

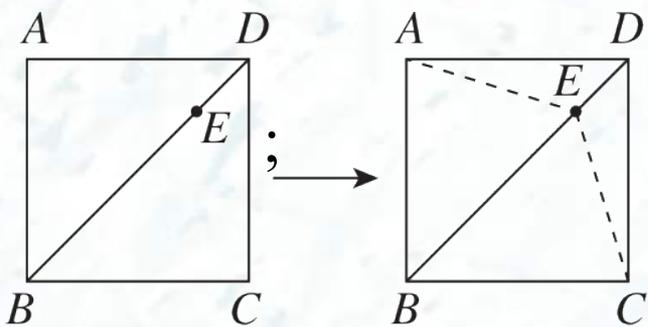


阶段拔尖专训10 正方形中的常见模型

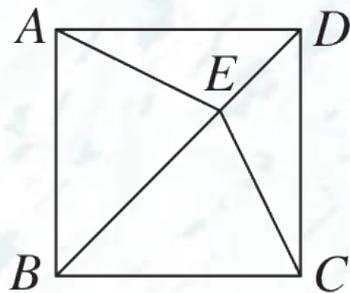


类型1 正方形中的“对称”模型

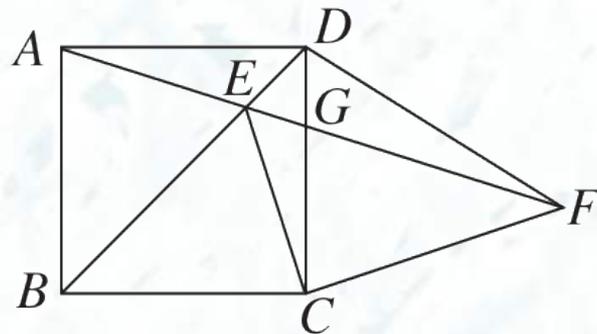
【高分秘籍】

图示	条件	结论
 <p>The diagram illustrates a square $ABCD$ with vertices A (top-left), B (bottom-left), C (bottom-right), and D (top-right). A diagonal BD is drawn, and a point E is located on it. An arrow points from the first diagram to a second diagram where dashed lines connect A to E and C to E.</p>	<p>四边形$ABCD$ 是正方形， 点E 在 BD 上 连结AE, CE</p>	

1. [2024随州期末] 已知正方形 $ABCD$, E 为对角线 BD 上一点.

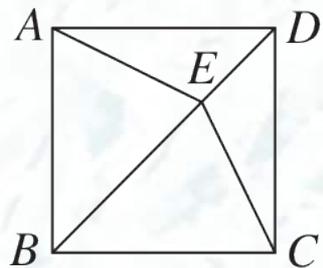


①

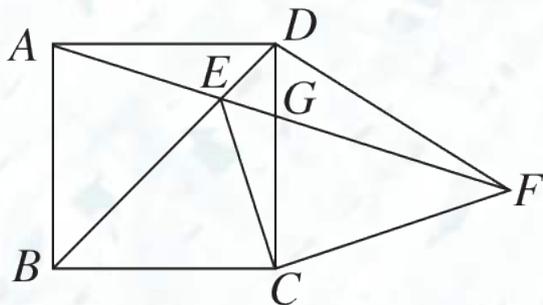


②

(1) 如图①, 连结 AE , CE , 求证: $\triangle ADE \cong \triangle CDE$;



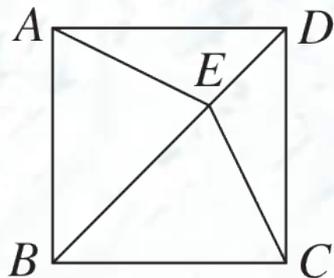
①



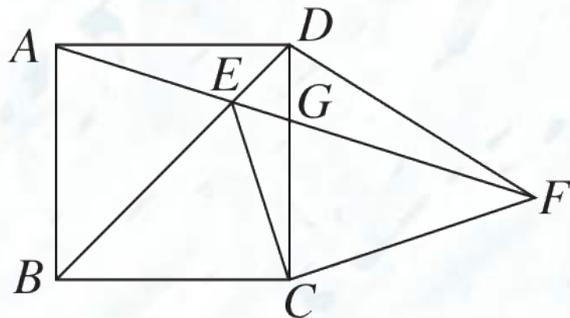
②

【证明】 \because 四边形 $ABCD$ 为正方形, $\therefore AD = CD$,
 $\angle ADE = \angle CDE = 45^\circ$ $\therefore DE = DE$, $\therefore \triangle ADE \cong \triangle CDE$.

