=y(-2)=C_{x_1}+\frac{1}{4}C_{x_2} \end{cases} \Rightarrow \begin{cases} C_{x_1}=1 \\ C_{x_2}=-4 \end{cases}$$

$\therefore y_x(k)=(-1)^k-4(-2)^k \quad k\geq 0$

令$y_{fp}=P$.则有$P_0+3P_0+2P_0=1, P_0=\frac{1}{6}$

由$y(k)=f(k)-3y(k-1)-2y(k-2)$得：

$$\begin{cases} y_f(0)=f(0)-y_f(-1)-2y_f(-2)=1 \\ y_f(1)=f(1)-3y_f(0)-2y_f(-1)=-2 \end{cases} \Rightarrow \begin{cases} y_f(0)=1 \\ y_f(1)=-2 \end{cases}$$

$$\begin{cases} y_f(0)=C_{f1}+C_{f2}+\frac{1}{6}=1 \\ y_f(1)=-C_{f1}-2C_{f2}+\frac{1}{6}=-2 \end{cases} \text{解之得:} \begin{cases} C_{f1}=-\frac{1}{2} \\ C_{f2}=\frac{4}{3} \end{cases}$$

$\therefore y_f(k)=[-\frac{1}{2}(-1)^k+\frac{4}{3}(-2)^k+\frac{1}{6}]\varepsilon(k)$

$y(k)=y_x(k)+y_f(k)=[(-1)^k-4(-2)^k-\frac{1}{2}(-1)^k+\frac{4}{3}(-2)^k+\frac{1}{6}]$

$=[\frac{1}{2}(-1)^k-\frac{8}{3}(-2)^k+\frac{1}{6}] \quad k\geq 0$

$$g_i(t)=\frac{1}{9}(1-e^{-st})\varepsilon(t)$$

2.7 (a)解：$h_i(t)=\frac{dg_i(t)}{dt}=\frac{5}{9}e^{-st}\varepsilon(t)$

$$h_u(t)=Rh_i(t)=\frac{10}{3}e^{-st}\varepsilon(t)$$

(b)解:由图知$i_c+i_r+i_l=i_s$

其中：$i_c=c\frac{du_c}{dt}=Lc\frac{d^2i_l}{dt^2}$　　　　$i_R=\frac{u_l}{R}=\frac{L}{R}\frac{di_l}{dt}$

故有：$LCi_L''+\frac{L}{R}i_L'+i_L=i_S$即:$\frac{1}{5}i_L''+\frac{2}{5}i_L'+i_L=i_s$

故$i_L''+2i_L'+5i_L=5i_S$　　　　$H(p)=\frac{5}{(p^2+2p+5)}=\frac{5}{(p+1)^2+4}$

$$h_{iL}(t)=\frac{5}{2}e^{-t}\sin 2t\varepsilon(t)$$

$$h_{u_L}=L\frac{dh_{iL}}{dt}=\frac{1}{5}\times\frac{d}{dt}[\frac{5}{2}e^{-t}\sin 2t\varepsilon(t)]$$

$$=\frac{1}{2}[-e^{-t}\sin 2t\varepsilon(t)+2e^{-t}\cos 2t\varepsilon(t)]$$

$$=[e^{-t}\cos 2t-\frac{1}{2}e^{-t}\sin 2t]\varepsilon(t)$$

$$\because u_L=L\frac{di_L}{dt}$$

$$\therefore h_{uL}=L\frac{dh_{iL}}{dt}$$

2．8 （1）

$$y'+2y=f'(t)-f(t)$$

$$H(p)=\frac{p-1}{p+2}=\frac{p+2-3}{p+2}=1-\frac{3}{p+2}$$

$$h(t)=\delta(t)-3e^{-2t}\varepsilon(t)$$

$$g(t)=\int_{0^-}^{t}h(\tau)d\tau=\int_{o^-}^{t}\delta(\tau)d\tau-3\int_{0^-}^{t}e^{-2\tau}d\tau=\varepsilon(t)+\frac{3}{2}(1-e^{-2t})\varepsilon(t)$$

