

Chapter 6 Frequency Response

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6.1 Introduction

- **The steady state response of a system to a sinusoidal input test signal.**

- **The frequency response** of a system is defined as the steady-state response of the system to a sinusoidal input signal.
- The sinusoid is a unique input signal, and the resulting output signal for a linear system is sinusoidal in the steady state; it differs from the input waveform only in **amplitude** and **phase angle**.

Input signal is $r(t) = R \sin \omega t$

or
$$R(s) = \frac{A\omega}{s^2 + \omega^2} = \frac{A\omega}{(s + j\omega)(s - j\omega)}$$

For a stable linear system, we have

$$\begin{aligned} T(s) &= \frac{Y(s)}{R(s)} = \frac{N(s)}{(s - p_1)(s - \dots)(s - p_n)} \\ \Rightarrow Y(s) &= \frac{N(s)}{(s - p_1)(s - \dots)(s - p_n)} \cdot \frac{A\omega}{(s + j\omega)(s - j\omega)} \\ &= \frac{A_1}{s + j\omega} + \frac{A_2}{s - j\omega} + \frac{B_1}{s - p_1} + \frac{-B_2}{s - p_2} + \dots + \frac{B_n}{s - p_n} \end{aligned}$$

All p_i have negative real parts, so the steady-state response of the system is

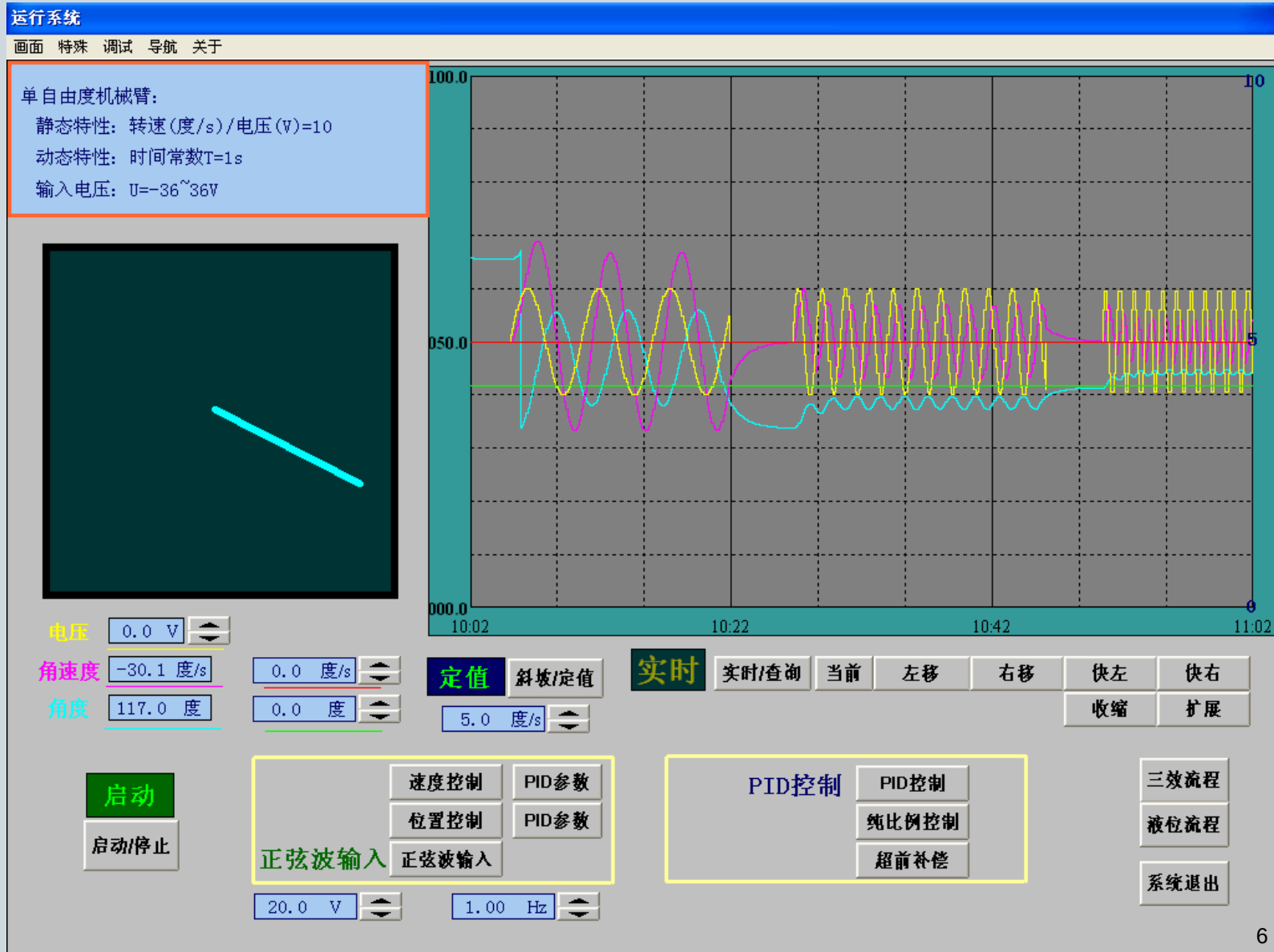
$$Y(s) = \frac{A_1}{s + j\omega} + \frac{A_2}{s - j\omega}$$