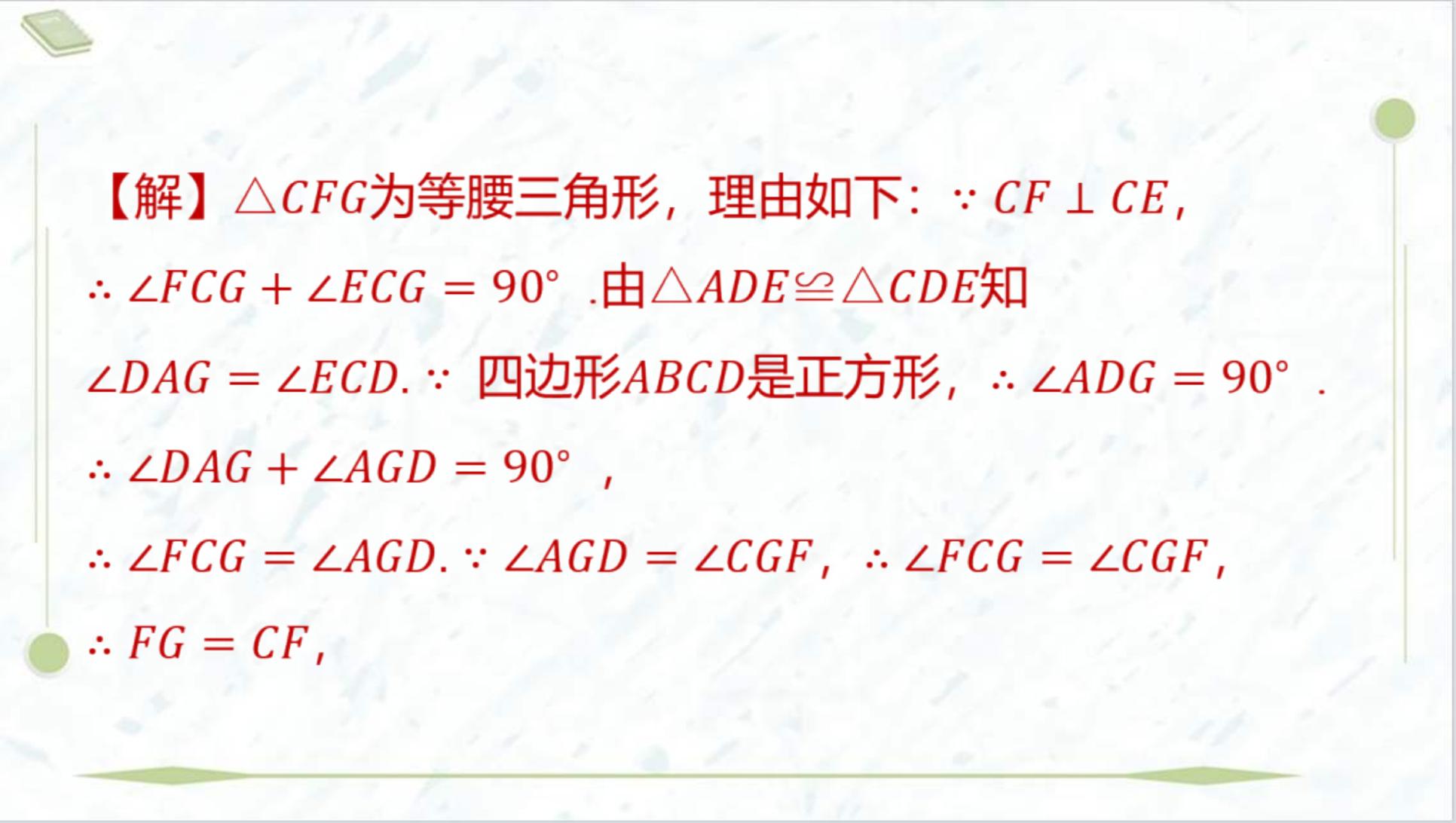
(2) 如图②, F 是 AE 延长线上的一点, $CF \perp CE$, EF 交 CD 于点 G , 判断 $\triangle CFG$ 的形状并说明理由;



