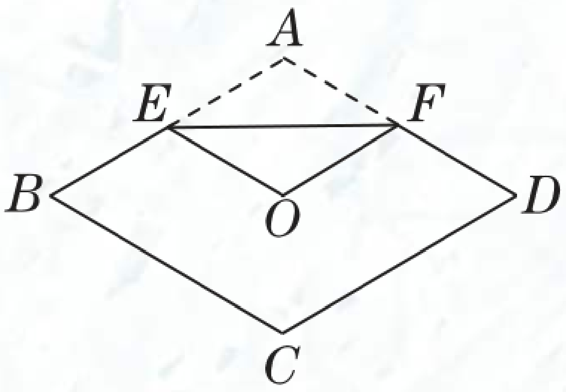
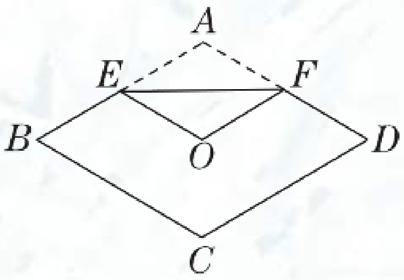
阶段拔尖专训6 特殊平行四边形中的折叠问题

题型1 菱形中的折叠问题

1.如图，将菱形纸片$ABCD$折叠，使点A恰好落在菱形的对角线交点O处，折痕为EF.若菱形的边长为2，$\angle A=120°$，求EF的长.



【解】连接BD，AC. ∵ 四边形$ABCD$是菱形，



∴ $AC \perp BD$，AC平分$\angle BAD$，

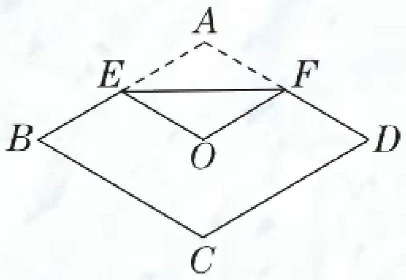
$BO = \frac{1}{2}BD$.

∵ $\angle BAD = 120°$ ，∴ $\angle BAC = 60°$.

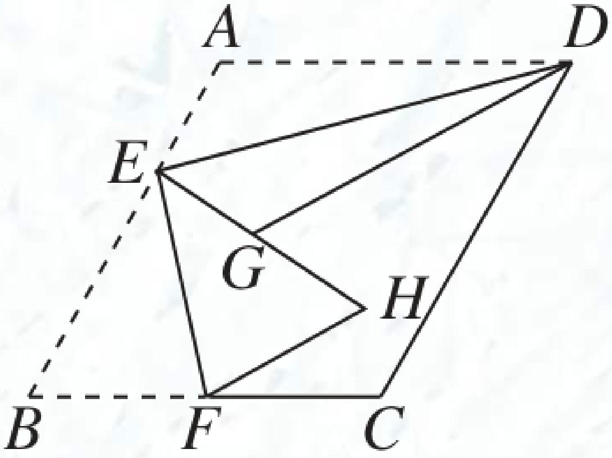
∴ $\angle ABO = 90° - 60° = 30°$.

$\because AB=2, \therefore AO=\frac{1}{2}AB=\frac{1}{2}\times 2=1$.由勾股定理得，$BO=\sqrt{AB^2-AO^2}=\sqrt{3}$. 易得$EF$为$\triangle ABD$的中位线，

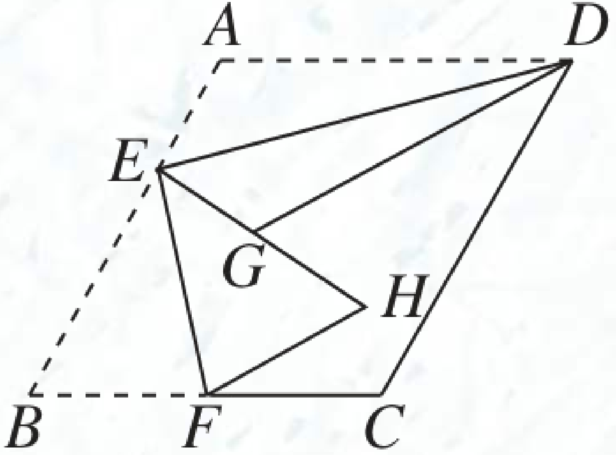
$\therefore EF=\frac{1}{2}BD=BO=\sqrt{3}$.



