Frequency Response

2. Bode plots (Logarithmic plots)

The use of semilog paper eliminates the need to take logarithms of very many numbers and also expands the low-frequency rang, which is of primary importance.

The advantages of logarithmic plot are following

1) **The mathematical operations of multiplication and division are transformed to addition and subtraction;**
2) **The work of obtaining the transfer function(fall into several categories) is largely graphical instead of analytical.**

Preliminary design------using the straight-line asymptotic approximations to obtain approximate performance characteristics

Specified design-------correcting the straight-line curves for greater accuracy.

Logarithmic plots $\xrightarrow{\text{enough data}}$ polar plots

Frequency Response

2. Bode plots (Logarithmic plots)

Basic definition of logarithmic terms:

Logarithm:

The logarithm of a complex number is itself a complex number. The abbreviation "log" is used to indicate the logarithm to the base 10

$$\log|G(j\omega)|e^{j\varphi(\omega)} = \log|G(j\omega)| + \log e^{j\varphi(\omega)}$$

$$= \log|G(j\omega)| + j0.434\varphi(\omega)$$

It is omitted in rest of this book

Decibel (dB): **the unit of the logarithm of the magnitude**

Frequency Response

2. Bode plots (Logarithmic plots)

Log magnitude. **The logarithm of the magnitude of a transfer function G(jω) (幅频特性) expressed in decibels is**

$$20\log|G(j\omega)| \qquad dB$$

The quantity is called the log magnitude, abbreviate Lm. Thus

$$LmG(j\omega) = 20\log|G(j\omega)| \qquad dB$$

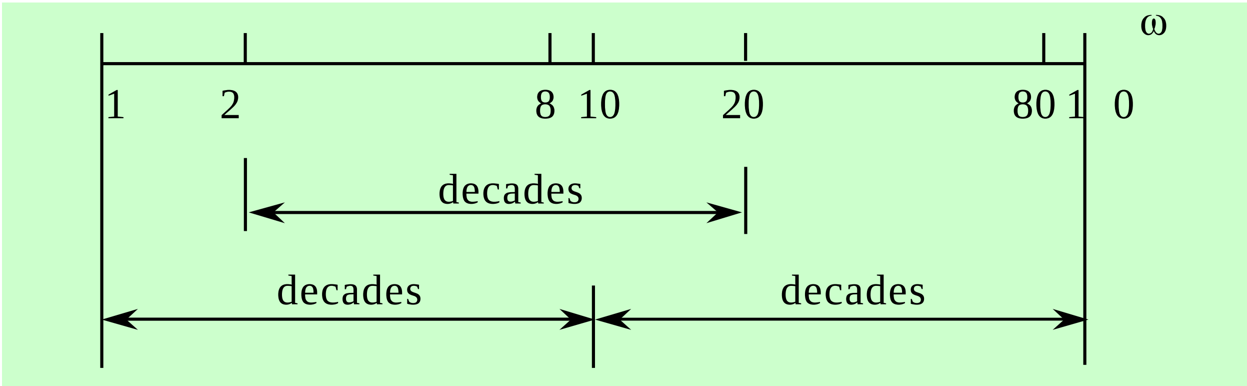
Since the transfer function is a function of frequency, the Lm is also a function of frequency.

Octave (倍频). **An octave is a frequency band from f_1 to f_2, where $f_2/f_1=2$. For example: the frequency band from 1 to 2 Hz is 1 octave in width, and the frequency band from 17.4 to 34.8 Hz is also 1 octave in width.**

Frequency Response

2. Bode plots (Logarithmic plots)

Decade(十倍频). **There is an increase of 1 decade from f_1 to f_2 when $f_2/f_1=10$. The frequency band from 1 to 10 Hz or from 2.5 to 25 Hz is 1 decade in width.**



The dB values of some common numbers are given in P.249 Table 8.1.

Note that the meaning of “decibel(分贝)”.