①

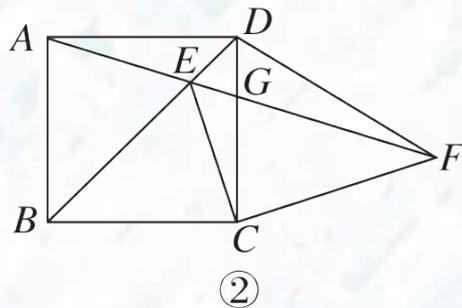
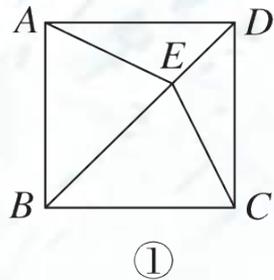


②

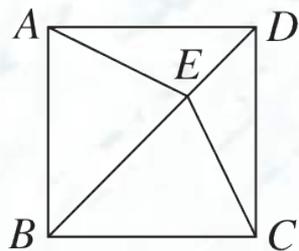


【解】 $\triangle CFG$ 为等腰三角形, 理由如下: $\because CF \perp CE$,
 $\therefore \angle FCG + \angle ECG = 90^\circ$. 由 $\triangle ADE \cong \triangle CDE$ 知
 $\angle DAG = \angle ECD$. \because 四边形 $ABCD$ 是正方形, $\therefore \angle ADG = 90^\circ$.
 $\therefore \angle DAG + \angle AGD = 90^\circ$,
 $\therefore \angle FCG = \angle AGD$. $\because \angle AGD = \angle CGF$, $\therefore \angle FCG = \angle CGF$,
 $\therefore FG = CF$,

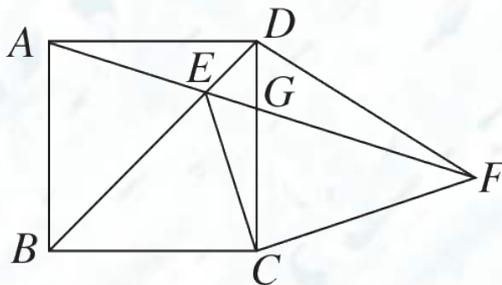
$\therefore \triangle CFG$ 为等腰三角形.



(3) 在 (2) 的条件下, 若 $AB = 3$, $CG = 2DG$, 连结 DF , 则 DF 的长为 $\sqrt{13}$.



①



②



【点拨】 过 F 作 $FH \perp CD$ 于点 H .

$\therefore \angle FHG = \angle ADG = 90^\circ \therefore \triangle CFG$ 为等腰三角形,

$\therefore CH = HG$.

$\therefore CG = 2DG, \therefore HG = DG$.

又 $\therefore \angle AGD = \angle HGF$,

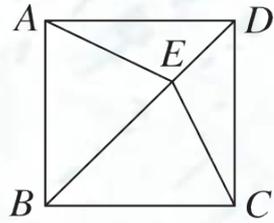
$\therefore \triangle ADG \cong \triangle FHG, \therefore FH = AD$.