2.如图，在菱形$ABCD$中，$\angle A = 120°$，$AB = 2$，点E是边AB上一点，以DE为对称轴将$\triangle DAE$折叠得到$\triangle DGE$，再折叠BE使BE落在直线EG上，点B的对应点为点H，折痕为EF且交BC于点F.



（1）求$\angle DEF$的度数；



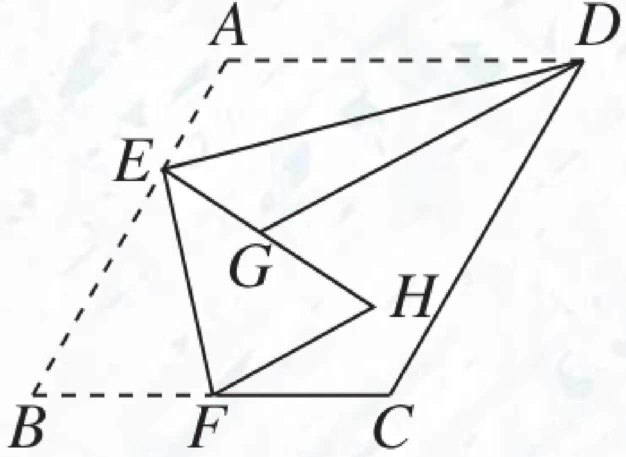
【解】由翻折可得$\angle AED=\angle DEG$，$\angle BEF=\angle HEF$，

$\therefore \angle DEG+\angle HEF=\angle AED+\angle BEF$.

$\because \angle DEG+\angle HEF+\angle AED+\angle BEF=180^\circ$，

$\therefore \angle DEG+\angle HEF=90^\circ$，即$\angle DEF=90^\circ$.

(2) 若点E是AB的中点，求DF的长.



$\because$ 四边形$ABCD$为菱形，$\therefore AD//BC$.

$\therefore \angle A+\angle B=180^{\circ}$.

由翻折可得$AE=EG$，$BE=EH$，

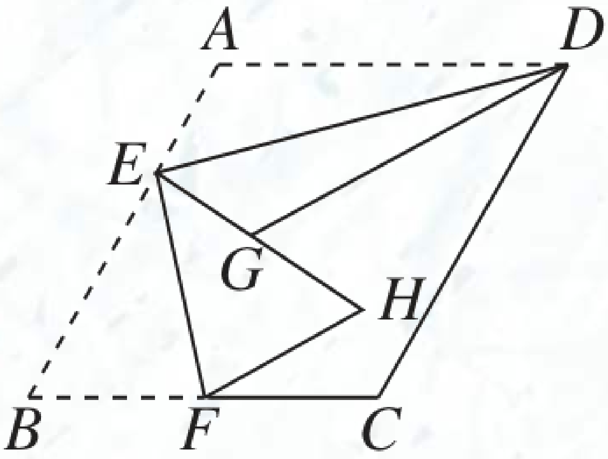
$\angle A=\angle EGD$，$\angle B=\angle EHF$.

$\because$ 点E是AB的中点，$\therefore AE=BE$.

$\therefore EG=EH$,即点G与点H重合.

$\because \angle EGD+\angle EHF=\angle A+\angle B=180^{\circ}$ ，

$\therefore$ 点D，G，F三点在同一条直线上.



过点D作$DM \perp BC$，交BC的延长线于点M.

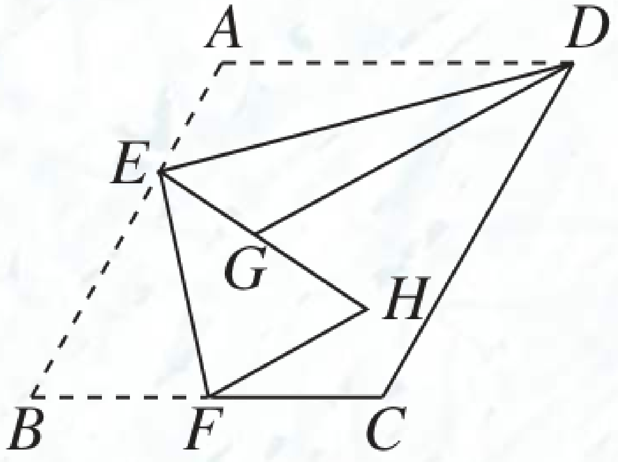
$\because \angle A = 120°$，$AB = 2$，

$\therefore$ 易得$\angle DCM = 60°$，

$CD = 2$. $\therefore \angle CDM = 30°$.