Frequency Response: 2. Bode plots

General frequency-transfer-function relationships

The frequency transfer function:

$$G(j\omega)=\frac{K_m(1+j\omega T_1)(1+j\omega T_2)^r}{(j\omega)^m(1+j\omega T_a)[1+(2\zeta/\omega_n)j\omega+(1/\omega_n^2)(j\omega)^2]} \qquad (8.11)$$

The log magnitude:

$$LmG(j\omega)=LmK_m+Lm(1+j\omega T_1)+rLm(1+j\omega T_2)+\cdots-mLm(j\omega)-Lm(1+j\omega T_a)-Lm\left[1+\frac{2\zeta}{\omega_n}j\omega+\frac{1}{\omega_n^2}(j\omega)^2\right]-\cdots \qquad (8.12)$$

The angle equation:

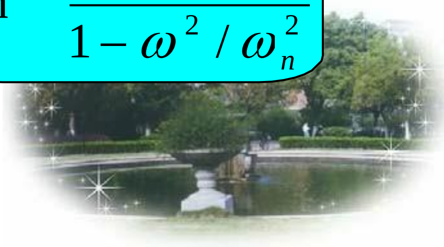
$\tan^{-1}\omega T_1$

$m90$

$$\angle G(j\omega)=\angle K_m+\angle(1+j\omega T_1)+r\angle(1+j\omega T_2)+\cdots-m\angle(j\omega)-\angle(1+j\omega T_a)-\angle\left[1+\frac{2\zeta}{\omega_n}j\omega+\frac{1}{\omega_n^2}(j\omega)^2\right]-\cdots$$

$0°/1$

$\tan^{-1}\dfrac{2\xi\omega/\omega_n}{1-\omega^2/\omega_n^2}$



Frequency Response: 2. Bode plot

Drawing the Bode plots

The generalized form of the transfer function

$$G(j\omega)=\frac{K_m(1+j\omega T_1)(1+j\omega T_2)^r}{(j\omega)^m(1+j\omega T_a)[1+(2\zeta/\omega_n)j\omega+(1/\omega_n^2)(j\omega)^2]}$$

The basic types of factors:

$$K_m \qquad (j\omega)^{\pm m} \qquad (1+j\omega T)^{\pm r} \qquad \left|1+\frac{2\zeta}{\omega_n}j\omega+\frac{1}{\omega_n^2}(j\omega)^2\right|^{\pm p}$$

$$K_m \qquad (j\omega)^{\pm 1} \qquad (1+j\omega T)^{\pm 1} \qquad \left|1+\frac{2\zeta}{\omega_n}j\omega+\frac{1}{\omega_n^2}(j\omega)^2\right|^{\pm 1}$$

These curves for each factor can be added together graphically to get the curves for the complete transfer function, especially when asymptotic straight lines are used.