（2） $y'(t)+2y(t)=f''(t)$

$$H(p)=\frac{p^2}{p+2}=\frac{p^2+2p-2p-4+4}{p+2}=p-2+\frac{4}{p+2}$$

$$h(t)=\delta'(t)-2\delta(t)+4e^{-2t}\varepsilon(t)$$

$$g(t)=\int_{0_-}^{t}h(\tau)d\tau=\int_{0_-}^{t}\delta'(\tau)d\tau-2\int_{0_-}^{t}\delta(\tau)d\tau+4\int_{0_-}^{t}e^{-2\tau}d\tau\bullet\varepsilon(t)$$

$$=\delta(t)-2\varepsilon(t)+2\varepsilon(t)-2e^{-2t}\varepsilon(t)$$

$$=\delta(t)-2e^{-2t}\varepsilon(t)$$

2.9，求 $h(t)$

(1) $2y''+8y=f$ $\qquad H(p)=\frac{1}{2p^2+8}=\frac{1}{2}\frac{1}{p^2+4}$

$$h(t)=\frac{1}{4}\sin 2t\varepsilon(t)$$

(2) $y''+y'+y=f'+f$

$$H(p)=\frac{p+1}{p^2+p+1}=\frac{p+\frac{1}{2}+\frac{1}{2}}{(p+\frac{1}{2})^2+\frac{3}{4}}=\frac{p+\frac{1}{2}}{(p+\frac{1}{2})^2+\frac{3}{4}}+\frac{1}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{(p+\frac{1}{2})^2+\frac{3}{4}}$$

$$h(t)=e^{-\frac{t}{2}}\cos\frac{\sqrt{3}}{2}t\varepsilon(t)+\frac{1}{\sqrt{3}}e^{-\frac{t}{2}}\sin\frac{\sqrt{3}}{2}t\varepsilon(t)$$

(3) $y''+2y'+y=f'+2f$

$$H(p)=\frac{p+2}{p^2+2p+1}=\frac{p+2}{(p+1)^2}=\frac{1}{(p+1)^2}+\frac{1}{(p+1)}$$

$$h(t)=(e^{-t}+te^{-t})\varepsilon(t)$$

(4) $y'''+6y''+11y'+6y=f'+2f$

H(p)= $\dfrac{p+2}{p^3+6p^2+11p+6}=\dfrac{p+2}{(p+1)(p+2)(p+3)}=\dfrac{1}{(p+1)(p+2)}$

h(t)= $\dfrac{e^{pt}}{p+3}\Big|_{p=-1}+\dfrac{e^{pt}}{p+1}\Big|_{p=-3}=(\frac{1}{2}e^{-t}-\frac{1}{2}e^{-3t})\varepsilon(t)$

2.10 求 h(k)

(1) y(k)+2y(k-1)=f(k-1)

解：H(E)= $\dfrac{E^{-1}}{1+2E^{-1}}=\dfrac{1}{E+2}\longleftrightarrow h(k)=(-2)^{k-1}\varepsilon(k-1)$

(2) y(k+2)+3y(k+1)+2y(k)=f(k+1)+f(k)

H(E)= $\dfrac{E+1}{E^2+3E+2}=\dfrac{E+1}{(E+1)(E+2)}=\dfrac{1}{E+2}$

$\longleftrightarrow h(k)=(-2)^{k-1}\varepsilon(k-1)$

(3) y(k)+y(k-1)+ $\frac{1}{4}$ y(k-2)=f(k)

解： H(E)= $\dfrac{E^2}{E^2+E+\frac{1}{4}}=\dfrac{E^2}{(E+\frac{1}{2})^2}$

h(k)= $\dfrac{d}{dE}\ (E+\frac{1}{2})^2$ H(E)E $^{k-1}\Big|_{E=\frac{1}{2}}=\dfrac{d}{dE}(\mathrm{E}^{k+1})\Big|_{E=\frac{1}{2}}\ \varepsilon(k)$

=(k+1)E $^{k}\Big|_{E=-\frac{1}{2}}\varepsilon(k)=(k+1)(-\frac{1}{2})^k\varepsilon(k).$

(4) y(k)-4y(k-1)+8y(k-2)=f(k)

解: h(k)-4y(k-1)+8y(k-2)= $\delta(k)$

k>0 时,有 h(k)-4h(k-1)+8h(k-2)=0

$\gamma^2-4\gamma+8=0$ γ_{12}=2 ± j2=2$\sqrt{2}$ $\angle\frac{\pi}{4}$

h(c)=δ (c)+4δ (k-1)-8h(k-2)

h(0)=δ (0)+4h(-1)-8h(-2)=1=p

h(1)=δ (1)+4h(0)-8h(-1)=4=2$\sqrt{2}\frac{1}{\sqrt{2}}Q+2\sqrt{2}\frac{p}{\sqrt{2}}$

故：p=1,Q=1.