$\therefore CM = \frac{1}{2}CD = 1$. $\therefore DM = \sqrt{CD^2 - CM^2} = \sqrt{3}$.

由翻折可得$BF = FG$，$AD = DG = 2$，



设$BF = x$，则

$MF = 2 - x + 1 = 3 - x$,

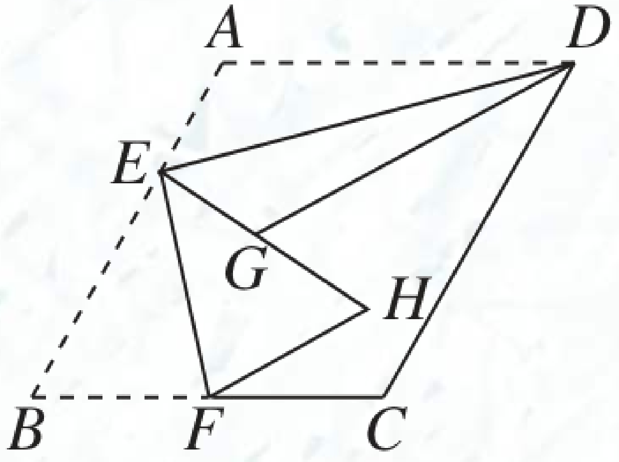
$DF = 2 + x$,

由勾股定理得

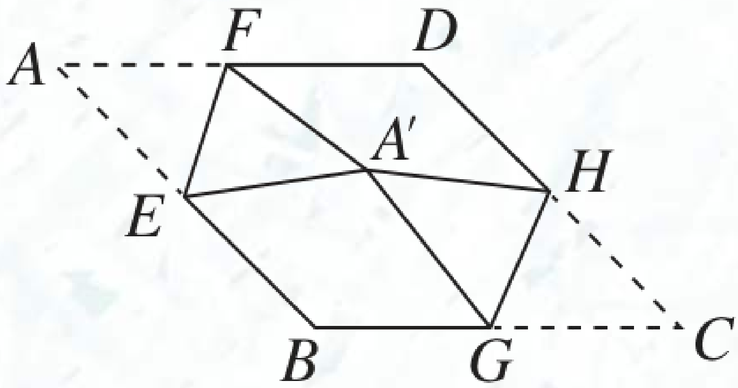
$(2 + x)^2 = (3 - x)^2 + (\sqrt{3})^2$，解得

$x = \frac{4}{5}$,

$\therefore DF = \frac{14}{5}$.

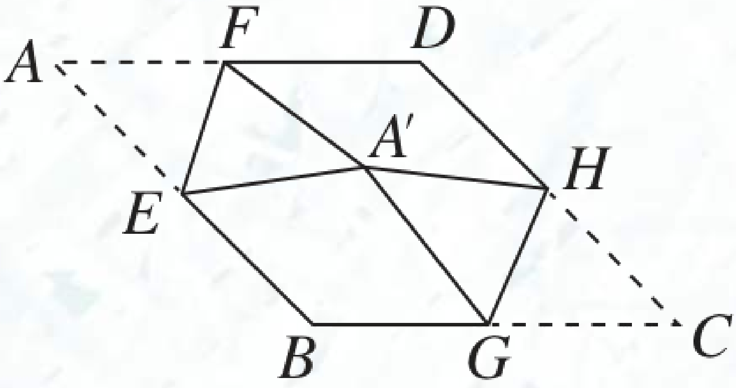


3. 新考法·分类讨论法 2024·烟台蓬莱区模拟 如图，在菱形$ABCD$中，$AB=4$，$\angle A=45^{\circ}$，点E为AB的中点，点F为AD上一动点，将$\triangle AEF$沿EF翻折，点A的对应点为点A'，再折叠菱形，使点C的对应点与点A'重合，折痕分别交BC，CD于点G，H.当$\triangle A'GH$是等腰三角形时，求AF的长.



