Linear-Quadratic Control of SPDEs with State Dependent Gaussian Noise

Tyrone E. Duncan

University of Kansas

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Outline

1. Some background results for LQ control of SPDEs with additive fractional Brownian motions
2. Linear equations with a Volterra type Gaussian noise
3. Some examples of these Gaussian noise processes
4. Solutions of linear equations with state dependent noise
5. Controlled linear equations with state dependent noise
6. Finite time LQ control with linear feedback controls
7. Riccati differential equations
8. Optimal feedback controls
9. Infinite time horizon optimal feedback controls

Joint work with B. Maslowski and B. Pasik-Duncan

A Motivating Example from LQ Control with Fractional Brownian Motion

$$
\begin{aligned}
dX(t) &= (AX(t) + Bu(t))dt + CdB_H(t) \\
X(0) &= x
\end{aligned}
$$

$(B_H(t), t \geq 0)$ is a fractional Brownian motion in V with $H \in (\frac{1}{2}, 1)$, $B \in \mathrm{L}(U, \mathrm{H})$, $C \in \mathrm{L}(V, \mathrm{H})$, A is the generator of a strongly continuous semigroup

Admissible controls denoted U are all progressively measurable processes in $L^2([0, T] \times \Omega, U)$

$$
\begin{aligned}
J(x, u) = {} & \frac{1}{2}\mathrm{E}\int_0^T (< QX(s), X(s) > + < Ru(s), u(s) >)ds \\
& + \frac{1}{2}\mathrm{E} < GX(T), X(T) >
\end{aligned}
$$

An optimal control $\tilde{u}$ without a measurability condition is

$$\tilde{u}(t) = -R^{-1}B^{*}P(t)X(t) + \varphi(t)$$
$$\varphi(t) = \int_t^T U_P(s,t)P(s)C\,dB_H(s)$$

and an optimal control $\hat{u}$ with the measurability condition is

$$\hat{u}(t) = -R^{-1}B^{*}P(t)X(t) + \psi(t)$$
$$\psi(t) = \mathbb{E}[\varphi(t)|\mathcal{F}(t)]$$
$$= \int_0^t s^{-(H-\frac{1}{2})}\left(I_{t-}\left(I_{T-}^{(H-\frac{1}{2})}u_{H-\frac{1}{2}}U_P(\cdot,t)P(\cdot)C\right)\right)(s)\,dB_H(s)$$

$$\begin{aligned}(I_{b-}^{\alpha}f)(x) &= \frac{1}{\Gamma(\alpha)}\int_x^b \frac{f(t)}{(t-x)^{1-\alpha}}\,dt \\ u_a(s) &= s^{a}\end{aligned}$$

Stochastic Evolution Equations

$$
\begin{aligned}
dX(t) &= (A(t)X(t) + B(t)K(t)X(t))dt + \sigma(t)X(t)dB(t) \\
X(0) &= x_0
\end{aligned}
$$

where $X(t) \in V$ a real, separable Hilbert space, $(B(t), t \geq 0)$ is a real-valued Volterra-type Gaussian process, $(A(t), t \geq 0)$ is a family of closed, unbounded operators on V such that

$Dom(A(t)) = Dom(A(0))$ and $Dom(A^*(t)) = Dom(A^*(0))$ for each $t \in \mathbb{R}_+$ and the family generates a strongly continuous evolution operator, $B \in C_s(\mathbb{R}_+, \mathcal{L}(U, V))$ and $K \in C_s(\mathbb{R}_+, \mathcal{L}(V, U))$, σ is a real-valued continuous function. The control is

$$u(t) = K(t)X(t)$$

that can be described as a Markov type control.

Noise Process

The noise B is generated from a Wiener process W as follows

(R2) $$B(t) = \int_0^t K(t, r)\,dW(r) \qquad t \in \mathbb{R}_+$$

There is a continuous version of the process B.

Assume that $K(\cdot, s)$ has bounded variation on (s, T) and

(R3) $$\int_0^T |K|^2((s, T], s)\,ds < \infty$$