$$\Rightarrow y(t) = A|T(j\omega)|\sin(\omega t + \varphi)$$

$$\varphi = \angle T(j\omega)$$

The steady-state output signal depends only on the magnitude and phase of $T(j\omega)$ at a specific frequency ω .

DC motor



■ $G(j\omega)$ 形式:

$$G(j\omega) = |G(j\omega)|e^{j\theta} = |G(j\omega)|[\cos\theta + j\sin\theta] = U(\omega) + jV(\omega)$$

$U(\omega)$ 是 $G(j\omega)$ 的实部, 称为实频特性, $V(\omega)$ 是
 $G(j\omega)$ 的虚部, 称为虚频特性

思考:

实际物理系统对频率越高的输入响应幅值越小,
说明其传递函数分母阶次高于分子阶次, 为什么?

The advantage of frequency response method:

The experimental determination of the system's frequency response is easily accomplished, and the transfer function can be deduced from the experimental frequency response.

The transfer function describing the sinusoidal steady-state behavior of a system can be obtained by replacing s with $j\omega$.

The **disadvantage** of frequency response method:

- The direct correlation between the frequency response and the corresponding transient response characteristics are somewhat tenuous.

The relationship between Fourier and Laplace transforms

■ Laplace:

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$
$$f(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s) e^{st} ds$$

■ Fourier:

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{j\omega t} dt$$
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

When $f(t)$ is defined only for $t \geq 0$, as is often the case, the two equations differ only in the complex variables. Thus the Fourier transform can easily be obtained from Laplace transform just by setting $s = j\omega$.

- The Laplace transform permits us to investigate the s -plane location of the poles and zeros of a transfer $G(s)$.
- However, the frequency response method allows us to consider the transfer function $G(j\omega)$ and to concern ourselves with the amplitude and phase characteristics of the system.

6.2 Diagram of Frequency Response

Polar plot (Nyquist diagram)

Polar plot: The plot of frequency response $G(j\omega)$ in the complex plane as ω increases from $0(-\infty)$ to $+\infty$.

$$G(j\omega) = G(s) \Big|_{s=j\omega} = R(\omega) + jX$$

where : $R(\omega) = \text{Re}$

■ Steps:

1. start point: $\omega \rightarrow 0$ magnitude and phase (angle).

