# CHAPTER 21: OPTION VALUATION

## PROBLEM SETS

1. The value of a put option also increases with the volatility of the stock. We see this from the put–call parity theorem as follows:

   $$P = C - S_0 + \text{PV}(X) + \text{PV}(\text{Dividends})$$

   Given a value for $S$ and a risk-free interest rate, then, if $C$ increases because of an increase in volatility, $P$ must also increase in order to maintain the equality of the parity relationship.

2. A $1 increase in a call option's exercise price would lead to a decrease in the option's value of less than $1. The change in the call price would equal $1 only if: (a) there is a 100% probability that the call would be exercised, and (b) the interest rate is zero.

3. Holding firm-specific risk constant, higher beta implies higher total stock volatility. Therefore, the value of the put option increases as beta increases.

4. Holding beta constant, the stock with a lot of firm-specific risk has higher total volatility. The option on the stock with higher firm-specific risk is worth more.

5. A call option with a high exercise price has a lower hedge ratio. This call option is less in the money. Both $d_1$ and $N(d_1)$ are lower when $X$ is higher.

6. a. Put A must be written on the stock with the lower price. Otherwise, given the lower volatility of Stock A, Put A would sell for less than Put B.

   b. Put B must be written on the stock with the lower price. This would explain its higher price.

   c. Call B must have the lower time to expiration. Despite the higher price of Stock B, Call B is cheaper than Call A. This can be explained by a lower time to expiration.

   d. Call B must be written on the stock with higher volatility. This would explain its higher price.

   e. Call A must be written on the stock with higher volatility. This would explain its higher price.


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7.

| Exercise Price | Hedge Ratio |
|---|---|
| 120 | 0/30 = 0.000 |
| 110 | 10/30 = 0.333 |
| 100 | 20/30 = 0.667 |
| 90 | 30/30 = 1.000 |

As the option becomes more in the money, the hedge ratio increases to a maximum of 1.0.

8.

| $S$ | $d_1$ | $N(d_1)$ |
|---|---|---|
| 45 | -0.2768 | 0.3910 |
| 50 | 0.2500 | 0.5987 |
| 55 | 0.7266 | 0.7662 |

9. a. $uS_0 = 130 \Rightarrow P_u = 0$

$dS_0 = 80 \Rightarrow P_d = 30$

The hedge ratio is: $H = \dfrac{P_u - P_d}{uS_0 - dS_0} = \dfrac{0-30}{130-80} = -\dfrac{3}{5}$

b.

| Riskless Portfolio | $S_T = 80$ | $S_T = 130$ |
|---|---|---|
| Buy 3 shares | 240 | 390 |
| Buy 5 puts | 150 | 0 |
| Total | 390 | 390 |

Present value = \$390/1.10 = \$354.545

c. The portfolio cost is: $3S + 5P = 300 + 5P$

The value of the portfolio is: \$354.545

Therefore: $300 + 5P = \$354.545 \rightarrow P = \$54.545/5 = \$10.91$


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10. The hedge ratio for the call is: $H = \frac{C_u - C_d}{uS_0 - dS_0} = \frac{20 - 0}{130 - 80} = \frac{2}{5}$

| Riskless Portfolio | $S = 80$ | $S = 130$ |
|---|---|---|
| Buy 2 shares | 160 | 260 |
| Write 5 calls | 0 | -100 |
| Total | 160 | 160 |

Present value = \$160/1.10 = \$145.455

The portfolio cost is: $2S - 5C = \$200 - 5C$

The value of the portfolio is \$145.455

Therefore: $C = \$54.545/5 = \$10.91$

Does $P = C + \text{PV}(X) - S$?

10.91 = 10.91 + 110/1.10 – 100 = 10.91

11. $d_1 = 0.2192 \Rightarrow N(d_1) = 0.5868$

$d_2 = -0.1344 \Rightarrow N(d_2) = 0.4465$

$Xe^{-rT} = 49.2556$

$C = \$50 \times 0.5868 - 49.2556 \times 0.4465 = \$7.34$

12. $P = \$6.60$

This value is derived from our Black-Scholes spreadsheet, but note that we could have derived the value from put-call parity:

$P = C + \text{PV}(X) - S_0 = \$7.34 + \$49.26 - \$50 = \$6.60$

13. a. $C$ falls to \$5.1443

b. $C$ falls to \$3.8801

c. $C$ falls to \$5.4043

d. $C$ rises to \$10.5356

e. $C$ rises to \$7.5636


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14. According to the Black-Scholes model, the call option should be priced at:

$[\$55 \times N(d_1)] - [50 \times N(d_2)] = (\$55 \times 0.6) - (\$50 \times 0.5) = \$8$

Since the option actually sells for more than \$8, implied volatility is greater than 0.30.

15. A straddle is a call and a put. The Black-Scholes value would be:

$$C + P = S_0 \times N(d_1) - Xe^{-rT} \times N(d_2) + Xe^{-rT} \times [1 - N(d_2)] - S_0 [1 - N(d_1)]$$

$$= S_0 \times [2N(d_1) - 1] + Xe^{-rT} \times [1 - 2N(d_2)]$$

On the Excel spreadsheet (Spreadsheet 21.1), the valuation formula would be:

=B5*(2*E4 – 1) + B6*EXP(–B4*B3)*(1 – 2*E5)

16. A. A delta-neutral portfolio is perfectly hedged against small price changes in the underlying asset. This is true both for price increases and decreases. That is, the portfolio value will not change significantly if the asset price changes by a small amount. However, large changes in the underlying asset will cause the hedge to become imperfect. This means that overall portfolio value can change by a significant amount if the price change in the underlying asset is large.

17. A. Delta is the change in the option price for a given instantaneous change in the stock price. The change is equal to the slope of the option price diagram.

18. The best estimate for the change in price of the option is

Change in asset price × Delta = −\$6 × (-0.65) = \$3.90

19. The number of call options necessary to delta hedge is $\frac{51,750}{0.69} = 75,000$ options, or 750 options contracts, each covering 100 shares. Since these are call options, the options should be sold short.

20. The number of calls needed to create a delta-neutral hedge is inversely proportional to the delta. The delta decreases when stock price decreases. Therefore the number of calls necessary would increase if the stock price falls.

21. A delta-neutral portfolio can be created with any of the following combinations: long stock and short calls, long stock and long puts, short stock and long calls, and short stock and short puts.


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22. The rate of return of a call option on a long-term Treasury bond should be more sensitive to changes in interest rates than is the rate of return of the underlying bond. The option elasticity exceeds 1.0. In other words, the option is effectively a levered investment and the rate of return on the option is more sensitive to interest rate swings.

23. Implied volatility has increased. If not, the call price would have fallen as a result of the decrease in stock price.

24. Implied volatility has increased. If not, the put price would have fallen as a result of the decreased time to expiration.

25. The hedge ratio approaches one. As $S$ increases, the probability of exercise approaches 1.0. $N(d_1)$ approaches 1.0.

26. The hedge ratio approaches 0. As $X$ decreases, the probability of exercise approaches 0. $[N(d_1) - 1]$ approaches 0 as $N(d_1)$ approaches 1.

27. A straddle is a call and a put. The hedge ratio of the straddle is the sum of the hedge ratios of the individual options: $0.4 + (-0.6) = -0.2$

28. a. The spreadsheet appears as follows:

| **INPUTS** | | | **OUTPUTS** | |
|---|---|---|---|---|
| Standard deviation (annual) | 0.3213 | | $d1$ | 0.0089 |
| Expiration (in years) | 0.5 | | $d2$ | -0.2183 |
| Risk-free rate (annual) | 0.05 | | $N(d1)$ | 0.5036 |
| Stock price | 100 | | $N(d2)$ | 0.4136 |
| Exercise price | 105 | | B/S call value | 8.0000 |
| Dividend yield (annual) | 0 | | B/S put value | 10.4076 |

The standard deviation is: 0.3213

b. The spreadsheet below shows the standard deviation has increased to: 0.3568

| **INPUTS** | | | **OUTPUTS** | |
|---|---|---|---|---|
| Standard deviation (annual) | 0.3568 | | $d1$ | 0.0318 |
| Expiration (in years) | 0.5 | | $d2$ | -0.2204 |
| Risk-free rate (annual) | 0.05 | | $N(d1)$ | 0.5127 |
| Stock price | 100 | | $N(d2)$ | 0.4128 |
| Exercise price | 105 | | B/S call value | 9.0000 |
| Dividend yield (annual) | 0 | | B/S put value | 11.4075 |

Implied volatility has increased because the value of an option increases with greater volatility.


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c. Implied volatility increases to 0.4087 when expiration decreases to four months. The shorter expiration decreases the value of the option; therefore, in order for the option price to remain unchanged at $8, implied volatility must increase.

| **INPUTS** | | | **OUTPUTS** | |
|---|---|---|---|---|
| Standard deviation (annual) | 0.4087 | | $d1$ | -0.0182 |
| Expiration (in years) | 0.33333 | | $d2$ | -0.2541 |
| Risk-free rate (annual) | 0.05 | | $N(d1)$ | 0.4928 |
| Stock price | 100 | | $N(d2)$ | 0.3997 |
| Exercise price | 105 | | B/S call value | 8.0001 |
| Dividend yield (annual) | 0 | | B/S put value | 11.2646 |

d. Implied volatility decreases to 0.2406 when exercise price decreases to $100. The decrease in exercise price increases the value of the call, so that, in order for the option price to remain at $8, implied volatility decreases.

| **INPUTS** | | | **OUTPUTS** | |
|---|---|---|---|---|
| Standard deviation (annual) | 0.2406 | | $d1$ | 0.2320 |
| Expiration (in years) | 0.5 | | $d2$ | 0.0619 |
| Risk-free rate (annual) | 0.05 | | $N(d1)$ | 0.5917 |
| Stock price | 100 | | $N(d2)$ | 0.5247 |
| Exercise price | 100 | | B/S call value | 8.0010 |
| Dividend yield (annual) | 0 | | B/S put value | 5.5320 |

e. The decrease in stock price decreases the value of the call. In order for the option price to remain at $8, implied volatility increases.

| **INPUTS** | | | **OUTPUTS** | |
|---|---|---|---|---|
| Standard deviation (annual) | 0.3566 | | $d1$ | -0.0484 |
| Expiration (in years) | 0.5 | | $d2$ | -0.3006 |
| Risk-free rate (annual) | 0.05 | | $N(d1)$ | 0.4807 |
| Stock price | 98 | | $N(d2)$ | 0.3819 |
| Exercise price | 105 | | B/S call value | 8.0000 |
| Dividend yield (annual) | 0 | | B/S put value | 12.4075 |

29. a. The delta of the collar is calculated as follows:

| Position | Delta |
|---|---|
| Buy stock | 1.0 |
| Buy put, $X = \$45$ | $N(d_1) - 1 = -0.40$ |
| Write call, $X = \$55$ | $-N(d_1) = -0.35$ |
| Total | 0.25 |

If the stock price increases by $1, then the value of the collar increases by $0.25. The stock will be worth $1 more, the loss on the purchased put will be $0.40, and the call written represents a *liability* that increases by $0.35.