Frequency Response: 2. Bode plot

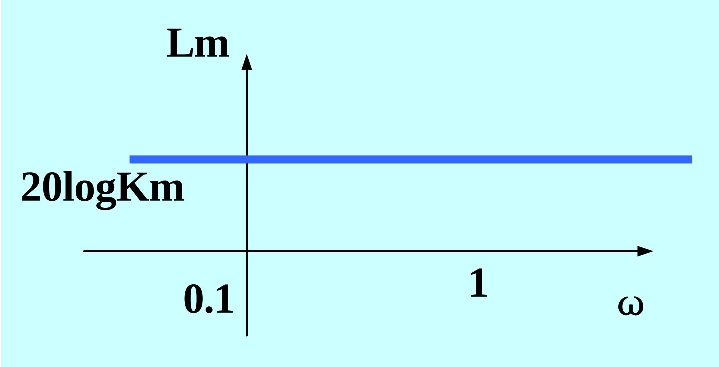
Drawing the Bode plots: : Constants

Constants:K_m

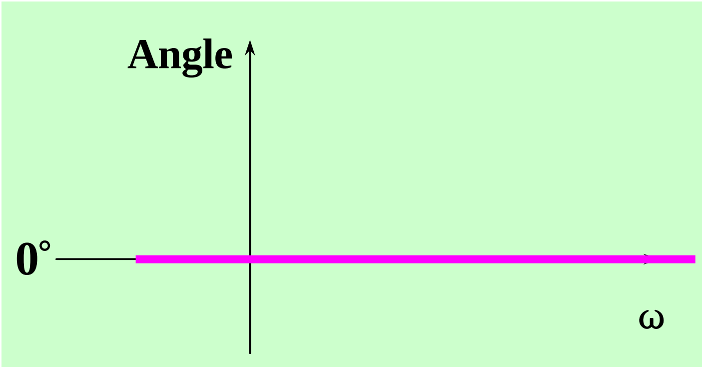
$$LmK_m = 20 \log K_m \quad dB$$

- **The log magnitude diagram is a horizontal straight line.**

The angle is zero as long as Km is positive.



The constant raises or lowers the Lm curve of the complete transfer function by a fixed amount.



Frequency Response: 2. Bode plot

Drawing the Bode plots: : jω Factors

jω Factors: there are 2 forms: $(j\omega)^{\pm 1}$

$$Lm(j\omega)^{-1} = 20\log\left|(j\omega)^{-1}\right| = -20\log\omega \qquad dB$$

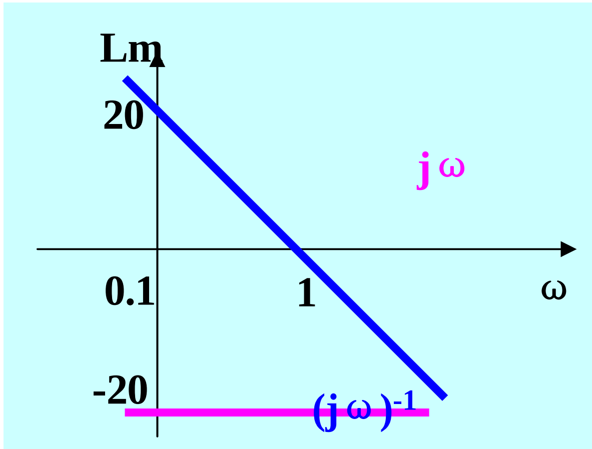
➢ The log magnitude curve is a straight line with a negative slope of 6dB/octave or 20dB/decade.

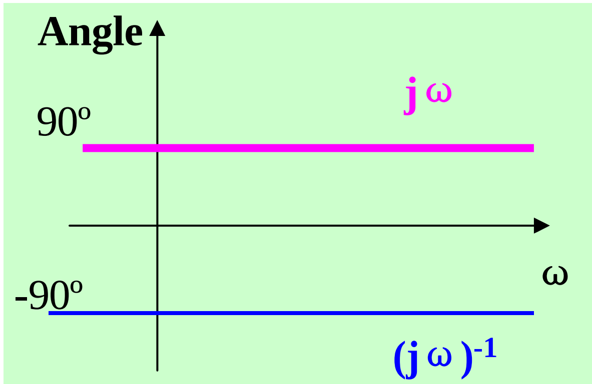
🕘The angle is constant and equal to -90º.

$$Lm(j\omega) = 20\log\left|(j\omega)\right| = 20\log\omega \qquad dB$$

➢ The log magnitude curve is a straight line with a positive slope of 6dB/octave or 20dB/decade.

🕘The angle is constant and equal to +90º.





Frequency Response: 2. Bode plot

Drawing the Bode plots: : 1+jωT Factors

1+jωT Factors: there are 2 forms $(1+j\omega T)^{\pm 1}$, too.