H(k)=(2$\sqrt{2}$)^k (sin$\frac{k\pi}{4}$+sin$\frac{k\pi}{4}$) ε (k)

=$\sqrt{2}$ (2$\sqrt{2}$)^k (sin$\frac{k\pi}{4}$+sin$\frac{k\pi}{4}$) ε (k)

(5)

y(k+2)+2y(k+1)+2y(k)=f(k+1)+2f(k)

解： h$_0$(k+2)+2 h$_0$(k+1)+2 h$_0$(k)=δ (k)

h$_o$ (k)=($\sqrt{2}^k$)(pcos$\frac{3k\pi}{4}$+Qsin$\frac{3k\pi}{4}$)

h$_o$ (2)=2[pcos$\frac{3k\pi}{2}$+Qsin$\frac{3k\pi}{2}$]=-2Q=1 所以 Q=-$\frac{1}{2}$

h$_o$ (1)= $\sqrt{2}$ [pcos$\frac{3k\pi}{4}$+Qsin$\frac{3k\pi}{4}$]=$\sqrt{2}$ [p$\frac{1}{\sqrt{2}}-\frac{1}{2}\frac{1}{\sqrt{2}}$]

=p-$\frac{1}{2}$=0 p=+$\frac{1}{2}$

所以 h$_o$ (k)=（$\sqrt{2}^k$） [-$\frac{1}{2}$ sin$\frac{3k\pi}{4}$+$\frac{1}{2}$ cos$\frac{3k\pi}{4}$] ε (k-1)

=$\sqrt{2}^{k-1}$ sin($\frac{3k\pi}{4}$-$\frac{3\pi}{4}$) ε (k-1)

h(k)= h$_o$ (k+1)+2 h$_0$ (k)

=$\sqrt{2}^k$ sin[$\frac{3\pi}{4}$ (k+1)- $\frac{3\pi}{4}$] ε (k)+ 2$\sqrt{2}^{k-1}$ sin($\frac{3k\pi}{4}$-$\frac{3\pi}{4}$) ε (k-1)

=$\sqrt{2}^k$ sin[$\frac{3\pi}{4}$ (k+1)- $\frac{3\pi}{4}$] ε (k)+ 2$\sqrt{2}^{k-1}$ sin($\frac{3k\pi}{4}$-$\frac{3\pi}{4}$) ε (k-1)

=-$\sqrt{2}^k$ cos$\frac{3k\pi}{4}$ ε (k-1)

2.11 (1) $y(k+2)+y(k+1)=2f(k+1)+f(k)$ (由图得)

移序得：$y(k+1)+y(k)=2f(k)-f(k-1)$

设$h_0(k+1)+h_0(k)=\delta(k)$,有$\gamma=-1$

$\therefore h_0(k)=C(-1)^k\varepsilon(k-1).$

又$\because h_0(1)=1 \therefore C=-1$

$\therefore h_0(k)=-(-1)^k\varepsilon(k-1)=(-1)^{k-1}\varepsilon(k-1)$

$h(k)=2h_0(k)-h_0(k-1)$

$=2(-1)^{k-1}\varepsilon(k-1)+(-1)^{k-2}\varepsilon(k-2)$

(2)由图可得差分方程

$y(k+2)-y(k+1)+0.24y(k)=f(k+2)-0.5f(k+1)$

$$H(E)=\frac{E^2-0.5E}{E^2-E+0.24}=\frac{E^2-0.5E}{(E-0.4)(E-0.6)}$$

$$\frac{H(E)}{E}=\frac{1}{2}\times\frac{1}{E-6}+\frac{1}{2}\times\frac{1}{E-0.4}$$

$$\therefore H(E)=\frac{1}{2}\times\frac{E}{E-6}+\frac{1}{2}\times\frac{E}{E-0.4}$$

$$\leftrightarrow h(k)=[\frac{1}{2}(0.6)^k+\frac{1}{2}(0.4)^k]\varepsilon(k)$$

2.12 图示法求解下列卷积

$t<0 \quad f(t)=f_1(t)*f_2(t)=0$

$0\le t<2 \quad f(t)=\frac{1}{2}\int_0^t \tau d\tau=\frac{1}{4}\tau\,|_0^t=\frac{1}{4}t$

