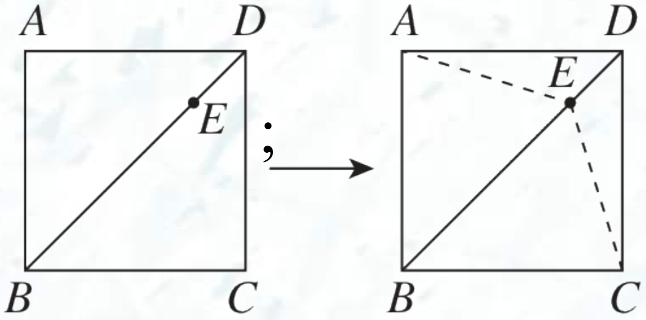
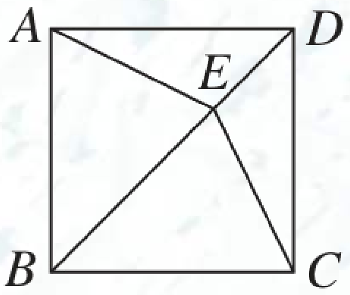
阶段拔尖专训10 正方形中的常见模型

类型1 正方形中的“对称”模型

【高分秘籍】

图示	条件	结论
	四边形ABCD是正方形，点E BD 连结AE,CE	

1.[2024随州期末] 已知正方形$ABCD$，E为对角线BD上一点.



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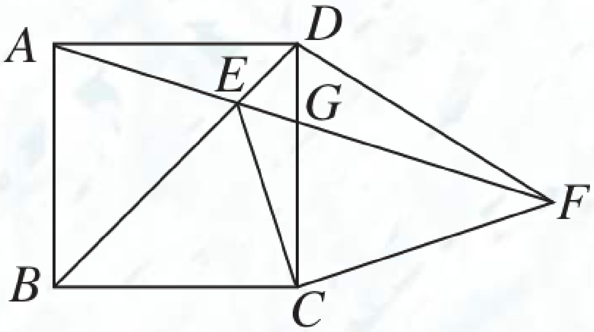

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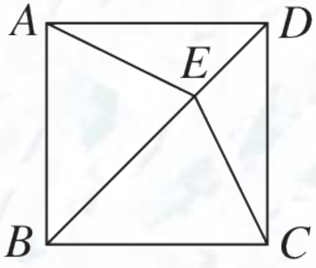
(1) 如图①，连结AE，CE，求证：$\triangle ADE \cong \triangle CDE$；



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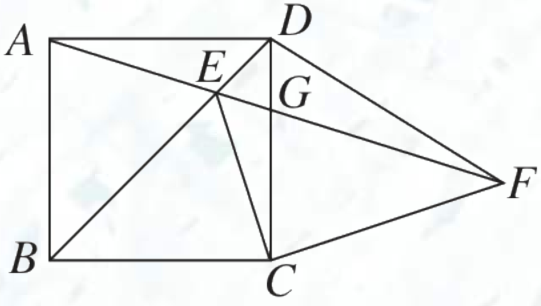

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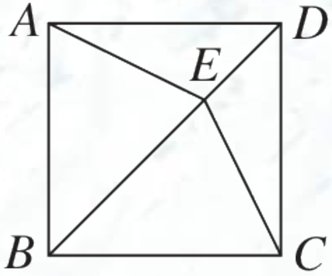
【证明】∵ 四边形$ABCD$为正方形，∴ $AD = CD$，

$\angle ADE = \angle CDE = 45^\circ$.∵ $DE = DE$，∴ $\triangle ADE \cong \triangle CDE$.