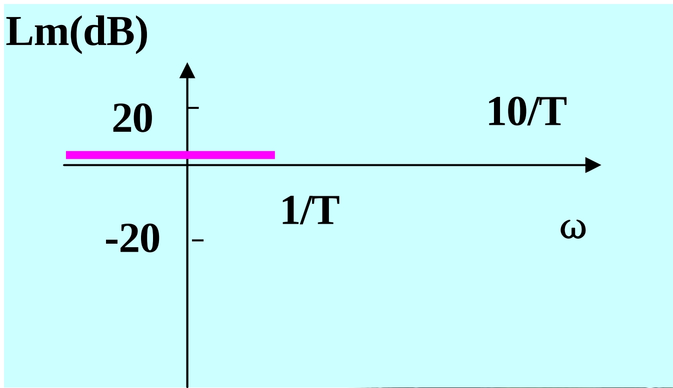
$$Lm(1+j\omega T)^{-1} = 20\log\left|1+j\omega T\right|^{-1} = -20\log\sqrt{1+\omega^2T^2} \qquad dB$$

We can calculate the values vs. ω and plot the frequency-response. However the asymptotic line is easier and more usually used.

For very small values of ω , that is, $\omega T \ll 1$

$$Lm(1+j\omega T)^{-1} \approx 20\log 1 = 0 \qquad dB$$

The plot of the Lm at small frequencies is the 0-dB line



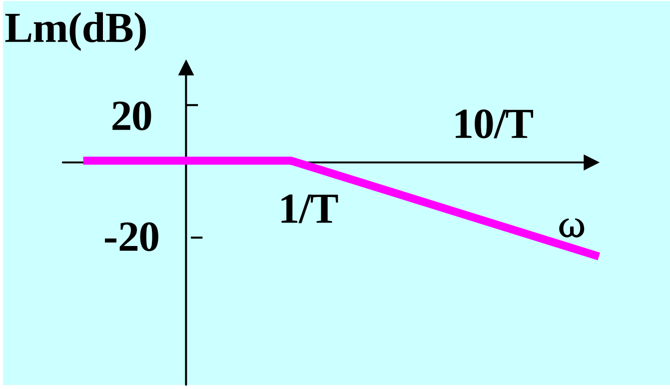
Frequency Response: 2. Bode plot

Drawing the Bode plots: : 1+jωT Factors

For very large values of ω, that is, ωT>>1

$$Lm(1+j\omega T)^{-1} \approx 20\log|j\omega T|^{-1} = -20\log\omega T \qquad dB$$

For values of ω >1/T this function is a straight line with –20dB/decade slope



- **The corner frequency ω_{cf}(转折频率): the frequency at which the asymptotes to the log magnitude curve intersect.**

- **The corner frequency for the function $(1+j\omega T)^{-r} = (1+j(\omega/\omega_{cf}))^{-r}$: $\omega_{cf}=1/T$**

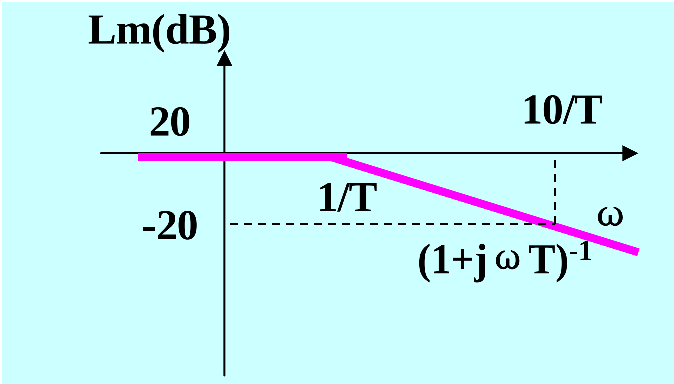
The asymptotes of the plot of $Lm(1+j\omega T)^{-1}$ are two straight lines, ones of zero slope below ω =1/T and ones of –6dB/octave or -20dB/decade slope above ω =1/T .