$2\le t<4 \qquad f(t)=f_1(t)*f_2(t)$

$$=\frac{1}{2}\int_t^2 \tau d\tau=\frac{1}{4}\tau\Big|_t^2=\frac{1}{4}(2-t)$$

$4\le t \qquad f=0$

$$\therefore f(t)=\begin{cases}0 & t<0\\ \frac{1}{4}t & 0\le t<2\\ \frac{1}{4}(2-t) & 2\le t<4\\ 0 & 4\le t\end{cases}$$

(2)$f_2(t-\tau)=e^{-(t-\tau+1)}\varepsilon(t-\tau+1)$

$t<1$

$$f(t)=\int_{-\infty}^{t+1}e^{-(t-\tau+1)}d\tau=e^{-(t+1)}\int_{-\infty}^{t+1}e^{\tau}d\tau$$

$$=e^{-(t+1)}e^{\tau}\Big|_{-\infty}^{t+1}=e^{-(t+1)}[e^{t+1}-e^{-\infty}]=1$$

$1\le t$

$$f(t)=\int_{-\infty}^{1}e^{-(t-\tau+1)}d\tau+2\int_1^{t+1}e^{-(t-\tau+1)}d\tau$$

$$=e^{-(t+1)}\int_{-\infty}^{1}e^{\tau}d\tau+2e^{-(t+1)}\int_1^{t+1}e^{\tau}d\tau$$

$$=e^{-(t+1)}e^{\tau}\Big|_{-\infty}^{1}+2e^{-(t+1)}e^{\tau}\Big|_1^{t+1}$$

$=e^{-(t+1)}[e-e^{-\infty}]+2e^{-(t+1)}[e^{t+1}-e]$

$=e^{-t}+2-2e^{-t}=2-e^{-t}$

$$\therefore f(t)=\begin{cases}1 & t<1\\ 2-e^{-t} & 1\le t\end{cases}$$

(3) $0\le t<\pi$

$f(t)=f_1(t)*f_2(t)=\int_0^t \sin\tau d\tau=-\cos\tau\Big|_0^t=1-\cos t$

$\pi\le t<2\pi$

$f(t)=\int_{t-1}^{\pi}\sin\tau d\tau=-\cos\tau\Big|_{t-1}^{\pi}=1-\cos(t-1)$

所以

$$f(t)=\begin{cases}0 & t<0\\ 1-\cos t & 0\le t<\pi\\ 1-\cos(t-1) & \pi\le t<2\pi\\ 0 & 2\pi\le t\end{cases}$$

(4)

$f(t)=\Delta_4(t+2)+\Delta_4(t-2)$

2.14 解：由右图知

当 $3\le t<5$

$y_f(t)=2\int_1^3 d\tau+4\int_3^t d\tau=2\tau\Big|_1^3+4\tau\Big|_3^t=2+4(t-3)$

所以 $y_f(4)=6$

2.15.证明：令$f(t)=\varepsilon(t-a_1)-\varepsilon(t-b_1), h(t)=\varepsilon(t-a_2)-\varepsilon(t-b_2)$

（作图略）

显然（a_1+a_2, b_1+b_2）以外的时间区间$y_f(t)=0$(证毕)

2.16 计算下列卷积

(1) $\varepsilon(t)*\varepsilon(t)=\varepsilon(t)\int_0^t d\tau=t\varepsilon(t)$

(2) $e^{-2t}\varepsilon(t)*\varepsilon(t)=\int_0^t e^{-2\tau}d\tau\varepsilon(t)=\frac{1}{2}(1-e^{-2t})\varepsilon(t)$

(3) $e^{-t}\varepsilon(t)*e^{-2t}\varepsilon(t)=\int_{-\infty}^{\infty}e^{-\tau}\varepsilon(\tau)e^{-2(t-\tau)}\varepsilon(t-\tau)d\tau$