（2）如图②，F是AE延长线上的一点，$CF\perp CE$，EF交CD于点G，判断$\triangle CFG$的形状并说明理由；



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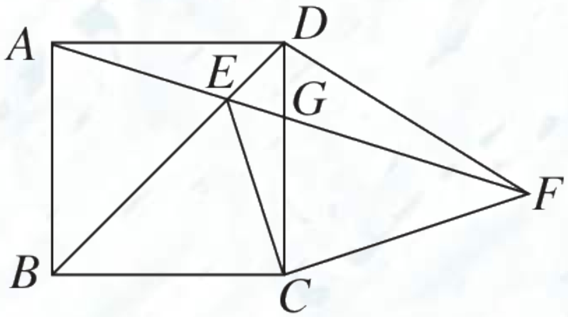

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【解】$\triangle CFG$为等腰三角形，理由如下：$\because CF\perp CE$，

$\therefore \angle FCG+\angle ECG=90°$.由$\triangle ADE\cong\triangle CDE$知

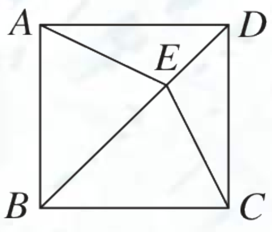
$\angle DAG=\angle ECD$. $\because$ 四边形$ABCD$是正方形，$\therefore \angle ADG=90°$.

$\therefore \angle DAG+\angle AGD=90°$，

$\therefore \angle FCG=\angle AGD$. $\because \angle AGD=\angle CGF$，$\therefore \angle FCG=\angle CGF$，

$\therefore FG=CF$，

$\therefore \triangle CFG$为等腰三角形.



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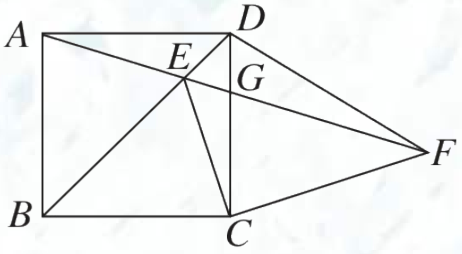

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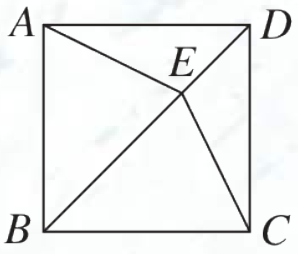
（3）在（2）的条件下，若$AB=3$，$CG=2DG$，连结DF，则DF的长为$\underline{\sqrt{13}}$.



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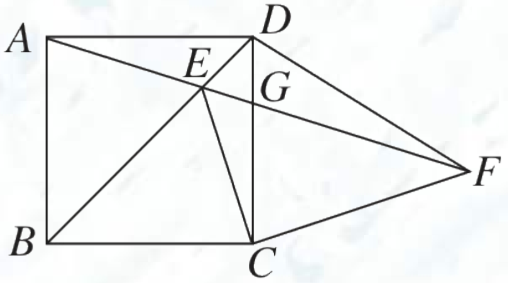

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【点拨】过F作$FH \perp CD$于点H.

$\therefore \angle FHG = \angle ADG = 90°$.$\because \triangle CFG$为等腰三角形，

$\therefore CH = HG$.

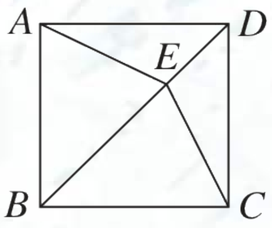
$\because CG = 2DG$，$\therefore HG = DG$.

又$\because \angle AGD = \angle HGF$，

$\therefore \triangle ADG \cong \triangle FHG$，$\therefore FH = AD$.