Frequency Response: 2. Bode plot

Drawing the Bode plots: : 1+jωT Factors

$$Lm(1+j\omega T)^{-1} = 20\log|1+j\omega T|^{-1} = -20\log\sqrt{1+\omega^2T^2} \qquad dB$$

$$\angle(1+j\omega T)^{-1} = -\tan^{-1}\omega T$$

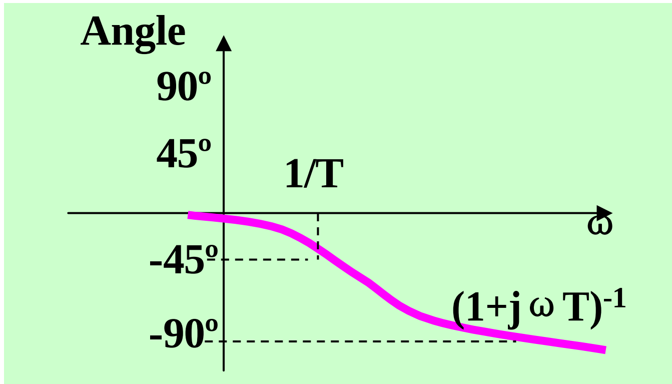


The phase angle:

at zero frequency , the angle is 0º;

at the corner frequency $\omega = \omega_{cf}$, the angle is -45º;

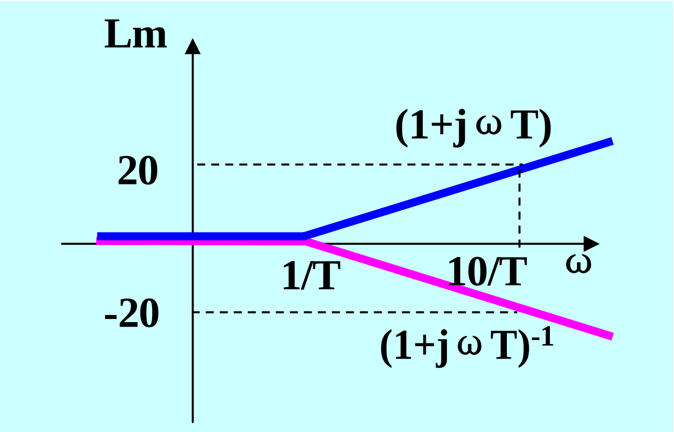
at the infinite frequency , the angle is -90º.

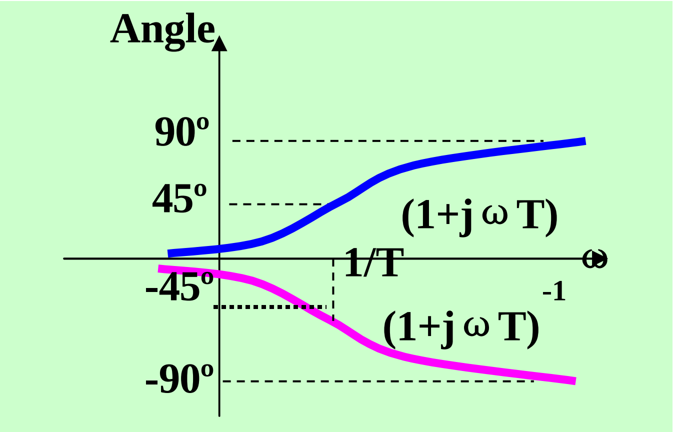


Frequency Response: 2. Bode plot

Drawing the Bode plots: : 1+jωT Factors

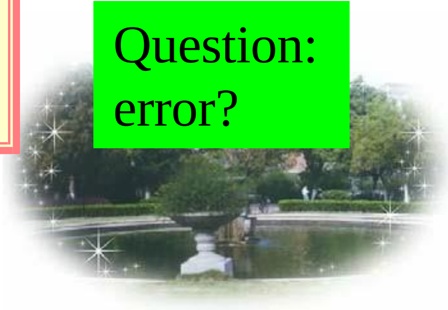
Similarly, the factor 1+jωT: $Lm(1+j\omega T)=20\log\sqrt{1+\omega^2T^2} \qquad dB$