$$G(s) = \frac{K(\tau_1 s + 1) \cdots}{s^v (T_1 s + 1) \cdots} = \frac{b_m s^m + \cdots}{a_n s^n + \cdots}$$

$$\lim_{\omega \rightarrow 0} G(j\omega) = \lim_{\omega \rightarrow 0} K$$

2. end point: $\omega \rightarrow +$

$$\lim_{\omega \rightarrow +} G(j\omega) = \lim_{\omega \rightarrow +} \frac{b_m / a_n}{\omega^{n-m}} \angle (n-m) \times 90^\circ$$

$$= \begin{cases} \left| \frac{b_m}{a_n} \right| \angle 0^\circ & n = m \\ \left| 0 \right| \angle (n-m) \times 90^\circ & n > m \end{cases}$$

3. circumrotate direction:

4. intersection (交点) on negative real axis

$$\operatorname{Im} G(j\omega) = 0 \Rightarrow \omega \Rightarrow \operatorname{Re} G(j\omega)$$

Example 6.1: Polar plot of a transfer function

$$G(s) = \frac{1}{Ts + 1} \quad \Rightarrow \quad G(j\omega) = \frac{1}{j\omega T + 1}$$

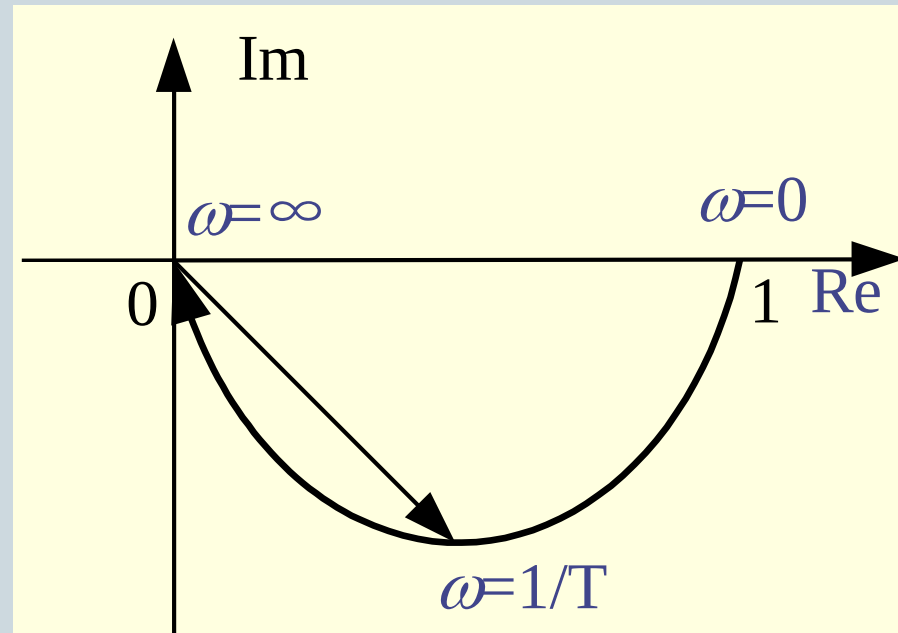
$$\angle G(j\omega) = -\arctan T\omega \quad |G(j\omega)| = \frac{1}{\sqrt{1 + T^2\omega^2}}$$