【解】由折叠的性质得

$A'G=CG$，$A'H=CH$，$\angle GA'H=\angle BCD=\angle BAD=45°$，$A'F=AF$，$A'E=AE$，$\angle EA'F=\angle BAD=45°$．



∵ $\triangle A'GH$是等腰三角形，∴ 分三种情况讨论：

当$A'G = A'H$时，连接$A'A$，$A'C$，

则$A'G = CG = A'H = CH$，

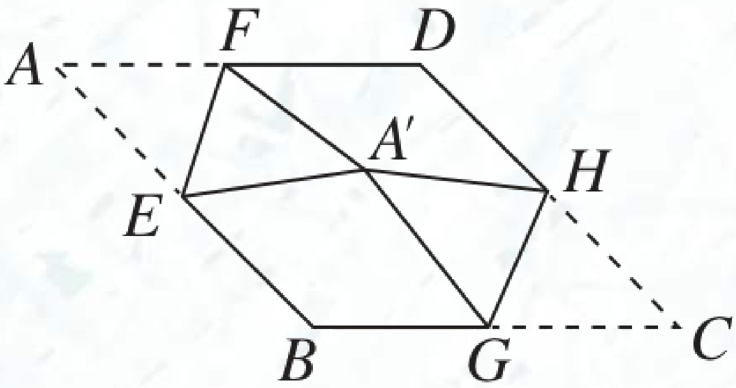
∴ 四边形$A'GCH$是菱形.∴ CA'是

$\angle BCD$的平分线.

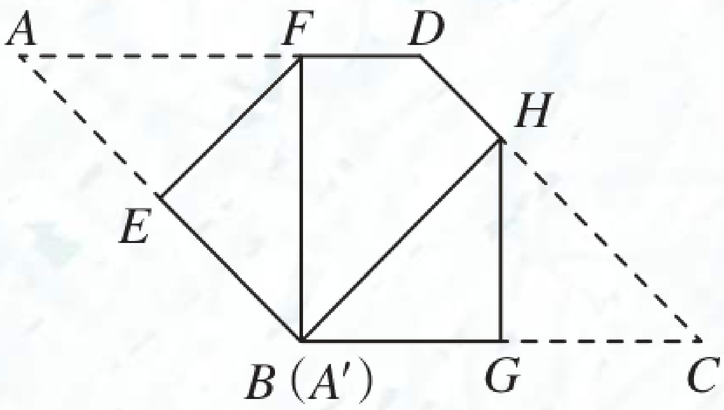
∴ $A'C$经过点A. ∴ 四边形$A'EAF$也

是菱形.

∴ $AF = AE = \frac{1}{2}AB = 2$;



当$A'G=GH$时，易知$\triangle A'GH$是等腰直角三角形，则点A'与点B重合，如图，

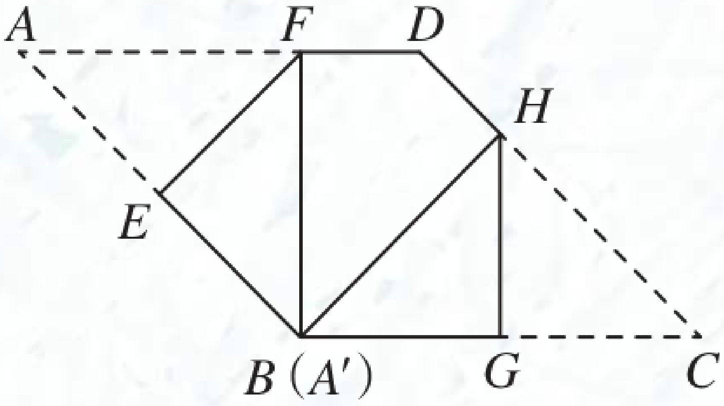


$\therefore \triangle AFB$是等腰直角三角形.

$\therefore$ 易得$AF=2\sqrt{2}$;

当$A'H=GH$，即$A'H=GH=CH$时，

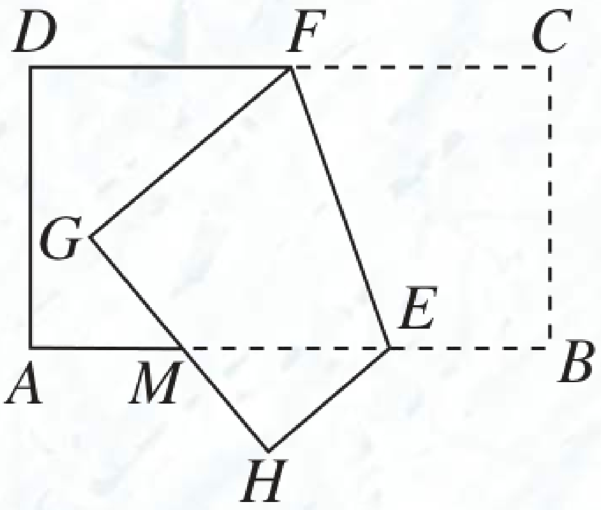
$\triangle A'GH$和$\triangle CGH$都是等腰直角三角形，且$\angle A'HG=\angle CHG=90°$，故点$A'$在直线$CD$上，此情况不存在.