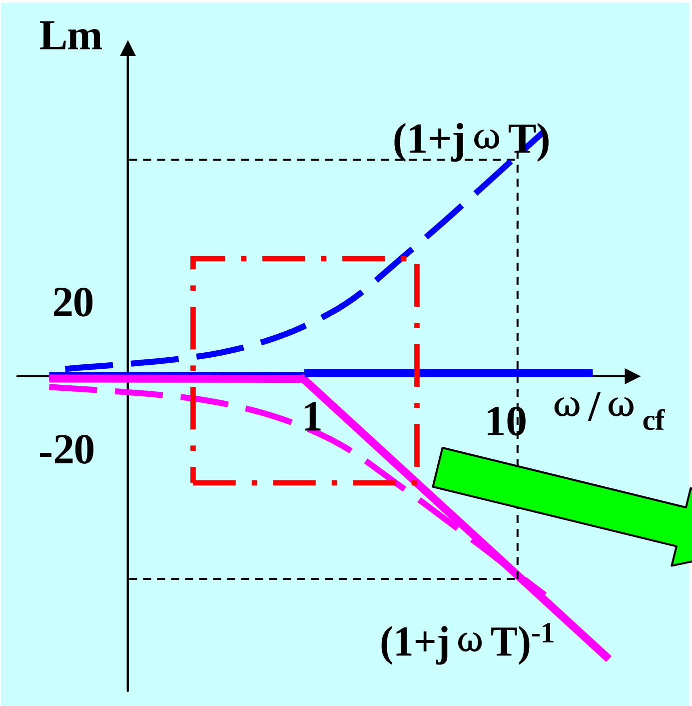
The corner frequency is the same, and the angle varies from 0 to 90° as the frequency increases from zeros to infinity.

Lm and angle curves for the function **(1+j ωT)** are symmetrical about the abscissa to the curves for **$(1+j\omega T)^{-1}$.**



Frequency Response: 2. Bode plot

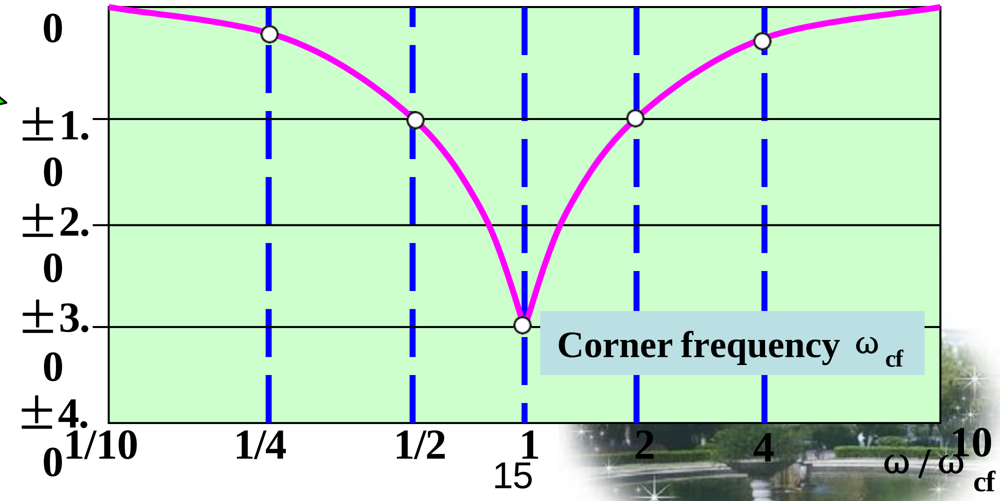
Drawing the Bode plots: error



The exact curve and the asymptotes line

The error between the exact curve and the asymptotes line are as following

- At the corner frequency: 3dB
- One octave above and below the corner frequency : 1dB
- Two octaves from the corner frequency: 0.26dB



Frequency Response: 2. Bode plot

Drawing the Bode plots: Examples 7-3

$$G_1(s) = \frac{1}{1+0.1s}$$

$$G_2(s) = 1 + 0.1s$$