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b. If $S$ becomes very large, then the delta of the collar approaches zero. Both $N(d_1)$ terms approach 1. Intuitively, for very large stock prices, the value of the portfolio is simply the (present value of the) exercise price of the call, and is unaffected by small changes in the stock price.

As $S$ approaches zero, the delta also approaches zero: both $N(d_1)$ terms approach 0. For very small stock prices, the value of the portfolio is simply the (present value of the) exercise price of the put and is unaffected by small changes in the stock price.

30.

| Put | $X$ | Delta |
|---|---|---|
| A | 10 | –0.1 |
| B | 20 | –0.5 |
| C | 30 | –0.9 |

31. a. Choice A: Calls have higher elasticity than shares. For equal dollar investments, a call's capital gain potential is greater than that of the underlying stock.

b. Choice B: Calls have hedge ratios less than 1.0, so the shares have higher profit potential. For an equal number of shares controlled, the dollar exposure of the shares is greater than that of the calls, and the profit potential is therefore greater.

32. a. $uS_0 = 110 \Rightarrow P_u = 0$

$dS_0 = 90 \Rightarrow P_d = 10$

The hedge ratio is $H = \dfrac{P_u - P_d}{uS_0 - dS_0} = \dfrac{0-10}{110-90} = -\dfrac{1}{2}$

A portfolio comprised of one share and two puts provides a guaranteed payoff of \$110, with present value: \$110/1.05 = \$104.76

Therefore:

$S + 2P = \$104.76$

$\$100 + 2P = \$104.76 \Rightarrow P = \$2.38$

b. Cost of protective put portfolio with a \$100 guaranteed payoff:

= \$100 + \$2.38 = \$102.38


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c. Our goal is a portfolio with the same exposure to the stock as the hypothetical protective put portfolio. Since the put's hedge ratio is –0.5, the portfolio consists of (1 – 0.5) = 0.5 shares of stock, which costs \$50, and the remaining funds (\$52.38) invested in T-bills, earning 5% interest.

| Portfolio | $S = 90$ | $S = 110$ |
|---|---|---|
| Buy 0.5 shares | 45 | 55 |
| Invest in T-bills | 55 | 55 |
| Total | 100 | 110 |

This payoff is identical to that of the protective put portfolio. Thus, the stock plus bills strategy replicates both the cost and payoff of the protective put.

33. The put values in the second period are:

$P_{uu} = 0$

$P_{ud} = P_{du} = 110 - 104.50 = 5.50$

$P_{dd} = 110 - 90.25 = 19.75$

To compute $P_u$, first compute the hedge ratio:

$$H = \frac{P_{uu} - P_{ud}}{uuS_0 - udS_0} = \frac{0 - 5.50}{121 - 104.50} = -\frac{1}{3}$$

Form a riskless portfolio by buying one share of stock and buying three puts.

The cost of the portfolio is: $S + 3P_u = \$110 + 3P_u$

The payoff for the riskless portfolio equals \$121:

| Riskless Portfolio | $S = 104.50$ | $S = 121$ |
|---|---|---|
| Buy 1 share | 104.50 | 121.00 |
| Buy 3 puts | 16.50 | 0.00 |
| Total | 121.00 | 121.00 |

Therefore, find the value of the put by solving:

$\$110 + 3P_u = \$121/1.05 \Rightarrow P_u = \$1.746$

To compute $P_d$, compute the hedge ratio:

$$H = \frac{P_{du} - P_{dd}}{duS_0 - ddS_0} = \frac{5.50 - 19.75}{104.50 - 90.25} = -1.0$$

Form a riskless portfolio by buying one share and buying one put.

The cost of the portfolio is: $S + P_d = \$95 + P_d$

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The payoff for the riskless portfolio equals \$110:

| Riskless Portfolio | $S = 90.25$ | $S = 104.50$ |
|---|---|---|
| Buy 1 share | 90.25 | 104.50 |
| Buy 1 put | 19.75 | 5.50 |
| Total | 110.00 | 110.00 |

Therefore, find the value of the put by solving:

$$\$95 + P_d = \$110/1.05 \Rightarrow P_d = \$9.762$$

To compute $P$, compute the hedge ratio:

$$H = \frac{P_u - P_d}{uS_0 - dS_0} = \frac{1.746 - 9.762}{110 - 95} = -0.5344$$

Form a riskless portfolio by buying 0.5344 of a share and buying one put.
The cost of the portfolio is: $0.5344S + P = \$53.44 + P$

The payoff for the riskless portfolio equals \$60.53:

| Riskless Portfolio | $S = 95$ | $S = 110$ |
|---|---|---|
| Buy 0.5344 share | 50.768 | 58.784 |
| Buy 1 put | 9.762 | 1.746 |
| Total | 60.530 | 60.530 |

Therefore, find the value of the put by solving:

$$\$53.44 + P = \$60.53/1.05 \Rightarrow P = \$4.208$$

Finally, we verify this result using put-call parity. Recall from Example 21.1 that:

$$C = \$4.434$$

Put-call parity requires that:

$$P = C + \text{PV(X)} - S$$

$$\$4.208 = \$4.434 + (\$110/1.05^2) - \$100$$

Except for minor rounding error, put-call parity is satisfied.

34. If $r = 0$, then one should never exercise a put early. There is no "time value cost" to waiting to exercise, but there is a "volatility benefit" from waiting. To show this more rigorously, consider the following portfolio: lend \$X and short one share of stock. The cost to establish the portfolio is $(X - S_0)$. The payoff at time $T$ (with zero interest earnings on the loan) is $(X - S_T)$. In contrast, a put option has a payoff at time $T$ of $(X - S_T)$ if that value is positive, and zero otherwise. The put's payoff is at least as large as the portfolio's, and therefore, the put must cost at least as much as the portfolio to purchase. Hence, $P \geq (X - S_0)$, and the put can be sold for more than the proceeds from immediate exercise. We conclude that it doesn't pay to exercise early.


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35. a. $Xe^{-rT}$

b. $X$

c. 0

d. 0

e. It is optimal to exercise immediately a put on a stock whose price has fallen to zero. The value of the American put equals the exercise price. Any delay in exercise lowers value by the time value of money.

36. Step 1: Calculate the option values at expiration. The two possible stock prices and the corresponding call values are:

$uS_0 = 120 \Rightarrow C_u = 20$

$dS_0 = 80 \Rightarrow C_d = 0$

Step 2: Calculate the hedge ratio.

$$H = \frac{C_u - C_d}{uS_0 - dS_0} = \frac{20 - 0}{120 - 80} = \frac{1}{2}$$

Therefore, form a riskless portfolio by buying one share of stock and writing two calls. The cost of the portfolio is $S - 2C = 100 - 2C$

Step 3: Show that the payoff for the riskless portfolio equals \$80:

| Riskless Portfolio | $S = 80$ | $S = 120$ |
|---|---|---|
| Buy 1 share | 80 | 120 |
| Write 2 calls | 0 | -40 |
| Total | 80 | 80 |

Therefore, find the value of the call by solving

$\$100 - 2C = \$80/1.10 \Rightarrow C = \$13.636$

Notice that we did not use the probabilities of a stock price increase or decrease. These are not needed to value the call option.


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37. The two possible stock prices and the corresponding call values are:

$uS_0 = 130 \Rightarrow C_u = 30$

$dS_0 = 70 \Rightarrow C_d = 0$

The hedge ratio is $H = \dfrac{C_u - C_d}{uS_0 - dS_0} = \dfrac{30 - 0}{130 - 70} = \dfrac{1}{2}$

Form a riskless portfolio by buying one share of stock and writing two calls. The cost of the portfolio is: $S - 2C = 100 - 2C$

The payoff for the riskless portfolio equals \$70:

| Riskless Portfolio | $S = 70$ | $S = 130$ |
|---|---|---|
| Buy 1 share | 70 | 130 |
| Write 2 calls | 0 | -60 |
| Total | 70 | 70 |

Therefore, find the value of the call by solving

$\$100 - 2C = \$70/1.10 \Rightarrow C = \$18.182$

Here, the value of the call is greater than the value in the lower-volatility scenario.

38. The two possible stock prices and the corresponding put values are:

$uS_0 = 120 \Rightarrow P_u = 0$

$dS_0 = 80 \Rightarrow P_d = 20$

The hedge ratio is $H = \dfrac{P_u - P_d}{uS_0 - dS_0} = \dfrac{0 - 20}{120 - 80} = -\dfrac{1}{2}$

Form a riskless portfolio by buying one share of stock and buying two puts. The cost of the portfolio is: $S + 2P = 100 + 2P$

The payoff for the riskless portfolio equals \$120:

| Riskless Portfolio | $S = 80$ | $S = 120$ |
|---|---|---|
| Buy 1 share | 80 | 120 |
| Buy 2 puts | 40 | 0 |
| Total | 120 | 120 |

Therefore, find the value of the put by solving

$\$100 + 2P = \$120/1.10 \Rightarrow P = \$4.545$

According to put-call parity $P + S = C + \text{PV}(X)$

Our estimates of option value satisfy this relationship:

$\$4.545 + \$100 = \$13.636 + \$100/1.10 = \$104.545$


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39. If we assume that the only possible exercise date is just prior to the ex-dividend date, then the relevant parameters for the Black-Scholes formula are:

$S_0 = 60$
$r = 0.5\%$ per month
$X = 55$
$\sigma = 7\%$
$T = 2$ months

In this case: $C = \$6.04$

If instead, one commits to foregoing early exercise, then we reduce the stock price by the present value of the dividends. Therefore, we use the following parameters:

$S_0 = 60 - 2e^{-(0.005 \times 2)} = 58.02$
$r = 0.5\%$ per month
$X = 55$
$\sigma = 7\%$
$T = 3$ months

In this case, $C = \$5.05$

The pseudo-American option value is the higher of these two values: \$6.04

40. True. The call option has an elasticity greater than 1.0. Therefore, the call's percentage rate of return is greater than that of the underlying stock. Hence the GM call responds more than proportionately when the GM stock price changes in response to broad market movements. Therefore, the beta of the GM call is greater than the beta of GM stock.