$=e^{-2t}\int_0^t e^{\tau}d\tau\varepsilon(t)=e^{-2t}(e^{t}-1)\varepsilon(t)=(e^{-t}-e^{-2t})\varepsilon(t)$

(4) $t\varepsilon(t)*e^{-2t}\varepsilon(t)=\int_{-\infty}^{\infty}\tau\varepsilon(\tau)e^{-2(t-\tau)}\varepsilon(t-\tau)d\tau$

$$=e^{-2t}\int_0^t \tau e^{\tau} d\tau = e^{-2t}[\frac{\tau}{2}e^{2\tau}|_0^t - \frac{1}{2}\int_0^t e^{2\tau}d\tau]\varepsilon(t)$$

$$=e^{-2t}[\frac{t}{2}e^{2t} - \frac{1}{4}e^{2\tau}|_0^t]\varepsilon(t)$$

$$=e^{-2t}[\frac{t}{2}e^{2t} - \frac{1}{4}e^{2t} + \frac{1}{4}]\varepsilon(t)$$

$$=[\frac{t}{2} + \frac{1}{4}e^{-2t} - \frac{1}{4}]\varepsilon(t)$$

(5) $e^{-t}\varepsilon(t) * \cos t\varepsilon(t) = \int_0^t \cos\tau e^{-(t-\tau)} d\tau\varepsilon(t)$

$$=e^{-t}[\int_0^t \cos\tau e^{\tau} d\tau]\varepsilon(t) = \frac{1}{2}e^{-t}[\cos te^{t} - 1 + \sin te^{t}]\varepsilon(t)$$

$$=\frac{1}{2}[\cos t + \sin t - e^{-t}]\varepsilon(t)$$

(6)...$t\varepsilon(t) * [\varepsilon(t) - \varepsilon(t-2)] = \int_{-\infty}^{\infty} \tau\varepsilon(\tau)[\varepsilon(t-\tau) - \varepsilon(t-\tau-2)]d\tau$

$$=\int_{-\infty}^{\infty} \tau\varepsilon(\tau)\varepsilon(t-2)d\tau - \int_{-\infty}^{\infty} \tau\varepsilon(\tau)\varepsilon(t-\tau-2)d\tau$$

$$=\frac{\tau^2}{2}|_0^t\ \varepsilon(t) - \frac{\tau^2}{2}|_0^{t-2}\ \varepsilon(t-2) = \frac{t^2}{2}\varepsilon(t) - \frac{1}{2}(t-2)^2\varepsilon(t-2)$$

(7) $e^{-2t}\varepsilon(t+1) * \varepsilon(t-3)$

$$=\int_{-\infty}^{\infty} e^{-2\tau}\varepsilon(\tau+1)\varepsilon(t-\tau-3)d\tau = \int_{-1}^{t-3} e^{-2\tau} d\tau\varepsilon(t-2) = -\frac{1}{2}[e^{-2(t-3)} - e^{2}]\varepsilon(t-2)$$

$$=\frac{1}{2}e^{2}\varepsilon(t-2) - \frac{1}{2}e^{-2(t-3)}\varepsilon(t-2)$$

(8) $e^{-t} * e^{-2t}\varepsilon(t) = \int_{-\infty}^{\infty} e^{-\tau}e^{-2(t-\tau)}\varepsilon(t-\tau)d\tau$

$$=e^{-2t}\int_{-\infty}^{t} e^{\tau} d\tau = e^{-t}$$

(9) 由图知，当 t-1<2 即 t<3 时

$\varepsilon(t-1) * e^{t}\varepsilon(2-t)$

$=\int_{-\infty}^{t-1} e^{\tau} d\tau = e^{t-1}$

当 $3 \le t$ 时，原式$=\int_{-\infty}^{2} e^{\tau} d\tau = e^{\tau}|_{-\infty}^{2} = \mathrm{e}^2$

2.17 $f_1(t) = \Delta_4(t)$． $f_2(t) = \delta(t+2) + \delta(t-2)$． $f_3(t) = \delta(t+1) + \delta(t-1)$