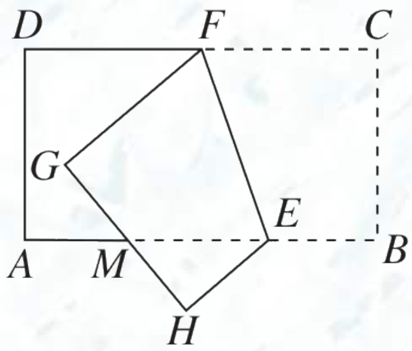
综上，AF的长为2或$2\sqrt{2}$.

题型2 矩形中的折叠问题

4.[2024·聊城模拟] 如图，将矩形$ABCD$沿EF对折，BC的对应边GH与AB交于点M，若$\angle CFE=70^\circ$，则$\angle GMA=$ <u>50</u> °.



【点拨】$\because$ 四边形$ABCD$是矩形，



$\therefore \angle B = 90°$，$AB // CD$.

$\therefore \angle AEF = \angle CFE = 70°$，

$\angle BEF = 180° - \angle CFE = 110°$.

由折叠得$\angle H = \angle B = 90°$，

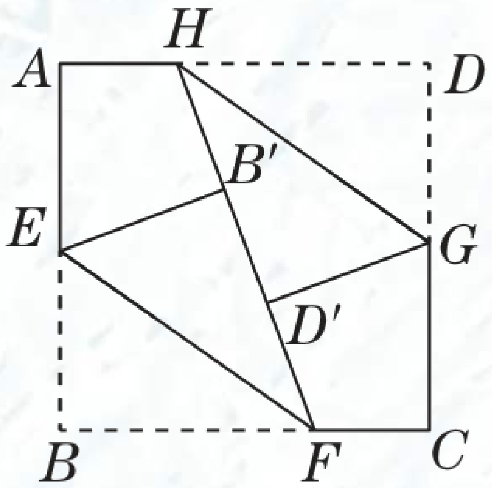
$\angle HEF = \angle BEF = 110°$，

$\therefore \angle MEH = \angle HEF - \angle AEF = 110° - 70° = 40°$.

$\therefore \angle HME = 90° - 40° = 50°$.

$\therefore \angle GMA = \angle HME = 50°$.

5.如图，把矩形$ABCD$的两角折叠，折痕分别为EF，HG，点B，D折叠后的对应点分别是B'，D'，并且点H，B'，D'，F在同一直线上，试说明两条折痕EF，GH相互平行.



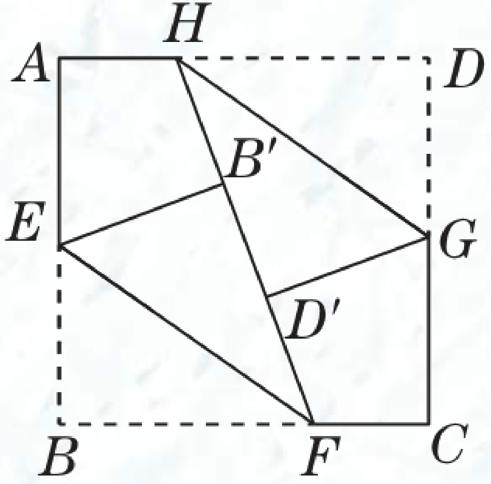
【解】∵ 四边形$ABCD$是矩形，

$\therefore AD//BC.\therefore \angle DHF=\angle BFH.$由折叠知

$\angle FHG=\angle DHG=\frac{1}{2}\angle DHF,$

$\angle HFE=\angle BFE=\frac{1}{2}\angle BFH,$

$\therefore \angle FHG=\angle HFE.\therefore EF//HG$，即两条折痕$EF$，$GH$相互平行.



6.[2024·泰安肥城市模拟] 如图，在矩形$ABCD$中，$AB = 3\ \mathrm{cm}, BC = 4\ \mathrm{cm}$,点$M$，$N$分别在边$AD$，$BC$上，沿着$MN$折叠矩形$ABCD$，使点$A$，$B$分别落在$E$，$F$处，且点$F$在线段$CD$上（可与点$C$，$D$重合），过点$M$作$MH \perp BC$于点$H$.