41. False. The elasticity of a call option is higher the more out of the money is the option. (Even though the delta of the call is lower, the value of the call is also lower. The proportional response of the call price to the stock price increases. You can confirm this with numerical examples.) Therefore, the rate of return of the call with the higher exercise price responds more sensitively to changes in the market index, and therefore it has the higher beta.

42. As the stock price increases, conversion becomes increasingly more assured. The hedge ratio approaches 1.0. The price of the convertible bond will move one-for-one with changes in the price of the underlying stock.


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43. Goldman Sachs believes that the market assessment of volatility is too high. It should sell options because the analysis suggests the options are overpriced with respect to true volatility. The delta of the call is 0.6, while the put is 0.6 – 1 = –0.4. Therefore, Goldman should sell puts and calls in the ratio of 0.6 to 0.4. For example, if Goldman sells 2 calls and 3 puts, the position will be delta neutral:

    Delta = (2 × 0.6) + [3 × (–0.4)] = 0

44. If the stock market index increases 1%, the 1 million shares of stock on which the options are written would be expected to increase by

    0.75% × \$5 × 1 million = \$37,500

    The options would increase by:

    Delta × \$37,500 = 0.8 × \$37,500 = \$30,000

    In order to hedge your market exposure, you must sell \$3,000,000 of the market index portfolio so that a 1% change in the index would result in a \$30,000 change in the value of the portfolio.

45. $S = 100$; current value of portfolio

    $X = 100$; floor promised to clients (0% return)

    $\sigma = 0.25$; volatility

    $r = 0.05$; risk-free rate

    $T = 4$ years; horizon of program

    a. Using the Black-Scholes formula, we find that

        $d_1 = 0.65$, $N(d_1) = 0.7422$, $d_2 = 0.15$, $N(d_2) = 0.5596$

        Put value = \$10.27

        Therefore, total funds to be managed equals \$110.27 million: \$100 million portfolio value plus the \$10.27 million fee for the insurance program.

        The put delta is $N(d_1) – 1 = 0.7422 – 1 = –0.2578$

        Therefore, sell off 25.78% of the equity portfolio, placing the remaining funds in T-bills. The amount of the portfolio in equity is therefore \$74.22 million, while the amount in T-bills is: \$110.27 million – \$74.22 million = \$36.06 million

    b. At the new portfolio value of 97, the put delta is

        $N(d_1) – 1 = 0.7221 – 1 = –0.2779$

        This means that you must reduce the delta of the portfolio by

        0.2779 – 0.2578 = 0.0201

        You should sell an additional 2.01% of the equity position and use the proceeds to buy T-bills. Since the stock price is now at only 97% of its original value, you need to sell

        \$97 million × 0.0201 = \$1.946 million of stock


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46. Using the true volatility (32%) and time to expiration T = 0.25 years, the hedge ratio for ExxonMobil is $N(d_1) = 0.5945$. Because you believe the calls are underpriced (selling at an implied volatility that is too low), you will buy calls and short 0.5945 shares for each call you buy.

47. The calls are cheap (implied $\sigma = 0.30$) and the puts are expensive (implied $\sigma = 0.34$). Therefore, buy calls and sell puts. Using the "true" volatility of $\sigma = 0.32$, the call delta is 0.5945 and the put delta is $0.5945 - 1.0 = -0.4055$. Therefore, for each call purchased, buy $0.5945/0.4055 = 1.466$ puts.

48. a. To calculate the hedge ratio, suppose that the market index increases by 1%. Then the stock portfolio would be expected to increase by:

$$1\% \times 1.5 = 1.5\% \text{ or } 0.015 \times \$1{,}250{,}000 = \$18{,}750$$

Given the option delta of 0.8, the option portfolio would increase by

$$\$18{,}750 \times 0.8 = \$15{,}000$$

JP Morgan Chase's liability from writing these options would increase by the same amount. The market index portfolio would increase in value by 1%. Therefore, JP Morgan should purchase $1,500,000 of the market index portfolio in order to hedge its position so that a 1% change in the index would result in a $15,000 change in the value of the portfolio.

b. The delta of a put option is

$$0.8 - 1 = -0.2$$

Therefore, for every 1% the market increases, the index will rise by 10 points and the value of the put option contract will change by

$$\text{Delta} \times 10 \times \text{Contract multiplier} = -0.2 \times 10 \times 100 = -\$200$$

Therefore, JP Morgan should write $15,000/$200 = 75 put contracts

49. a,b,c

| Subperiods | $\Delta t = T/n$ | $u = \exp(\sigma\sqrt{\Delta t})$ | $d = \exp(-\sigma\sqrt{\Delta t})$ |
|---|---|---|---|
| 1 | 1/1=1 | 1.4918 | 0.6703 |
| 4 | 1/4=.25 | 1.2214 | 0.8187 |
| 12 | 1/12=.0833 | 1.1224 | 0.8909 |


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50. Since the spread between $u$ and $d$ reflects the volatility of the rate of return and $u$ and $d$ depend on that volatility, next year's rate of return should increase accordingly by the $\sigma\sqrt{\Delta t}$

51. $P = C - S_0 + \text{PV}(X)$. When at-the-money, $X = S_0$. PV($X$) will always be less than $S_0$ and due to discounting PV($X$) can never be large enough to raise $P$ over the price of $C$. If $X=X(1+r)^T$ then $X = S_0$ and $P=C$.

52. Using the risk-neutral shortcut, we must first calculate the risk-neutral probability $p$.

$$p = \frac{1 + rf - d}{u - d} = \frac{1 + .1 - .8}{1.2 - .8} = .75$$

After one year, the stock price will have either a 75% chance of rising to \$120 (a \$20 payoff) or a 25% chance of falling to \$80 (\$0 payoff). Discounting the weighted average of these payoffs by the risk-free rate gives us

$$\frac{.75 \times 20 + .25 \times 0}{(1 + .10)^1} = 13.636$$

which is exactly equal to the two-state approach.

53. If the stock price rises, the payoff will be zero as profit from a put is only made when the stock price is less than the exercise price. If the stock price falls, it falls below the \$100 exercise price and the payoff will be the difference between the two. Using the risk-neutral shortcut

$$p = \frac{1 + rf - d}{u - d} = \frac{1 + .1 - .8}{1.2 - .8} = .75$$

After one year, the stock price will have either a 75% chance of rising to \$120 (a \$0 payoff) or a 25% chance of falling to \$80 (\$20 payoff). Discounting the weighted average of these payoffs by the risk-free rate gives us

$$\frac{.75 \times 0 + .25 \times 20}{(1 + .10)^1} = 4.545$$

Equal to the two-stage approach.


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## CFA PROBLEMS

1. Statement a: The hedge ratio (determining the number of futures contracts to sell) ought to be adjusted by the beta of the equity portfolio, which is 1.20. The correct hedge ratio would be

$$\frac{\$100 \text{ million}}{\$100 \times 250} \times \beta = 4,000 \times \beta = 4,000 \times 1.2 = 4,800$$

Statement b: The portfolio will be hedged and should therefore earn the risk-free rate, not zero, as the consultant claims. Given a futures price of 100 and an equity price of 99, the rate of return over the three-month period is

(100 – 99)/99 = 1.01% = approximately 4.1% annualized

2. a. The value of the call option decreases if underlying stock price volatility decreases. The less volatile the underlying stock price, the less the chance of extreme price movements—the lower the probability that the option expires in the money. This makes the participation feature on the upside less valuable.

The value of the call option is expected to increase if the time to expiration of the option increases. The longer the time to expiration, the greater the chance that the option will expire in the money resulting in an increase in the time premium component of the option's value.

b. i. When European options are out of the money, investors are essentially saying that they are willing to pay a premium for the right, but not the obligation, to buy or sell the underlying asset. The out-of-the-money option has no intrinsic value, but, since options require little capital (just the premium paid) to obtain a relatively large potential payoff, investors are willing to pay that premium even if the option may expire worthless. The Black-Scholes model does not reflect investors' demand for any premium above the time value of the option. Hence, if investors are willing to pay a premium for an out-of-the-money option above its time value, the Black-Scholes model does not value that excess premium.

ii. With American options, investors have the right, but not the obligation, to exercise the option prior to expiration, even if they exercise for noneconomic reasons. This increased flexibility associated with American options has some value but is not considered in the Black-Scholes model because the model only values options to their expiration date (European options).

3. a. American options should cost more (have a higher premium). American options give the investor greater flexibility than European options since the investor can *choose* whether to exercise early. When the stock pays a dividend, the option to exercise a call early can be valuable. But regardless of the dividend, a European option (put or call) never sells for more than an otherwise-identical American option.


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b. $C = S_0 + P - \text{PV}(X) = \$43 + \$4 - \$45/1.055 = \$4.346$

Note: we assume that Abaco does not pay any dividends.

c. i. An increase in short-term interest rate ⇒ PV(exercise price) is lower, and call value increases.

ii. An increase in stock price volatility ⇒ the call value increases.

iii. A decrease in time to option expiration ⇒ the call value decreases.

4. a. The two possible values of the index in the first period are:

$uS_0 = 1.20 \times 50 = 60$

$dS_0 = 0.80 \times 50 = 40$

The possible values of the index in the second period are:

$uuS_0 = (1.20)^2 \times 50 = 72$

$udS_0 = 1.20 \times 0.80 \times 50 = 48$

$duS_0 = 0.80 \times 1.20 \times 50 = 48$

$ddS_0 = (0.80)^2 \times 50 = 32$

b. The call values in the second period are:

$C_{uu} = 72 - 60 = 12$

$C_{ud} = C_{du} = C_{dd} = 0$

Since $C_{ud} = C_{du} = 0$, then $C_d = 0$.

To compute $C_u$, first compute the hedge ratio:

$$H = \frac{C_{uu} - C_{ud}}{uuS_0 - udS_0} = \frac{12 - 0}{72 - 48} = \frac{1}{2}$$

Form a riskless portfolio by buying one share of stock and writing two calls.