$\because AB = 3$，$\therefore FH = 3$，$HD = 2$，

$$\therefore DF = \sqrt{DH^2 + FH^2} = \sqrt{13}.$$



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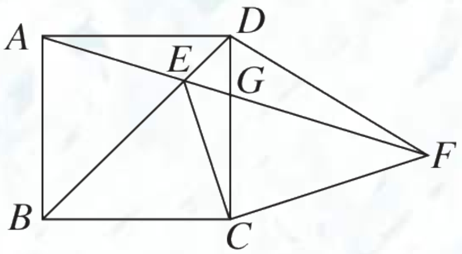

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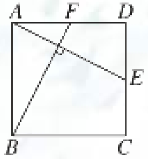
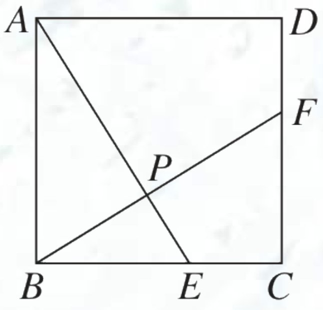
类型2 正方形中的“十字架”模型

a.过两个顶点型

【高分秘籍】

图示	条件	结论
	在正方形ABCD中，点E,F 边CD，AD上，AE⊥BF	

2. 母题·新视角·动点探究题·教材P121 习题T2 如图，在正方形$ABCD$中，E，F分别是BC，CD上的点，AE，BF相交于点P，并且$AE=BF$.



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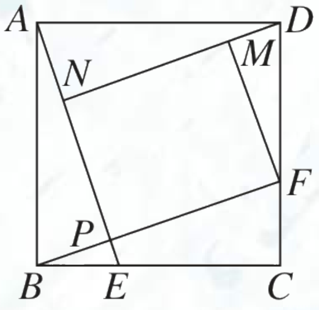

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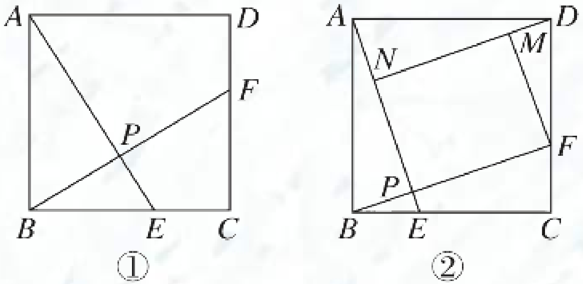
(1) 如图①，判断AE和BF的位置关系并说明理由；

【解】$AE\perp BF$.理由如下：∵ 四边形$ABCD$是正方形，

∴ $AB=BC$，$\angle ABC=\angle C=90°$.



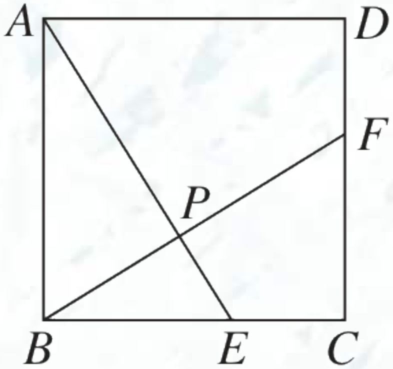
在Rt△ABE和Rt△BCF中，$\begin{cases}AE=BF,\\AB=BC,\end{cases}$

∴ Rt△ABE≌Rt△BCF，∴ $\angle BAE=\angle CBF$.

∵ $\angle BAE+\angle BEA=90°$ ，∴ $\angle CBF+\angle BEA=90°$ ，

∴ $\angle BPE=90°$ ，∴ $AE\perp BF$.

(2) 若$AB=8$，$BE=6$，求BP的长度；



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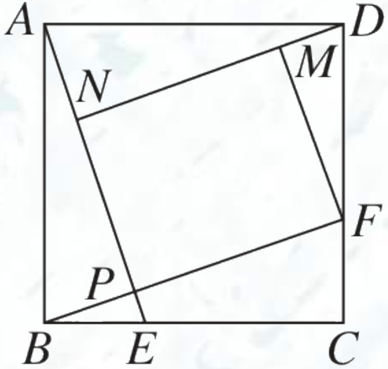