$\therefore AB = 3, \therefore FH = 3, HD = 2,$

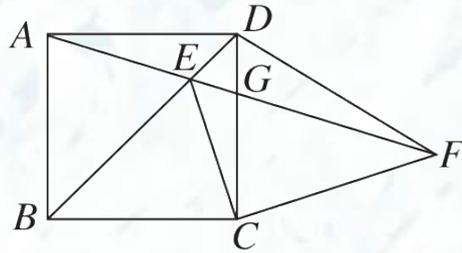




$\therefore DF = \sqrt{DH^2 + FH^2} = \sqrt{13}.$



①

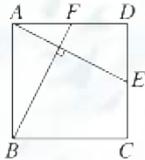


②

类型2 正方形中的“十字架”模型

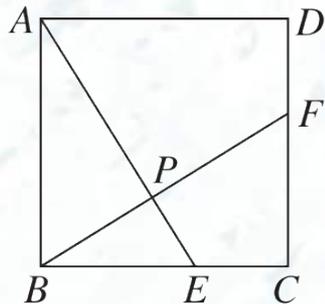
a.过两个顶点型

【高分秘籍】

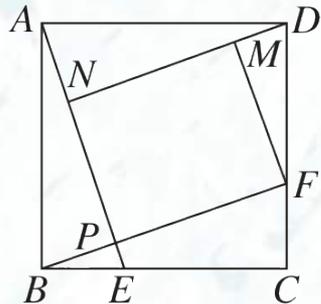
图示	条件	结论
	在正方形ABCD中，点E,F 边CD，AD上， $AE \perp BF$	

新视角·动点探究题

2. **母题**·教材P121习题T2 如图, 在正方形 $ABCD$ 中, E, F 分别是 BC, CD 上的点, AE, BF 相交于点 P , 并且 $AE = BF$.



①



②

(1) 如图①, 判断 AE 和 BF 的位置关系并说明理由;

【解】 $AE \perp BF$.理由如下: \because 四边形 $ABCD$ 是正方形,

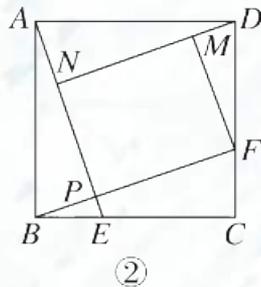
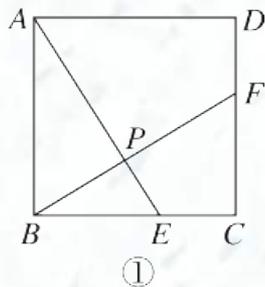
$\therefore AB = BC, \angle ABC = \angle C = 90^\circ$.

在 $\text{Rt}\triangle ABE$ 和 $\text{Rt}\triangle BCF$ 中, $\begin{cases} AE = BF, \\ AB = BC, \end{cases}$

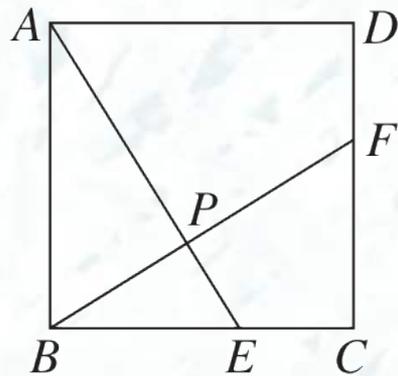
$\therefore \text{Rt}\triangle ABE \cong \text{Rt}\triangle BCF, \therefore \angle BAE = \angle CBF$.

$\because \angle BAE + \angle BEA = 90^\circ, \therefore \angle CBF + \angle BEA = 90^\circ,$

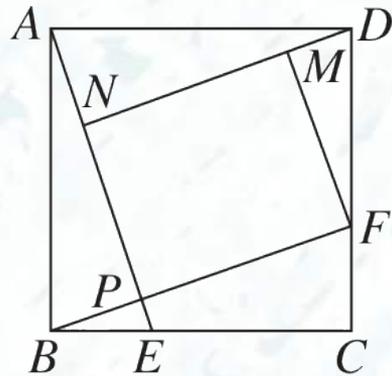
$\therefore \angle BPE = 90^\circ, \therefore AE \perp BF$.