The cost of the portfolio is: $S - 2C_u = \$60 - 2C_u$

The payoff for the riskless portfolio equals \$48:

| Riskless Portfolio | $S = 48$ | $S = 72$ |
|---|---|---|
| Buy 1 share | 48 | 72 |
| Write 2 calls | 0 | -24 |
| Total | 48 | 48 |

Therefore, find the value of the call by solving:

$\$60 - 2C_u = \$48/1.06 \Rightarrow C_u = \$7.358$

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To compute $C$, compute the hedge ratio:

$$H = \frac{C_u - C_d}{uS_0 - dS_0} = \frac{7.358 - 0}{60 - 40} = 0.3679$$

Form a riskless portfolio by buying 0.3679 of a share and writing one call. The cost of the portfolio is 0.3679S – $C$ = \$18.395 – $C$

The payoff for the riskless portfolio equals \$14.716:

| Riskless Portfolio | $S = 40$ | $S = 60$ |
|---|---|---|
| Buy 0.3679 share | 14.716 | 22.074 |
| Write 1 call | 0.000 | −7.358 |
| Total | 14.716 | 14.716 |

Therefore, find the value of the call by solving:

\$18.395 – $C$ = \$14.716/1.06 $\Rightarrow$ $C$ = \$4.512

c. The put values in the second period are:

$P_{uu} = 0$

$P_{ud} = P_{du} = 60 - 48 = 12$

$P_{dd} = 60 - 32 = 28$

To compute $P_u$, first compute the hedge ratio:

$$H = \frac{P_{uu} - P_{ud}}{uuS_0 - udS_0} = \frac{0 - 12}{72 - 48} = -\frac{1}{2}$$

Form a riskless portfolio by buying one share of stock and buying two puts. The cost of the portfolio is: $S + 2P_u$ = \$60 + $2P_u$

The payoff for the riskless portfolio equals \$72:

| Riskless Portfolio | $S = 48$ | $S = 72$ |
|---|---|---|
| Buy 1 share | 48 | 72 |
| Buy 2 puts | 24 | 0 |
| Total | 72 | 72 |

Therefore, find the value of the put by solving:

\$60 + $2P_u$ = \$72/1.06 $\Rightarrow$ $P_u$ = \$3.962

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To compute $P_d$, compute the hedge ratio:

$$H = \frac{P_{du} - P_{dd}}{duS_0 - ddS_0} = \frac{12 - 28}{48 - 32} = -1.0$$

Form a riskless portfolio by buying one share and buying one put.
The cost of the portfolio is: $S + P_d = \$40 + P_d$

The payoff for the riskless portfolio equals \$60:

| Riskless Portfolio | $S = 32$ | $S = 48$ |
|---|---|---|
| Buy 1 share | 32 | 48 |
| Buy 1 put | 28 | 12 |
| Total | 60 | 60 |

Therefore, find the value of the put by solving

$$\$40 + P_d = \$60/1.06 \Rightarrow P_d = \$16.604$$

To compute $P$, compute the hedge ratio:

$$H = \frac{P_u - P_d}{uS_0 - dS_0} = \frac{3.962 - 16.604}{60 - 40} = -0.6321$$

Form a riskless portfolio by buying 0.6321 of a share and buying one put.
The cost of the portfolio is: $0.6321S + P = \$31.605 + P$

The payoff for the riskless portfolio equals \$41.888:

| Riskless Portfolio | $S = 40$ | $S = 60$ |
|---|---|---|
| Buy 0.6321 share | 25.284 | 37.926 |
| Buy 1 put | 16.604 | 3.962 |
| Total | 41.888 | 41.888 |

Therefore, find the value of the put by solving:

$$\$31.605 + P = \frac{\$41.888}{1.06} \Rightarrow P = \$7.912$$

d. According to put-call-parity:

$$C = S_0 + P - \text{PV}(X) = \$50 + \$7.912 - \frac{\$60}{1.06^2} = \$4.512$$

This is the value of the call calculated in part (b) above.


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5. a.(i) Index increases to 1,193. The combined portfolio will suffer a loss. The written calls expire in the money; the protective put purchased expires worthless. Let's analyze the outcome on a per-share basis. The payout for each call option is $43, for a total cash outflow of $86. The stock is worth $1,190. The portfolio will thus be worth:

$$\$1,190 - \$86 = \$1,104.00$$

The net cost of the portfolio when the option positions are established is:

$$\$1,136 + \$16.10 \text{ (put)} - [2 \times \$8.60] \text{ (calls written)} = \$1,134.90$$

The portfolio experiences a small loss of $30.90

(ii) Index remains at 1,136. Both options expire out of the money. The portfolio will thus be worth $1,136 (per share), compared to an initial cost 30 days earlier of $1,134.90. The portfolio experiences a very small gain of $1.10.

(iii) Index declines to 1,080. The calls expire worthless. The portfolio will be worth $1,130, the exercise price of the protective put. This represents a very small loss of $4.90 compared to the initial cost 30 days earlier of $1,134.90

b.(i) Index increases to 1,193. The delta of the call approaches 1.0 as the stock goes deep into the money, while expiration of the call approaches and exercise becomes essentially certain. The put delta approaches zero.

(ii) Index remains at 1,136. Both options expire out of the money. Delta of each approaches zero as expiration approaches and it becomes certain that the options will not be exercised.

(iii) Index declines to 1,080. The call is out of the money as expiration approaches. Delta approaches zero. Conversely, the delta of the put approaches −1.0 as exercise becomes certain.

c. The call sells at an implied volatility (22.00%) that is less than recent historical volatility (23.00%); the put sells at an implied volatility (24.00%) that is greater than historical volatility. The call seems relatively cheap; the put seems expensive.


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# CHAPTER 22: FUTURES MARKETS

## PROBLEM SETS

1. There is little hedging or speculative demand for cement futures, since cement prices are fairly stable and predictable. The trading activity necessary to support the futures market would not materialize. Only those commodities and financial securities with significant volatility tend to have futures contracts available for hedgers and speculators.

2. The ability to buy on margin is one advantage of futures. Another is the ease with which one can alter one's holdings of the asset. This is especially important if one is dealing in commodities, for which the futures market is far more liquid than the spot market.

3. Short selling results in an immediate cash inflow, whereas the short futures position does not:

| Action | Initial CF | Final CF |
|---|---|---|
| Short sale | $+P_0$ | $-P_T$ |
| Short futures | 0 | $F_0 - P_T$ |

4. a. False. For any given level of the stock index, the futures price will be lower when the dividend yield is higher. This follows from spot-futures parity:

$$F_0 = S_0 (1 + r_f - d)^T$$

   b. False. The parity relationship tells us that the futures price is determined by the stock price, the interest rate, and the dividend yield; it is not a function of beta.

   c. True. The short futures position will profit when the S&P 500 Index falls. This is a negative beta position.

5. The futures price is the agreed-upon price for deferred delivery of the asset. If that price is fair, then the value of the agreement ought to be zero; that is, the contract will be a zero-NPV agreement for each trader. Over time, however, the price of the underlying asset will change and this will affect the value of the contract.

6. Because long positions equal short positions, futures trading must entail a "canceling out" of bets on the asset. Moreover, no cash is exchanged at the inception of futures trading. Thus, there should be minimal impact on the spot market for the asset, and futures trading should not be expected to reduce capital available for other uses.


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7. a. The closing futures price for the March contract was 1,491.80, which has a dollar value of:

$$\$250 \times 1{,}491.80 = \$372{,}950$$

Therefore, the required margin deposit is: \$37,295

b. The futures price increases by: \$1,498.00 – 1,491.80 = \$6.2

The credit to your margin account would be: $6.2 \times \$250 = \$1{,}550$

This is a percent gain of: \$1,550/\$37,295 = 0.04 = 4%

Note that the futures price itself increased by only 0.42%.

c. Following the reasoning in part (b), any change in $F$ is magnified by a ratio of (1 /margin requirement). This is the leverage effect. The return will be –10%.

8. a. $F_0 = S_0(1 + r_f) = \$150 \times 1.03 = \$154.50$

b. $F_0 = S_0(1 + r_f)^3 = \$150 \times 1.03^3 = \$163.91$

c. $F_0 = 150 \times 1.06^3 = \$178.65$

9. a. Take a short position in T-bond futures, to offset interest rate risk. If rates increase, the loss on the bond will be offset to some extent by gains on the futures.

b. Again, a short position in T-bond futures will offset the interest rate risk.

c. You want to protect your cash outlay when the bond is purchased. If bond prices increase, you will need extra cash to purchase the bond. Thus, you should take a long futures position that will generate a profit if prices increase.

10. $F_0 = S_0 \times (1 + r_f – d) = 1{,}400 \times (1 + 0.03 – 0.02) = 1{,}414$

If the T-bill rate is less than the dividend yield, then the futures price should be less than the spot price.

11. The put-call parity relation states that: But spot-futures parity tells us that:

$$C = P + S_0 - \frac{X}{(1+r_f)^T} \qquad F = S_0 \times (1+r_f)^T$$

Substituting, we find that:

$$P = C - S_0 + \frac{[S_0 \times (1+r_f)^T]}{(1+r_f)^T} = C - S_0 + S_0 = C$$


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12. According to the parity relation, the proper price for December futures is:

$$F_{\text{Dec}} = F_{\text{June}}(1 + r_f)^{1/2} = 1500 \times 1.02^{1/2} = 1{,}514.93$$

The actual futures price for December is low relative to the June price. You should take a long position in the December contract and short the June contract.

13. a. $120 \times 1.06 = \$127.20$

b. The stock price falls to: $120 \times (1 - 0.03) = \$116.40$

The futures price falls to: $116.4 \times 1.06 = \$123.384$

The investor loses: $(127.20 - 123.384) \times 1{,}000 = \$3{,}816$

c. The percentage loss is: $\$3{,}816/\$12{,}000 = 0.318 = 31.8\%$

14. a. The initial futures price is $F_0 = 1300 \times (1 + 0.005 - 0.002)^{12} = \$1{,}347.58$

In one month, the futures price will be:

$$F_0 = 1320 \times (1 + 0.005 - 0.002)^{11} = \$1{,}364.22$$

The increase in the futures price is 16.64, so the cash flow will be:

$16.64 \times \$250 = \$4{,}160.00$

b. The holding period return is: $\$4{,}160.00/\$13{,}000 = 0.3200 = 32.00\%$

15. The treasurer would like to buy the bonds today but cannot. As a proxy for this purchase, T-bond futures contracts can be purchased. If rates do in fact fall, the treasurer will have to buy back the bonds for the sinking fund at prices higher than the prices at which they could be purchased today. However, the gains on the futures contracts will offset this higher cost to some extent.