$f_4(t) = \delta(t-2) - \delta(t-3) + \delta(t-4)$ 则

(1) $f_1(t) * f_2(t) = \Delta_4(t) * [\delta(t+2) + \delta(t-2)]$

$$= \Delta_4(t+2) + \Delta_4(t-2)$$

$$(2) f_1(t) * f_3(t) = \Delta_4(t) * [\delta(t+1) + \delta(t-1)]$$

$$= \Delta_4(t+1) + \Delta_4(t-1)$$

$$(3) f_1(t) * f_4(t) = \Delta_4(t) * [\delta(t-2) - \delta(t-3) + \delta(t-4)]$$

$$= \Delta_4(t-2) - \Delta_4(t-3) + \Delta_4(t-4)$$

$$(4) f_1(t) * f_2(t) * f_2(t) = [\Delta_4(t+2) + \Delta_4(t-2)] * [\delta(t+2) + \delta(t-2)]$$

$$= \Delta_4(t+4) + 2\Delta_4(t) + \Delta_4(t-4)$$

$$(5) f_1(t) * [2 f_4(t) - f_3(t-3)] = \Delta_4(t) * [2\delta(t-2) - 2\delta(t-3) + 2\delta(t-4)$$

$$- \delta(t-2) - \delta(t-4)]$$

$$= \Delta_4(t) * [\delta(t-2) - 2\delta(t-3) + \delta(t-4)]$$

$$= \Delta_4(t-2) - 2\Delta_4(t-3) + \Delta_4(t-4)$$

2.18 解：$(1)\delta_T(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT)$ $\qquad y_1(t) = f(t) * \delta_T(t) * g_\tau(t)$

$$(2) y_2(t) = [\delta_T(t) * g_\tau(t)] \times f(t)$$

2.19. (1) $f(t) = t^2 * \delta(t) = t^2$

(2) $f(t) = e^{-2t}\varepsilon(t) * \delta(3t-2) = e^{-2t}\varepsilon(t) * \frac{1}{3}\delta(t-\frac{2}{3})$

$$= \frac{1}{3} e^{-2(t-\frac{2}{3})} \varepsilon(t-\frac{2}{3})$$

(3)

$$f(t) = \sin \pi t[\varepsilon(t) - \varepsilon(t-1)] * [\delta(t) + \delta(t-1)]$$
$$= \sin \pi t[\varepsilon(t) - \varepsilon(t-1)] + \sin \pi(t-1)[\varepsilon(t-1) - \varepsilon(t-2)]$$

(4)

$$f(t) = e^{-t}\varepsilon(t) * \delta'(t) * \varepsilon(t) = e^{-t}\varepsilon(t) * [t\delta(t)]$$
$$= e^{-t}\varepsilon(t)$$

(5)

$$f(t)=e^{-2t}\varepsilon(t)*\delta''(t)*t\varepsilon(t)$$
$$=e^{-2t}\varepsilon(t)*\delta(t)=e^{-2t}\varepsilon(t)$$

2.20 解: (1) $y_f(t)=\int_{-\infty}^{t}e^{-(t-\tau)}f(\tau-2)d\tau$

令 $x=\tau-2$ 则

$$y_f(t)=\int_{-\infty}^{t-2}f(x)e^{-(t-x-2)}dx$$

$$=\int_{-\infty}^{\infty}f(x)e^{-(t-x-2)}\varepsilon(t-2-x)dx$$

所以 $h(t)=e^{-(t-2)}\varepsilon(t-2)$ ， $f(t)=\varepsilon(t+1)-\varepsilon(t-2)$

(2)

$$y_f(t)=\varepsilon(t+1)*h(t)$$
$$=\int_{-\infty}^{\infty}[\varepsilon(t+1)-\varepsilon(t-2)]e^{-(t-\tau-2)}\varepsilon(t-\tau-2)d\tau$$
$$=\int_{-\infty}^{\infty}\varepsilon(\tau+1)e^{-(t-\tau-2)}\varepsilon(t-\tau-2)d\tau-\int_{-\infty}^{\infty}\varepsilon(\tau-2)e^{-(t-\tau-2)}\varepsilon(t-\tau-2)d\tau$$

$$y_f(t)=\int_{-1}^{t-2}e^{-(t-2)}e^{\tau}d\tau\varepsilon(t-1)-\int_{2}^{t-2}e^{-(t-2)}e^{\tau}d\tau\varepsilon(t-4)$$
$$=e^{-(t-2)}e^{\tau}\Big|_{-1}^{t-2}\ \varepsilon(t-1)-e^{-(t-2)}e^{\tau}\Big|_{2}^{t-2}\ \varepsilon(t-4)$$
$$=e^{-(t-2)}[e^{(t-2)}-e^{-1}]\varepsilon(t-1)-e^{-(t-2)}[e^{(t-2)}-e^{2}]\varepsilon(t-4)$$
$$=[1-e^{-(t-1)}]\varepsilon(t-1)-[1-e^{-(t-4)}]\varepsilon(t-4)$$