(2) 若 $AB = 8$, $BE = 6$, 求 BP 的长度;



①

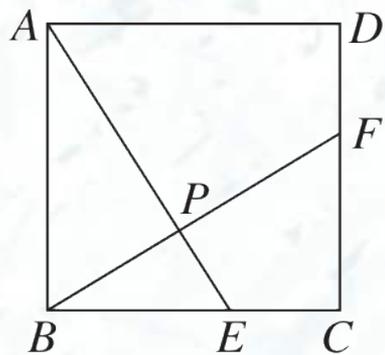


②

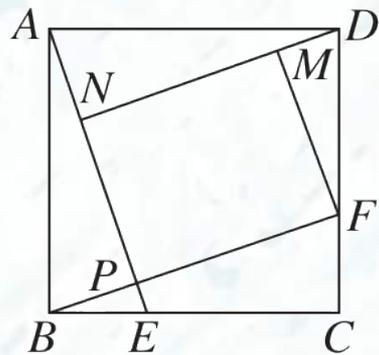
在Rt $\triangle ABE$ 中, $AB = 8$, $BE = 6$, 根据勾股定理得

$$AE = \sqrt{AB^2 + BE^2} = 10. \therefore S_{\triangle ABE} = \frac{1}{2} AB \cdot BE = \frac{1}{2} AE \cdot BP,$$

$$\therefore 8 \times 6 = 10BP, \therefore BP = 4.8.$$

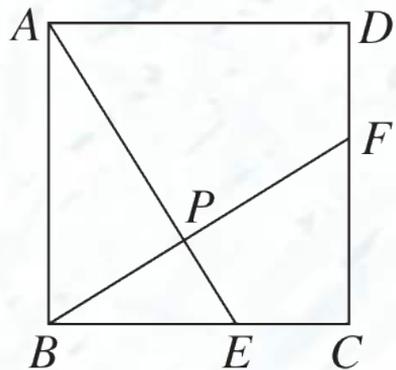


①

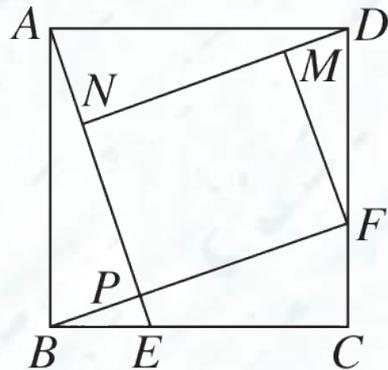


②

(3) 如图②, $DN \perp AE$, $FM \perp DN$, 当点 F 在线段 CD 上运动时 (点 F 不与点 C, D 重合), 四边形 $FMNP$ 能否成为正方形? 请说明理由.



①



②

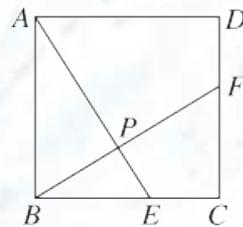
四边形 $FMNP$ 不能成为正方形.

理由如下: 由 (1) 知 $AE \perp BF$, $\therefore \angle APF = 90^\circ$.

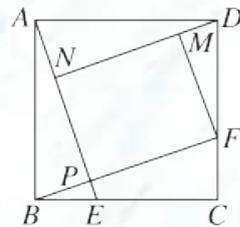
$\therefore FM \perp DN$, $DN \perp AE$, $\therefore \angle FMN = \angle MNP = 90^\circ$, \therefore 四边形 $FMNP$ 是矩形.

$\therefore \angle BAP + \angle NAD = \angle NAD + \angle ADN = 90^\circ$,

$\therefore \angle BAP = \angle ADN$. 在 $\triangle BAP$ 和 $\triangle ADN$ 中,



①



②



$$\begin{cases} \angle BAP = \angle ADN, \\ AB = DA, \\ \angle APB = \angle DNA = 90^\circ, \end{cases} \therefore \triangle BAP \cong \triangle ADN,$$

$\therefore AN = BP. \therefore AE = BF, \therefore AE - AN = BF - BP,$

$\therefore EN = PF. \therefore$ 点 F 在线段 CD 上运动 (点 F 不与点 C, D 重合), $\therefore P, E$ 不重合, $\therefore PN \neq PF, \therefore$ 四边形 $FMNP$ 不能成为正方形.



以上内容仅为本文档的试下载部分，为可阅读页数的一半内容。如要下载或阅读全文，请访问：<https://d.book118.com/557053124044010010>