16. The parity value of $F$ is: $1{,}550 \times (1 + 0.04 - 0.01) = 1{,}597$

The actual futures price is 1,550, too low by 47.

| Arbitrage Portfolio | CF now | CF in 1 year |
|---|---|---|
| Short index | 1,500 | $-S_T - (0.01 \times 1{,}500)$ |
| Buy futures | 0 | $S_T - 1{,}550$ |
| Lend | −1,500 | $1{,}500 \times 1.04$ |
| Total | 0 | 47 |

17. a. Futures prices are determined from the spreadsheet as follows:

| **Spot Futures Parity and Time Spreads** |
|---|


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| | | | |
|---|---|---|---|
| Spot price | 1,500 | | |
| Income yield (%) | 1.5 | Futures prices versus maturity | |
| Interest rate (%) | 3.0 | | |
| Today's date | 1/1/2013 | Spot price | 1,500.00 |
| Maturity date 1 | 2/14/2013 | Futures 1 | 1,502.67 |
| Maturity date 2 | 5/21/2013 | Futures 2 | 1,508.71 |
| Maturity date 3 | 11/18/2013 | Futures 3 | 1,519.79 |
| | | | |
| Time to maturity 1 | 0.12 | | |
| Time to maturity 2 | 0.39 | | |
| Time to maturity 3 | 0.88 | | |
| | | **LEGEND:** | |
| | | **Enter data** | |
| | | **Value calculated** | |
| | | **See comment** | |

b. The spreadsheet demonstrates that the futures prices now decrease with increased income yield:

| **Spot Futures Parity and Time Spreads** | | | |
|---|---|---|---|
| Spot price | 1,500 | | |
| Income yield (%) | 4.0 | Futures prices versus maturity | |
| Interest rate (%) | 3.0 | | |
| Today's date | 1/1/2013 | Spot price | 1,500.00 |
| Maturity date 1 | 2/14/2013 | Futures 1 | 1,498.20 |
| Maturity date 2 | 5/21/2013 | Futures 2 | 1,494.15 |
| Maturity date 3 | 11/18/2013 | Futures 3 | 1,486.78 |
| | | | |
| Time to maturity 1 | 0.12 | | |
| Time to maturity 2 | 0.39 | | |
| Time to maturity 3 | 0.88 | | |
| | | **LEGEND:** | |
| | | **Enter data** | |
| | | **Value calculated** | |
| | | **See comment** | |

18. a. The current yield for Treasury bonds (coupon divided by price) plays the role of the dividend yield.

b. When the yield curve is upward sloping, the current yield exceeds the short rate. Hence, T-bond futures prices on more distant contracts are lower than those on near-term contracts.


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19. a.

| | Cash Flows | | |
|---|---|---|---|
| Action | Now | $T_1$ | $T_2$ |
| Long futures with maturity $T_1$ | 0 | $P_1 - F(T_1)$ | 0 |
| Short futures with maturity $T_2$ | 0 | 0 | $F(T_2) - P_2$ |
| Buy asset at $T_1$, sell at $T_2$ | 0 | $-P_1$ | $P_2$ |
| At $T_1$, borrow $F(T_1)$ | 0 | $F(T_1)$ | $-F(T_1) \times (1+r_f)^{(T_2-T_1)}$ |
| Total | 0 | 0 | $F(T_2) - F(T_1) \times (1+r_f)^{(T_2-T_1)}$ |

b. Since the $T_2$ cash flow is riskless and the net investment was zero, then any profits represent an arbitrage opportunity.

c. The zero-profit no-arbitrage restriction implies that

$$F(T_2) = F(T_1) \times (1+r_f)^{(T_2-T_1)}$$

## CFA PROBLEMS

1. a. The strategy that would take advantage of the arbitrage opportunity is a "reverse cash and carry." A reverse cash and carry opportunity results when the following relationship does not hold true:

$$F_0 \geq S_0 (1+C)$$

If the futures price is less than the spot price plus the cost of carrying the goods to the futures delivery date, then an arbitrage opportunity exists. A trader would be able to sell the asset short, use the proceeds to lend at the prevailing interest rate, and then buy the asset for future delivery. At the future delivery, the trader would then collect the proceeds of the loan with interest, accept delivery of the asset, and cover the short position in the commodity.


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b.

| | Cash Flows | |
|---|---|---|
| Action | Now | One year from now |
| Sell the spot commodity short | +$120.00 | −$125.00 |
| Buy the commodity futures expiring in 1 year | $0.00 | $0.00 |
| Contract to lend $120 at 8% for 1 year | −$120.00 | +$129.60 |
| Total cash flow | $0.00 | +$4.60 |

2. a. The call option is distinguished by its asymmetric payoff. If the Swiss franc rises in value, then the company can buy francs for a given number of dollars to service its debt and thereby put a cap on the dollar cost of its financing. If the franc falls, the company will benefit from the change in the exchange rate. The futures and forward contracts have symmetric payoffs. The dollar cost of the financing is locked in regardless of whether the franc appreciates or depreciates. The major difference from the firm's perspective between futures and forwards is in the mark-to-market feature of futures. The consequence of this is that the firm must be ready for the cash management issues surrounding cash inflows or outflows as the currency values and futures prices fluctuate.

   b. The call option gives the company the ability to benefit from depreciation in the franc but at a cost equal to the option premium. Unless the firm has some special expertise in currency speculation, it seems that the futures or forward strategy, which locks in a dollar cost of financing without an option premium, may be the better strategy.

3. The important distinction between a futures contract and an options contract is that the futures contract is an obligation. When an investor purchases or sells a futures contract, the investor has an obligation to either accept or deliver, respectively, the underlying commodity on the expiration date. In contrast, the buyer of an option contract is not obligated to accept or deliver the underlying commodity but instead has the right, or choice, to accept delivery (for call holders) or make delivery (for put holders) of the underlying commodity anytime during the life of the contract.

   Futures and options modify a portfolio's risk in different ways. Buying or selling a futures contract affects a portfolio's upside risk and downside risk by a similar magnitude. This is commonly referred to as *symmetrical impact*. On the other hand, the addition of a call or put option to a portfolio does not affect a portfolio's upside risk and downside risk to a similar magnitude. Unlike futures contracts, the impact of options on the risk profile of a portfolio is asymmetric.

4. a. The investor should sell the forward contract to protect the value of the bond against rising interest rates during the holding period. Because the investor intends to take a long position in the underlying asset, the hedge requires a short position in the derivative instrument.


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b. The value of the forward contract on expiration date is equal to the spot price of the underlying asset on expiration date minus the forward price of the contract:

$$\$978.40 - \$1{,}024.70 = -\$46.30$$

The contract has a negative value. This is the value to the holder of a long position in the forward contract. In this example, the investor should be *short* the forward contract, so that the value to this investor would be +$46.30 since this is the cash flow the investor expects to receive.

c. The value of the combined portfolio at the end of the six-month holding period is:

$$\$978.40 + \$46.30 = \$1{,}024.70$$

The change in the value of the combined portfolio during this six-month period is $24.70

The value of the combined portfolio is the sum of the market value of the bond and the value of the *short* position in the forward contract. At the start of the six-month holding period, the bond is worth $1,000 and the forward contract has a value of zero (because this is not an off-market forward contract, no money changes hands at initiation). Six months later, the bond value is $978.40 and the value of the *short* position in the forward contract is $46.30, as calculated in part (b).

The fact that the combined value of the long position in the bond and the short position in the forward contract at the forward contract's maturity date is equal to the forward price on the forward contract at its initiation date is not a coincidence. By taking a long position in the underlying asset and a short position in the forward contract, the investor has created a fully hedged (and hence risk-free) position and should earn the risk-free rate of return. The six-month risk-free rate of return is 5% (annualized), which produces a return of $24.70 over a six-month period:

$$(\$1{,}000 \times 1.05^{(1/2)}) - \$1{,}000 = \$24.70$$

These results support VanHusen's statement that selling a forward contract on the underlying bond protects the portfolio during a period of rising interest rates. The loss in the value of the underlying bond during the six-month holding period is offset by the cash payment made at expiration date to the holder of the short position in the forward contract; that is, a short position in the forward contract protects (hedges) the long position in the underlying asset.

5. a. Accurate. Futures contracts are marked to the market daily. Holding a short position on a bond futures contract during a period of rising interest rates (declining bond prices) generates positive cash inflow from the daily mark to market. If an investor in a futures contract has a long position when the price of the underlying asset increases, then the daily mark to market generates a positive cash inflow that can be reinvested. Forward contracts settle only at expiration date and do not generate any cash flow prior to expiration.


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b. Inaccurate. According to the cost-of-carry model, the futures contract price is adjusted upward by the cost of carry for the underlying asset. Bonds (and other financial instruments), however, do not have any significant storage costs. Moreover, the cost of carry is reduced by any coupon payments paid to the bondholder during the life of the futures contract. Any "convenience yield" from holding the underlying bond also reduces the cost of carry. As a result, the cost of carry for a bond is likely to be negative.


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# CHAPTER 23: FUTURES, SWAPS, AND RISK MANAGEMENT

## PROBLEM SETS

1. In formulating a hedge position, a stock's beta and a bond's duration are used similarly to determine the expected percentage gain or loss in the value of the underlying asset for a given change in market conditions. Then, in each of these markets, the expected percentage change in value is used to calculate the expected dollar change in value of the stock or bond portfolios, respectively. Finally, the dollar change in value of the underlying asset, along with the dollar change in the value of the futures contract, determines the hedge ratio.

   The major difference in the calculations necessary to formulate a hedge position in each market lies in the manner in which the first step identified above is computed. For a hedge in the equity market, the product of the equity portfolio's beta with respect to the given market index and the expected percentage change in the index for the futures contract equals the expected percentage change in the value of the portfolio. Clearly, if the portfolio has a positive beta and the investor is concerned about hedging against a decline in the index, the result of this calculation is a decrease in the value of the portfolio. For a hedge in the fixed income market, the product of the bond's modified duration and the expected change in the bond's yield equals the expected percentage change in the value of the bond. Here, the investor who has a long position in a bond (or a bond portfolio) is concerned about the possibility of an increase in yield, and the resulting change in the bond's value is a loss.