2.21 (1) $y_f(t)=\varepsilon(t)+\varepsilon(t-1)-\varepsilon(t-2)-\varepsilon(t-3), f(t)=\varepsilon(t)-\varepsilon(t-2)$

(图略)

$\therefore h(t)=\delta(t)+\delta(t-1)$

$y_{f_2}(t)=\varepsilon(t)-\varepsilon(t-2)$

2.22(图略)

（2） $h(t)=t\varepsilon(t)-2(t-1)\varepsilon(t-1)+(t-2)\varepsilon(t-2)$

2．23 $y^{''}+3y^{'}+2y=f^{'}+3f$

解 （1）求 $h(t)$

$$H(p)=\frac{p+3}{p^2+3p+2}=\frac{p+3}{(p+1)(p+2)}=\frac{2}{p+1}-\frac{1}{p+2}$$

$$h(t)=2e^{-t}-e^{-2t}\varepsilon(t)$$

$(2)f_1(t)=\varepsilon(t)$

$$y_{f_1}=\varepsilon(t)*h(t)$$

$$\begin{aligned}
&=\varepsilon(t)*[2e^{-t}\varepsilon(t)-e^{-2t}\varepsilon(t)]\\
&=2\int_{-\infty}^{\infty}\varepsilon(\tau)e^{-(t-\tau)}d\tau-\int_{-\infty}^{\infty}\varepsilon(\tau)e^{-2(t-\tau)}\varepsilon(t-\tau)d\tau\\
&=2\int_0^t e^{-t}e^{\tau}d\tau\varepsilon(t)-\int_0^t e^{-2t}e^{2\tau}d\tau\varepsilon(t)\\
&=2e^{-t}[e^t-1]\varepsilon(t)-\frac{e^{-2t}}{2}[e^{2t}-1]\varepsilon(t)\\
&=(1.5-2e^{-t}+0.5e^{-2t})\varepsilon(t)
\end{aligned}$$

$$f_2(t)=e^{-3t}\varepsilon(t)$$

$$y_{f_2}(t)=e^{-3t}\varepsilon(t)*[2e^{-t}\varepsilon(t)-e^{-2t}\varepsilon]$$

$$=2e^{-3t}\varepsilon(t)*e^{-t}\varepsilon(t)-e^{-3t}\varepsilon(t)*e^{-2t}\varepsilon(t)$$

$$y_{f_2}(t)=2\int_0^t e^{-3\tau}e^{-(t-2)}d\tau\varepsilon(t)-\int_0^t e^{-3\tau}e^{-2(t-\tau)}d\tau\varepsilon(t)$$

$$=2\int_0^t e^{-t}e^{-2\tau}d\tau\varepsilon(t)-e^{-2t}\int_0^t e^{-\tau}d\tau\varepsilon(t)$$

$$=2(-\frac{1}{2})e^{-t}e^{-2\tau}\Big|_0^t\varepsilon(t)-e^{-2t}(-e^{-\tau})\Big|_0^t\varepsilon(t)$$

$$=(-1)e^{-t}(e^{-2t}-1)\varepsilon(t)-e^{-2t}(e^{-t}-1)\varepsilon(t)$$

$$=(e^{-t}-e^{-2t})\varepsilon(t)$$

2.24解： $y''+3y'+2y=2f'+f.$

$$\mathrm{H}(p)=\frac{2p+1}{p^2+3p+2}=\frac{2p+1}{(p+1)(p+2)}$$

$$\therefore h(t)=(3e^{-2t}-e^{-t})\varepsilon(t)$$

$$y_f(t)=f(t)*h(t)=\varepsilon(t)*[3e^{-2t}-e^{-t}]\varepsilon(t)$$

$$=3\int_0^t e^{-2(t-\tau)}d\tau\varepsilon(t)-\int_0^t e^{-(t-\tau)}d\tau\varepsilon(t)$$