# Functional-Dependency Theory

- We now consider the formal theory that tells us which functional dependencies are implied logically by a given set of functional dependencies.
- We then develop algorithms to generate lossless decompositions into BCNF and 3NF and higher normal form.

# Closure of a Set of Functional Dependencies

- **Given a set *F* set of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.**
  - **For e.g.: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$**

- **The set of all functional dependencies logically implied by *F* is the closure of *F*, we denote the *closure* of *F* by $F^+$.**

# Closure of a Set of Functional Dependencies

- **We can find $F^{+}$, the closure of F, by repeatedly applying**

  **Armstrong's Axioms:**

  - **if $\subseteq \alpha$, then $\alpha \rightarrow$ (reflexivity 自反律)**
  - **if $\alpha \rightarrow \beta$, then $\gamma\, \alpha \rightarrow \gamma\, \beta$ (augmentation 增广律)**
  - **if $\alpha \rightarrow$ , and $\rightarrow \gamma$, then $\alpha \rightarrow \gamma$ (transitivity 传递律)**

- **These rules are**

  - **sound (do not generate any incorrect functional dependencies)**
  - **complete (generate all functional dependencies that hold).**

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- some members of $F^+$
  - $A \rightarrow H$,
    by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$
    and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$,
    and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$,
    and then transitivity

# Procedure for Computing $F^+$

- **To compute the closure of a set of functional dependencies F:**

**$F^+ = F$**

**repeat**

**for each functional dependency $f$ in $F^+$**

**apply reflexivity and augmentation rules on $f$**

**add the resulting functional dependencies to $F^+$**

**for each pair of functional dependencies $f_1$ and $f_2$ in $F^+$**

**if $f_1$ and $f_2$ can be combined using transitivity**

**then add the resulting functional dependency to $F^+$**

**until $F^+$ does not change any further**

**NOTE: We shall see an alternative procedure for this task later**

# Closure of Functional Dependencies (Cont.)

- **Additional rules:**
  - If $\alpha \rightarrow$ holds *and* $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \gamma$ holds (**union**)
  - If $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow$ holds and $\alpha \rightarrow \gamma$ holds (**decomposition**)
  - If $\alpha \rightarrow$ holds and $\gamma \rightarrow \delta$ holds, then $\alpha \gamma \rightarrow \delta$ holds (**pseudotransitivity**)

The above rules can be inferred from Armstrong's axioms.

◆ **Example: 已知关系模式R中**

**U={A, B, C, D, E, G},**

**F={AB→C, C→A, BC→D, ACD→B,**

**D→EG, BE→C, CG→BD, CE→AG},**

**判断BD→AC是否为F逻辑蕴含**

◆ **解: 由D→EG知D→E, BD→BE …①**

**又知BE→C, C→A 所以BE→A, BE→AC …②**

**由①、②知, BD→AC, 所以BD→AC被F所蕴涵**