   A secondary difference in the calculations necessary to formulate a hedge position in each market arises in the calculation of the hedge ratio. In the equity market, the hedge ratio is typically calculated by dividing the total expected dollar change in the value of the portfolio (for a given change in the index) by the profit (i.e., the dollar change in value) on one futures contract (for the given change in the index). In the bond market, the comparable calculation is generally thought of in terms of the price value of a basis point (PVBP) for the bond and the PVBP for the futures contract, rather than in terms of the total dollar change in both the value of the portfolio and the value of a single futures contract.

2. One of the considerations that would enter into the hedging strategy for a U.S. exporting firm with outstanding bills to its customers denominated in foreign currency is whether the U.S. firm also has outstanding payables denominated in the same foreign currency. Since the firm receives foreign currency when its customers' bills are paid, the firm hedges by taking a short position in the foreign currency. The U.S. firm would reduce its short position in futures to the extent that outstanding payables offset outstanding receivables with the same maturity because the outstanding payables effectively hedge the exchange rate risk of the outstanding receivables. Equivalently, if the U.S. firm expects to incur ongoing obligations denominated in the same foreign currency in order to meet expenses required to deliver additional products to its customers, then the firm would reduce its short


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position in the foreign currency futures. In general, if the U.S. firm incurs expenses in the same foreign currency, then the firm would take a short position in the currency futures to hedge its profits measured in the foreign currency. If the U.S. firm incurs all of its expenses in U.S. dollars, but bills its customers in the foreign currency, then the firm would take a position to hedge the outstanding receivables, not just the profit. Another consideration that affects the U.S. exporting firm's willingness to hedge its exchange rate risk is the impact of depreciation of the foreign currency on the firm's prices for its products. For a U.S. firm that sets its prices in the foreign currency, the dollar-equivalent price of the firm's products is reduced when the foreign currency depreciates, so that the firm is likely to find it desirable to increase its short position in currency futures to hedge against this risk. If the U.S. firm is not able to increase the price of its products in the foreign currency due to competition, the depreciation of the foreign currency has an impact on profits similar to the impact of foreign currency depreciation on the U.S. firm's receivables.

3. The hedge will be much more effective for the gold-producing firm. Prices for distant maturity oil futures contracts have surprisingly low correlation with current prices because convenience yields and storage costs for oil can change dramatically over time. When near-term oil prices fall, there may be little or no change in longer-term prices, since oil prices for very distant delivery generally respond only slightly to changes in the current market for short-horizon oil. Because the correlation between short- and long-maturity oil futures is so low, hedging long-term commitments with short-maturity contracts does little to eliminate risk; that is, such a hedge eliminates very little of the variance entailed in uncertain future oil prices.

   In contrast, both convenience yields and storage costs for gold are substantially smaller and more stable; the result is that the correlation between short-term and more distant gold futures prices is substantially greater. In other words, the basis between near and distant maturity gold futures prices is far less variable, so hedging long-term prices with short-term gold contracts results in a substantially greater percentage reduction in volatility.
4. Municipal bond yields, which are below T-bond yields (tax-exempt status), are expected to close in on Treasury yields. Because yields and prices are inversely related, municipal bond prices will perform poorly compared to Treasuries. Establish a spread position, buying Treasury-bond futures and selling municipal bond futures. The net bet on the general level of interest rates is approximately zero. You have made a bet on relative performances in the two sectors.


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5. a. $S_0 \times (1 + r_M) - D = (1{,}425 \times 1.06) - 15 = 1{,}495.50$

   b. $S_0 \times (1 + r_f) - D = (1{,}425 \times 1.03) - 15 = 1{,}452.75$

   c. The futures price is too low. Buy futures, short the index, and invest the proceeds of the short sale in T-bills:

| | CF Now | CF in 6 months |
|---|---|---|
| Buy futures | 0 | $S_T - 1{,}422$ |
| Short index | 1,425 | $-S_T - 15$ |
| Buy T-bills | −1,425 | 1,467.75 |
| Total | 0 | 30.75 |

6. a. The value of the underlying stock is:

   $250 × 1,600 = $400,000

   $25/$400,000 = 0.0000625 = 0.0063% of the value of the stock

   b. $40 × 0.000063 = $0.00250

   c. $0.10/$0.0025 = 40.00

   The transaction cost in the stock market is 40.00 times the transaction cost in the futures market.

7. a. You should be short the index futures contracts. If the stock value falls, you need futures profits to offset the loss.

   b. Each contract is for $250 times the index, currently valued at 1,650. Therefore, each contract controls stock worth: $250 × 1,650 = $412,500
   In order to hedge a $16.5 million portfolio, you need:

$$\frac{16{,}500{,}000}{412{,}500} = 40 \text{ contracts}$$

   c. Now, your stock swings only 0.6 as much as the market index. Hence, you need 0.6 as many contracts as in (b): 0.6 × 40 = 24 contracts.

8. If the beta of the portfolio were 1.0, she would sell $1 million of the index. Because beta is 1.25, she should sell $1.25 million of the index.

9. You would short $0.50 of the market index contract and $0.75 of the computer industry contract for each dollar held in IBM.


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10. The dollar is depreciating relative to the euro. To induce investors to invest in the U.S., the U.S. interest rate must be higher.

11. a. From parity: $F_0 = E_0 \times \frac{1+r_{US}}{1+r_{UK}} = 2.00 \times \frac{1.04}{1.06} = 1.962$

    b. Let $F_0$ = \$2.03/£. Dollars are relatively too cheap in the forward market ↔ pounds are too expensive. Borrow the present value of £1, use the proceeds to buy pound-denominated bills in the spot market, and sell £1 forward:

| Action Now | CF in \$ | Action at period-end | CF in \$ |
|---|---|---|---|
| Sell £1 forward for \$2.03 | 0 | Collect \$2.03, deliver £1 | \$2.03 – \$$E_1$ |
| Buy £1/1.06 in spot market; invest at the British risk-free rate | –2.00/1.06 = –\$1.887 | Exchange £1 for \$$E_1$ | \$$E_1$ |
| Borrow \$1.887 | \$1.887 | Repay loan; U.S. interest rate = 4% | –\$1.962 |
| Total | 0 | Total | \$0.068 |

12. a. Lend in the U.K.

    b. Borrow in the U.S.

    c. Lending in the U.S. offers a 4% rate of return. Lending in the U.K. and covering interest rate risk with futures or forwards offers a rate of return of

$$r_{US} = \left[ (1+r_{UK}) \times \frac{F_0}{E_0} \right] - 1 = \left[ 1.07 \times \frac{1.98}{2.00} \right] - 1 = 0.0593 = 5.93\%$$

    An arbitrage strategy involves simultaneous lending (UK) and borrowing (US) with the covering of interest rate risk:

| Action Now | CF in \$ | Action at period-end | CF in \$ |
|---|---|---|---|
| Borrow \$2.00 in U.S. | \$2.00 | Repay loan | –\$2.00 × 1.04 |
| Convert borrowed dollars to pounds; lend £1 pound in U.K. | –\$2.00 | Collect repayment; exchange proceeds for dollars | 1.07 × $E_1$ |
| Sell forward £1.07 at $F_0$ = \$1.98 | 0 | Unwind forward | 1.07 × (\$1.98 – $E_1$) |
| Total | 0 | Total | \$0.0386 |

13. The farmer must sell forward

    100,000 × (1/0.90) = 111,111 bushels of yellow corn

    This requires selling 111,111/5,000 = 22.2 contracts

14. The closing futures price will be: 100 – 0.40 = 99.60


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The initial futures price was 99.7400, so the loss to the long side is 14 basis points or

14 basis points × \$25 per basis point = \$350

The loss can also be computed as

0.0014 × ¼ × \$1,000,000 = \$350

15. Suppose the yield on your portfolio increases by 1.5 basis points. Then the yield on the T-bond contract is likely to increase by 1 basis point. The loss on your portfolio will be

\$1 million × Δy × $D^*$ = \$1,000,000 × 0.00015 × 4 = \$600

The change in the futures price (per \$100 par value) will be

\$95 × 0.0001 × 9 = \$0.0855

This is a change of \$85.50 on a \$100,000 par value contract. Therefore you should sell:

\$600/\$85.50 = 7 contracts

16. She must sell: $\$1 \text{ million} \times \frac{8}{10} = \$0.8 \text{ million}$ of T-bonds

17. If yield changes on the bond and the contracts are each 1 basis point, then the bond value will change by

\$10,000,000 × 0.0001 × 8 = \$8,000

The contract will result in a cash flow of

\$100,000 × 0.0001 × 6 = \$60

Therefore, the firm should sell: 8,000/60 = 133 contracts
The firm sells the contracts because you need profits on the contract to offset losses as a bond issuer if interest rates increase.

18. $F_0 = S_0(1 + r_f)^T = 1500 \times 1.02 = 1530$

If $F_0 = 1545$, you could earn arbitrage profits as follows:

| | CF Now | CF in 1 year |
|---|---|---|
| Buy gold | −1500 | $S_T$ |
| Short futures | 0 | $1545 - S_T$ |
| Borrow \$980 | 1500 | −1530 |
| Total | 0 | 15 |

The forward price must be 1530 in order for this strategy to yield no profit.


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19. If a poor harvest today indicates a worse than average harvest in future years, then the futures prices will rise in response to today's harvest, although presumably the two-year price will change by less than the one-year price. The same reasoning holds if corn is stored across the harvest. Next year's price is determined by the available supply at harvest time, which is the actual harvest plus the stored corn. A smaller harvest today means less stored corn for next year which can lead to higher prices.

    Suppose first that corn is never stored across a harvest, and second that the quality of a harvest is not related to the quality of past harvests. Under these circumstances, there is no link between the current price of corn and the expected future price of corn. The quantity of corn stored will fall to zero before the next harvest, and thus the quantity of corn and the price in one year will depend solely on the quantity of next year's harvest, which has nothing to do with this year's harvest.

20. The required rate of return on an asset with the same risk as corn is

    .5% + 0.5(.9% – .5%) = .7% per month

    Thus, in the absence of storage costs, three months from now corn would sell for

    $\$5.50 \times 1.007^3 = \$5.616$

    The future value of three month's storage costs is

    $\$0.03 \times FA(.5\%, 3) = \$0.090$ (PMT = \$0.03; N = 3; I = 0.5; PV = \$0; Solve for FV = \$0.09).

    where FA stands for the future value factor for a level annuity with a given interest rate and number of payments. Thus, in order to induce storage, the expected price would have to be

    $\$5.616 + \$0.090 = \$5.707$

    Because the expected spot price is \$5.88, you would store corn.