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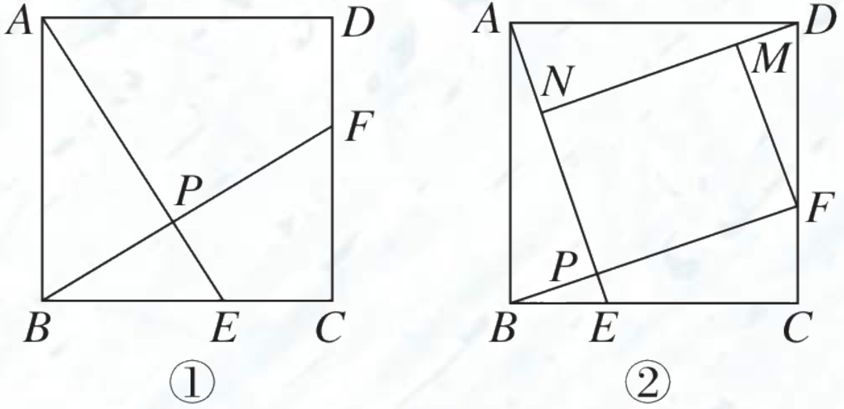
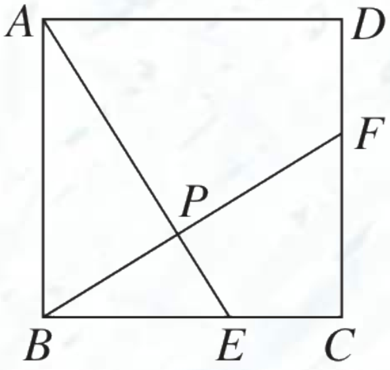
在Rt△ABE中，$AB=8$，$BE=6$，根据勾股定理得

$AE=\sqrt{AB^2+BE^2}=10.\because S_{\triangle ABE}=\frac{1}{2}AB\cdot BE=\frac{1}{2}AE\cdot BP$，

$\therefore 8\times 6=10BP$，$\therefore BP=4.8$.



（3）如图②，$DN \perp AE$，$FM \perp DN$，当点F在线段CD上运动时（点F不与点C，D重合），四边形$FMNP$能否成为正方形？请说明理由.



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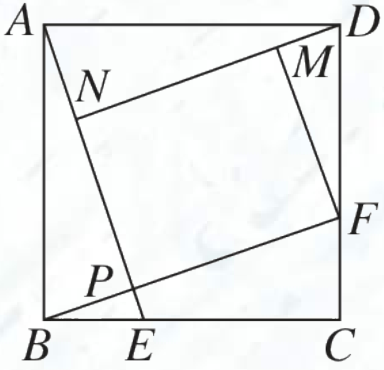

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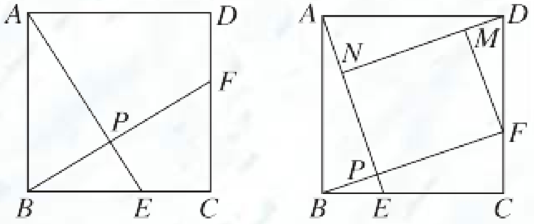
四边形$FMNP$不能成为正方形.

理由如下：由（1）知$AE \perp BF$，$\therefore \angle APF = 90°$．

$\because FM \perp DN$，$DN \perp AE$，$\therefore \angle FMN = \angle MNP = 90°$，$\therefore$ 四边形$FMNP$是矩形.

$\because \angle BAP + \angle NAD = \angle NAD + \angle ADN = 90°$，

$\therefore \angle BAP = \angle ADN$.在$\triangle BAP$和$\triangle ADN$中，



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$$\left\{\begin{array}{c}\angle BAP=\angle ADN,\\ AB=DA,\\ \angle APB=\angle DNA=90^{\circ},\end{array}\right.$$ $\therefore \triangle BAP\cong\triangle ADN$,

$\therefore AN=BP$. $\because AE=BF$，$\therefore AE-AN=BF-BP$，

$\therefore EN=PF$. $\because$ 点F在线段CD上运动（点F不与点C，D重合），$\therefore P$，E不重合，$\therefore PN\neq PF$，$\therefore$ 四边形$FMNP$不能成为正方形.