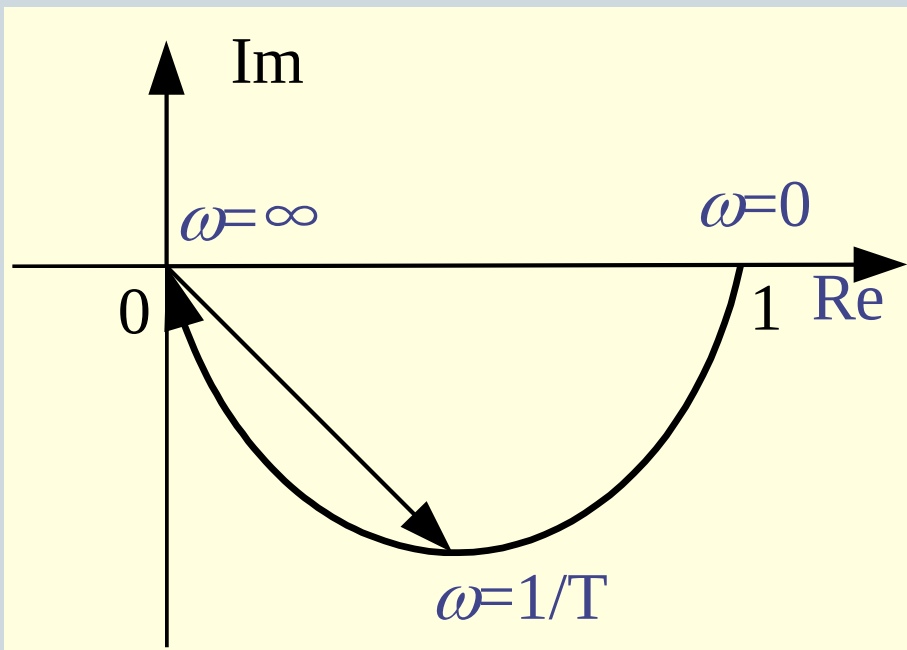
start: $\lim_{\omega \rightarrow 0} G(j\omega) = 1 \angle 0^\circ$

end: $\lim_{\omega \rightarrow +\infty} G(j\omega) = 0 \angle -90^\circ$

direction:

$\omega \uparrow \quad \angle G(j\omega) \downarrow (0^\circ \sim -90^\circ),$
clockwise



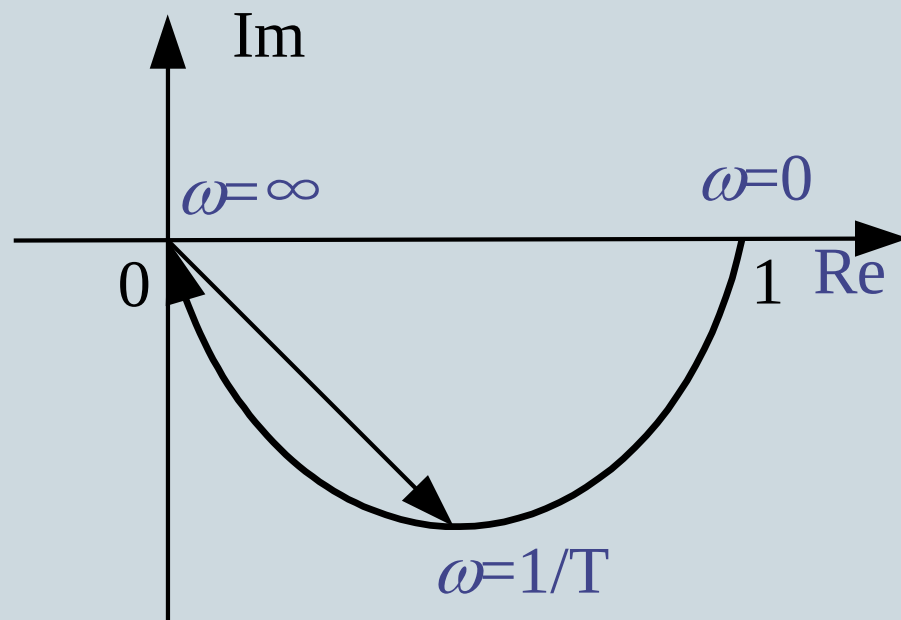


1、惯性环节

$$G(s) = \frac{1}{sT+1} \Rightarrow G(j\omega) = \frac{1}{jT\omega+1}$$

$$\angle G(j\omega) = -\arctan T\omega$$

$$|G(j\omega)| = \frac{1}{\sqrt{T^2\omega^2+1}}$$



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