21. If the exchange of currencies were structured as three separate forward contracts, the forward prices would be determined as follows:

    Forward exchange rate × \$1 million euros = Dollars to be delivered

    Year 1: $1.50 \times (1.04/1.03) \times 1$ million euros = \$1.5146 million

    Year 2: $1.50 \times (1.04/1.03)^2 \times 1$ million euros = \$1.5293 million

    Year 3: $1.50 \times (1.04/1.03)^3 \times 1$ million euros = \$1.5441 million

    Instead, we deliver the same number of dollars ($F^*$) each year. The value of $F^*$ is determined by first computing the present value of this obligation:

$$\frac{F^*}{1.04^1} + \frac{F^*}{1.04^2} + \frac{F^*}{1.04^3} = \frac{1.5146}{1.04^1} + \frac{1.5293}{1.04^2} + \frac{1.5441}{1.04^3} = 4.2430$$


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$F^*$ equals \$1.5290 million per year (PV = 4.243; N = 3; I = 4; FV = \$0; Solve for PMT = 1.529).

22. The firm's overall cost of the fund will equal the spread between the LIBOR rate and the fixed rate on debt times the notional principal. In this case (0.06-0.07) × \$100,000,000 = -\$1,000,000.

23. a. The swap rate moved in favor of firm ABC. ABC should have received 1% more per year than it could receive in the current swap market. Based on notional principal of \$10 million, the loss is

0.01 × \$10 million = \$100,000 per year.

b. The market value of the fixed annual loss is obtained by discounting at the current 5% rate on three-year swaps. The loss is

\$100,000 × Annuity factor (5%, 3) = \$272,325
(PMT = \$100,000; N = 3; I = 5; FV = \$0; Solve for PV = \$272,325)

c. If ABC had become insolvent, XYZ would not be harmed. XYZ would be happy to see the swap agreement canceled. However, the swap agreement ought to be treated as an asset of ABC when the firm is reorganized.

24. The firm receives a fixed rate that is 2% higher than the market rate. The extra payment of (0.02 × \$10 million) has present value equal to
\$200,000 × Annuity factor (8%, 5) = \$798,542
(PMT = \$200,000; N = 5; I = 8; FV = \$0; Solve for PV = \$798,542)

25. a. From parity: $F_0 = 1{,}600 \times (1 + 0.03) - 20 = 1{,}628$

Actual $F_0$ is 1,624; so the futures price is 4 below the "proper" level.

b. Buy the relatively cheap futures, sell the relatively expensive stock and lend the proceeds of the short sale:

| | CF Now | CF in 6 months |
|---|---|---|
| Buy futures | 0 | $S_T - 1{,}624$ |
| Sell shares | 1,600 | $-S_T - 20$ |
| Lend \$1,600 | −1,600 | 1,648 |
| Total | 0 | 4 |

c. If you do not receive interest on the proceeds of the short sales, then the \$1,600 you receive will not be invested but will simply be returned to you. The proceeds from the strategy in part (b) are now negative: an arbitrage opportunity no longer exists.

| CF Now | CF in 6 months |
|---|---|


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| | | |
|---|---|---|
| Buy futures | 0 | $S_T - 1{,}624$ |
| Sell shares | 1,600 | $-S_T - 20$ |
| Place \$1,200 in margin account | −1,600 | 1,600 |
| Total | 0 | −44 |

d. If we call the original futures price $F_0$, then the proceeds from the long-futures, short-stock strategy are

| | CF Now | CF in 6 months |
|---|---|---|
| Buy futures | 0 | $S_T - F_0$ |
| Sell shares | 1,600 | $-S_T - 20$ |
| Place \$1,200 in margin account | −1,600 | 1,600 |
| Total | 0 | $1{,}580 - F_0$ |

$F_0$ can be as low as 1,580 without giving rise to an arbitrage opportunity.

On the other hand, if $F_0$ is higher than the parity value (1,628), then an arbitrage opportunity (buy stocks, sell futures) will exist.

| | CF Now | CF in 6 months |
|---|---|---|
| Sell futures | 0 | $F_0 - S_T$ |
| Buy shares | −1,600 | $S_T + 20$ |
| Borrow \$1,200 | 1,600 | −1,648 |
| Total | 0 | $F_0 - 1{,}628$ |

Therefore, the no-arbitrage range is

$$1{,}580 \le F_0 \le 1{,}628$$

26. a. Call $p$ the fraction of proceeds from the short sale to which we have access. Ignoring transaction costs, the lower bound on the futures price that precludes arbitrage is the following usual parity value (except for the factor $p$):

$$S_0 (1 + r_f p) - D$$

The dividend ($D$) equals: $0.012 \times 1{,}350 = \$16.20$

The factor $p$ arises because only this fraction of the proceeds from the short sale can be invested in the risk-free asset. We can solve for $p$ as follows:

$$1{,}350 \times (1 + 0.022p) - 16.20 = 1{,}351 \Rightarrow p = 0.579$$

b. With $p = 0.9$, the no-arbitrage lower bound on the futures price is

$$1{,}350 \times [1 + (0.022 \times 0.9)] - 16.20 = 1{,}360.53$$

The actual futures price is 1,351. The departure from the bound is therefore 9.53. This departure also equals the potential profit from an arbitrage strategy. The strategy is to short the stock, which currently sells at 1,350. The investor receives


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90% of the proceeds (1,215) and the remainder (135) remains in the margin account until the short position is covered in six months. The investor buys futures and lends 1,215:

| | CF Now | CF in 6 months |
|---|---|---|
| Buy futures | 0 | $S_T - 1{,}351$ |
| Sell shares | 1350 – 135 | $135 - S_T - 16.20$ |
| Lend | –1,215 | $1{,}215 \times 1.022 = 1{,}241.73$ |
| Total | 0 | 9.53 |

The profit is: 9.53 × \$250 per contract = \$2,382.50

## CFA PROBLEMS

1. a. By spot-futures parity:

$$F_0 = S_0 \times (1 + r_f) = 185 \times [1 + (0.06/2)] = 190.55$$

b. The lower bound is based on the reverse cash-and-carry strategy.

| Action Now | CF in \$ | Action at period-end | CF in \$ |
|---|---|---|---|
| Buy one TOBEC index futures contract | 0 | Sell one TOBEC index futures contract | $\$100 \times (F_1 - F_0)$ |
| Sell spot TOBEC index | +\$18,500 | Buy spot TOBEC index | $-\$100 \times S_1$ |
| Lend \$18,500 | −\$18,500 | Collect loan repayment | $\$18{,}500 \times 1.03 = +\$19{,}055$ |
| | | Pay transaction costs | −\$15.00 |
| Total | 0 | Total | $-\$100F_0 + \$19{,}040$ |

(Note that $F_1 = S_1$ at expiration.)
$-\$100F_0 + \$19{,}040 = 0 \rightarrow$
The lower bound for $F_0$ is: 19,040/100 = 190.40

2. a. The strategy would be to sell Japanese stock index futures to hedge the market risk of Japanese stocks, and to sell yen futures to hedge the currency exposure.

b. Some possible practical difficulties with this strategy include
- Contract size on futures may not match size of portfolio.
- Stock portfolio may not closely track index portfolios on which futures trade.
- Cash flow management issues from marking to market.
- Potential mispricing of futures contracts (violations of parity).

3. a. The hedged investment involves converting the \$1 million to foreign currency, investing in that country, and selling forward the foreign currency in order to lock in the dollar value of the investment. Because the interest rates are for


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90-day periods, we assume they are quoted as bond equivalent yields, annualized using simple interest. Therefore, to express rates on a per quarter basis, we divide these rates by 4:

| | Japanese government | Swiss government |
|---|---|---|
| Convert $1 million to local currency | $1,000,000 × 133.05 = ¥133,050,000 | $1,000,000 × 1.5260 = SF1,526,000 |
| Invest in local currency for 90 days | ¥133,050,000 × [1 + (0.076/4)] = ¥135,577,950 | SF1,526,000 × [1 + (0.086/4)] = SF1,558,809 |
| Convert to $ at 90-day forward rate | 135,577,950/133.47 = $1,015,793 | 1,558,809/1.5348 = $1,015,643 |

b. The results in the two currencies are nearly identical. This near-equality reflects the interest rate parity theorem. This theory asserts that the pricing relationships between interest rates and spot and forward exchange rates must make covered (that is, fully hedged and riskless) investments in any currency equally attractive.

c. The 90-day return in Japan is 1.5793%, which represents a bond-equivalent yield of 1.5793% × 365/90 = 6.405%. The 90-day return in Switzerland is 1.5643%, which represents a bond-equivalent yield of 1.5643% × 365/90 = 6.344%. The estimate for the 90-day risk-free U.S. government money market yield is in this range.

4. The investor can buy $X$ amount of pesos at the (indirect) spot exchange rate and invest the pesos in the Mexican bond market. Then, in one year, the investor will have

$$X \times (1 + r_{MEX}) \text{ pesos}$$

These pesos can then be converted back into dollars using the (indirect) forward exchange rate. Interest rate parity asserts that the two holding period returns must be equal, which can be represented by the formula:

$$(1 + r_{US}) = E_0 \times (1 + r_{MEX}) \times (1/F_0)$$


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The left side of the equation represents the holding-period return for a U.S. dollar-denominated bond. If interest rate parity holds, then this term also corresponds to the U.S. dollar holding-period return for the currency-hedged Mexican one-year bond. The right side of the equation is the holding-period return, in dollar terms, for a currency-hedged peso-denominated bond. Solving for $r_{US}$:

$(1 + r_{US}) = 9.5000 \times (1 + 0.065) \times (1/9.8707)$

$(1 + r_{US}) = 1.0250$

$r_{US} = 2.50\%$

Thus $r_{US} = 2.50\%$, which is the same as the yield for the one-year U.S. bond.

5. a. From parity: $F_0 = E_0 \times \left(\frac{1 + r_{Japan}}{1 + r_{US}}\right)^{0.5} = 124.30 \times \left(\frac{1.0010}{1.0380}\right)^{0.5} = 122.06453$

b.

| Action Now | CF in $ | Action at Period- | CF in ¥ |
|---|---|---|---|
| Borrow $1,000,000 in U.S. | $1,000,000 | Repay loan | $-(\$1{,}000{,}000 \times 1.035^{0.25}) =$ $-\$1{,}008{,}637.446$ |
| Sell forward $1,008,637.446 at $F_0$ = ¥123.2605 | 0 | Unwind forward | $-(1{,}008{,}637.446 \times ¥123.2605) =$ $-¥124{,}325{,}155.9127$ |
| Convert borrowed dollars to yen; lend ¥124,300,000 in Japan | –$1,000,000 | Collect repayment in yen | $¥124{,}300{,}000 \times 1.005^{0.25} =$ ¥124,455,084.5187 |
| Total | 0 | Total | ¥129,928.61 |

The arbitrage profit is ¥129,928.61

6. a. Delsing should sell stock index futures contracts and buy bond futures contracts. This strategy is justified because buying the bond futures and selling the stock index futures provides the same exposure as buying the bonds and selling the stocks. This strategy assumes high correlation between the bond futures and the bond portfolio, as well as high correlation between the stock index futures and the stock portfolio.

b. The number of contracts in each case is:

i. $5 \times \$200{,}000{,}000 \times 0.0001 = \$100{,}000$

$\$100{,}000/97.85 = 1022$ contracts

ii. $\$200{,}000{,}000/(\$1{,}378 \times 250) = 581$ contracts


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7. Situation A. The market value of the portfolio to be hedged is $20 million. The market value of the bonds controlled by one futures contract is $63,330. If we were to equate the market values of the portfolio and the futures contract, we would sell

$$\$20{,}000{,}000/\$63{,}330 = 315.806 \text{ contracts}$$

However, we must adjust this "naive" hedge for the price volatility of the bond portfolio relative to the futures contract. Price volatilities differ according to both the duration and the yield volatility of the bonds. In this case, the yield volatilities may be assumed equal, because any yield spread between the Treasury portfolio and the Treasury bond underlying the futures contract is likely to be stable. However, the duration of the Treasury portfolio is less than that of the futures contract. Adjusting the naive hedge for relative duration and relative yield volatility, we obtain the adjusted hedge position

$$315.806 \times \frac{7.6}{8.0} \times 1.0 = 300 \text{ contracts}$$

Situation B. Here, the treasurer seeks to hedge the purchase price of the bonds; this requires a long hedge. The market value of the bonds to be purchased is

$$\$20 \text{ million} \times 0.93 = \$18.6 \text{ million}$$

The duration ratio is 7.2/8.0, and the relative yield volatility is 1.25. Therefore, the hedge requires the treasurer to take a long position in

$$\frac{18{,}600{,}000}{63{,}330} \times \frac{7.2}{8.0} \times 1.25 = 330 \text{ contracts}$$

8. a. % change in T-bond price = Modified duration × Change in YTM

$$= 7.0 \times 0.50\% = 3.5\%$$

b. When the YTM of the T-bond changes by 50 basis points, the predicted change in the yield on the KC bond is 1.22 × 50 = 61 basis points. Therefore

$$\text{\% change in KC price} = \text{Modified duration} \times \text{Change in YTM}$$

$$= 6.93 \times 0.61\% = 4.23\%$$


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# CHAPTER 24: PORTFOLIO PERFORMANCE EVALUATION

## PROBLEM SETS

1. The dollar-weighted average will be the internal rate of return between the initial and final value of the account, including additions and withdrawals. Using Excel's XIRR function, utilizing the given dates and values, the dollar-weighted average return is as follows:

| Date | Account |
|---|---|
| 1/1/2010 | -$148,000.00 |
| 1/3/2010 | $2,500.00 |
| 3/20/2010 | $4,000.00 |
| 7/5/2010 | $1,500.00 |
| 12/2/2010 | $14,360.00 |
| 3/10/2011 | -$23,000.00 |
| 4/7/2011 | $3,000.00 |
| 5/3/2011 | $198,000.00 |
| | 26.99% |
| | =XIRR(C13:C20,B13:B20) |

Since the dates of additions and withdrawals are not equally spaced, there really is no way to solve this problem using a financial calculator. Excel can solve this very quickly.

2. As established in the following result from the text, the Sharpe ratio depends on both alpha for the portfolio ( $\alpha_P$ ) and the correlation between the portfolio and the market index (ρ):

$$\frac{E(r_P - r_f)}{\sigma_P} = \frac{\alpha_P}{\sigma_P} + \rho S_M$$

Specifically, this result demonstrates that a lower correlation with the market index reduces the Sharpe ratio. Hence, if alpha is not sufficiently large, the portfolio is inferior to the index. Another way to think about this conclusion is to note that, even for a portfolio with a positive alpha, if its diversifiable risk is sufficiently large, thereby reducing the correlation with the market index, this can result in a lower Sharpe ratio.

3. The IRR (i.e., the dollar-weighted return) cannot be ranked relative to either the geometric average return (i.e., the time-weighted return) or the arithmetic average return. Under some conditions, the IRR is greater than each of the other two averages, and similarly, under other conditions, the IRR can also be less than each of the other averages. A number of scenarios can be developed to illustrate this conclusion. For


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example, consider a scenario where the rate of return each period consistently increases over several time periods. If the amount invested also increases each period, and then all of the proceeds are withdrawn at the end of several periods, the IRR is greater than either the geometric or the arithmetic average because more money is invested at the higher rates than at the lower rates. On the other hand, if withdrawals gradually reduce the amount invested as the rate of return increases, then the IRR is less than each of the other averages. (Similar scenarios are illustrated with numerical examples in the text, where the IRR is shown to be less than the geometric average, and in Concept Check 1, where the IRR is greater than the geometric average.)

4. It is not necessarily wise to shift resources to timing at the expense of security selection. There is also tremendous potential value in security analysis. The decision as to whether to shift resources has to be made on the basis of the macro, compared to the micro, forecasting ability of the portfolio management team.

5. a. Arithmetic average: $\bar{r}_{ABC} = 10\%$; $\bar{r}_{XYZ} = 10\%$

   b. Dispersion: $\sigma_{ABC} = 7.07\%$; $\sigma_{XYZ} = 13.91\%$

   Stock XYZ has greater dispersion.
   (Note: We used 5 degrees of freedom in calculating standard deviations.)

   c. Geometric average:

   $$r_{ABC} = (1.20 \times 1.12 \times 1.14 \times 1.03 \times 1.01)^{1/5} - 1 = 0.0977 = 9.77\%$$

   $$r_{XYZ} = (1.30 \times 1.12 \times 1.18 \times 1.00 \times 0.90)^{1/5} - 1 = 0.0911 = 9.11\%$$

   Despite the fact that the two stocks have the same arithmetic average, the geometric average for XYZ is less than the geometric average for ABC. The reason for this result is the fact that the greater variance of XYZ drives the geometric average further below the arithmetic average.

   d. In terms of "forward-looking" statistics, the arithmetic average is the better estimate of expected rate of return. Therefore, if the data reflect the probabilities of future returns, 10 percent is the expected rate of return for both stocks.

6. a. Time-weighted average returns are based on year-by-year rates of return:

| Year | Return = (Capital gains + Dividend)/Price |
|---|---|
| 2013 − 2014 | [($120 – $100) + $4]/$100 = 24.00% |
| 2014 − 2015 | [($90 – $120) + $4]/$120 = –21.67% |
| 2015 − 2016 | [($100 – $90) + $4]/$90 = 15.56% |

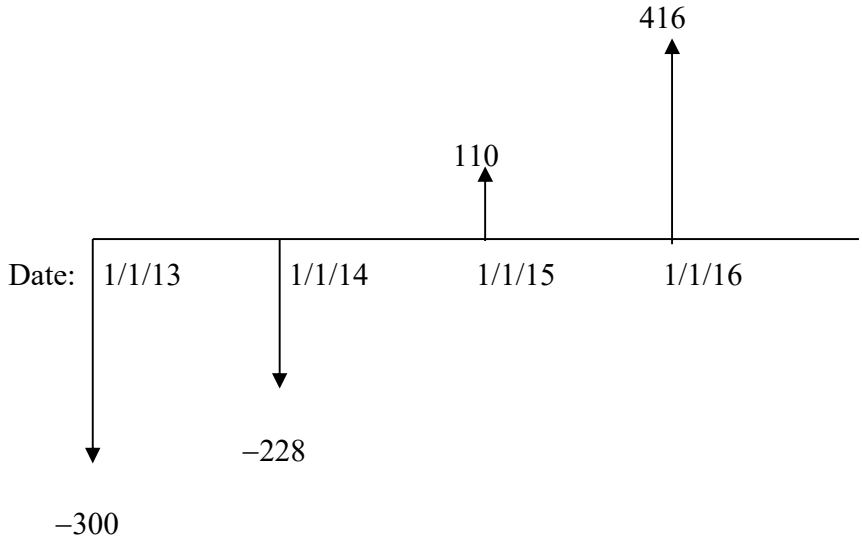
   Arithmetic mean: (24% – 21.67% + 15.56%)/3 = 5.96%

   Geometric mean: $(1.24 \times 0.7833 \times 1.1556)^{1/3} - 1 = 0.0392 = 3.92\%$


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b.

| Date | Cash Flow | Explanation |
|---|---|---|
| 1/1/13 | –$300 | Purchase of three shares at $100 each |
| 1/1/14 | –$228 | Purchase of two shares at $120 less dividend income on three shares held |
| 1/1/15 | $110 | Dividends on five shares plus sale of one share at $90 |
| 1/1/16 | $416 | Dividends on four shares plus sale of four shares at $100 each |




Dollar-weighted return = Internal rate of return = –0.1607%
(CF 0 = -$300; CF 1 = -$228; CF 2 = $110; CF 3 = $416; Solve for IRR = 16.07%.)

7.

| Time | Cash Flow | Holding Period Return |
|---|---|---|
| 0 | 3×(–$90) = –$270 | |
| 1 | $100 | (100–90)/90 = 11.11% |
| 2 | $100 | 0% |
| 3 | $100 | 0% |

a. Time-weighted geometric average rate of return =

$$(1.1111 \times 1.0 \times 1.0)^{1/3} - 1 = 0.0357 = 3.57\%$$

b. Time-weighted arithmetic average rate of return = (11.11% + 0 + 0)/3 = 3.70%

The arithmetic average is always greater than or equal to the geometric average; the greater the dispersion, the greater the difference.

c. Dollar-weighted average rate of return = IRR = 5.46%
[Using a financial calculator, enter: $n$ = 3, PV = –270, FV = 0, PMT = 100. Then compute the interest rate, or use the $CF_0$=−300, $CF_1$=100, $F_1$=3, then compute IRR]. The IRR exceeds the other averages because the investment fund was the largest when the highest return occurred.


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