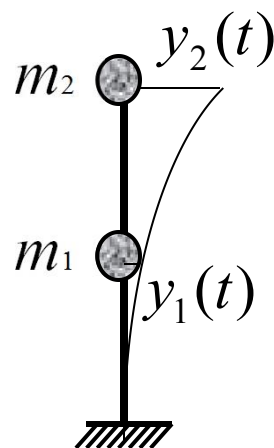


§3.1 自由振动分析

自由振动分析的目的: 是确定体系的动力特性. 可不计阻尼。

一. 运动方程及其解



$$\text{运动方程} \begin{cases} k_{11}y_1 + k_{12}y_2 = -m_1\ddot{y}_1 \\ k_{21}y_1 + k_{22}y_2 = -m_2\ddot{y}_2 \end{cases} \quad \begin{cases} (k_{11} - m_1\omega^2)X_1 + k_{12}X_2 = 0 \\ k_{21}X_1 + (k_{22} - m_2\omega^2)X_2 = 0 \end{cases}$$

$$\text{或} \quad [m]\{\ddot{y}\} + [k]\{y\} = \{0\}$$

$$\text{设方程的特解为} \quad \begin{cases} y_1 = X_1 \sin(\omega t + \alpha) \\ y_2 = X_2 \sin(\omega t + \alpha) \end{cases}$$

$$\left(\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \omega^2 \right) \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$([k] - \omega^2 [m])\{X\} = \{0\}$$

代入方程, 得

$$\begin{cases} k_{11}X_1 + k_{12}X_2 - m_1\omega^2 X_1 = 0 \\ k_{21}X_1 + k_{22}X_2 - m_2\omega^2 X_2 = 0 \end{cases}$$

$$| [k] - \omega^2 [m] | = 0$$

---频率方程

解频率方程得 ω^2 的两个根
 值小者记作 ω_1 称作**第一频率**
 也称作**基本频率**;
 值大者记作 ω_2
 称为**第二频率或高阶频率**.

将 ω_1 频率代入振型方程

$$(k_{11} - m_1 \omega_1^2) X_{11} + k_{12} X_{21} = 0$$

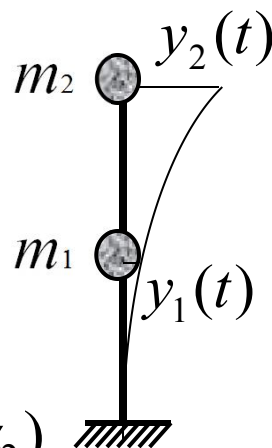
特解1

$$\begin{cases} y_{11} = X_{11} \sin(\omega_1 t + \alpha_1) \\ y_{21} = X_{21} \sin(\omega_1 t + \alpha_1) \end{cases}$$

特解2

$$\begin{cases} y_{12} = X_{12} \sin(\omega_2 t + \alpha_2) \\ y_{22} = X_{22} \sin(\omega_2 t + \alpha_2) \end{cases}$$

$$\frac{X_{11}}{X_{21}} = \frac{k_{12}}{m_1 \omega_1^2 - k_{11}}$$



运动方程

$$\begin{cases} k_{11} y_1 + k_{12} y_2 = -m_1 \ddot{y}_1 \\ k_{21} y_1 + k_{22} y_2 = -m_2 \ddot{y}_2 \end{cases}$$

或

$$[m] \{\ddot{y}\} + [k] \{y\} = \{0\}$$

设方程的特解为

$$\begin{cases} y_1 = X_1 \sin(\omega t + \alpha) \\ y_2 = X_2 \sin(\omega t + \alpha) \end{cases}$$

代入方程, 得

$$\begin{cases} k_{11} X_1 + k_{12} X_2 - m_1 \omega^2 X_1 = 0 \\ k_{21} X_1 + k_{22} X_2 - m_2 \omega^2 X_2 = 0 \end{cases}$$

$$\begin{cases} (k_{11} - m_1 \omega^2) X_1 + k_{12} X_2 = 0 \\ k_{21} X_1 + (k_{22} - m_2 \omega^2) X_2 = 0 \end{cases}$$

$$\left(\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \omega^2 \right) \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$([k] - \omega^2 [m]) \{X\} = \{0\}$$

---振型方程

$$| [k] - \omega^2 [m] | = 0$$

---频率方程

解频率方程得 ω^2 的两个根
值小者记作 ω_1 称作第一频率
也称作基本频率;值大者记作
 ω_2 称为第二频率或高阶频率.

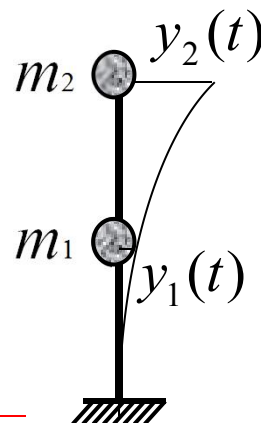
将 ω_1 频率代入振型方程

$$(k_{11} - m_1 \omega_1^2) X_{11} + k_{12} X_{21} = 0$$

$$\frac{X_{11}}{X_{21}} = \frac{k_{12}}{m_1 \omega_1^2 - k_{11}}$$

特解1
$$\begin{cases} y_{11} = X_{11} \sin(\omega_1 t + \alpha_1) \\ y_{21} = X_{21} \sin(\omega_1 t + \alpha_1) \end{cases}$$

特解2
$$\begin{cases} y_{12} = X_{12} \sin(\omega_2 t + \alpha_2) \\ y_{22} = X_{22} \sin(\omega_2 t + \alpha_2) \end{cases}$$



$$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}_1 = \begin{Bmatrix} X_{11} \\ X_{21} \end{Bmatrix} \sin(\omega_1 t + \alpha_1)$$

$$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}_2 = \begin{Bmatrix} X_{12} \\ X_{22} \end{Bmatrix} \sin(\omega_2 t + \alpha_2)$$

通解

$$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} X_{11} \\ X_{21} \end{Bmatrix} \sin(\omega_1 t + \alpha_1) + \begin{Bmatrix} X_{12} \\ X_{22} \end{Bmatrix} \sin(\omega_2 t + \alpha_2)$$

二. 频率与振型

$$\begin{cases} y_{11} = X_{11} \sin(\omega_1 t + \alpha_1) \\ y_{21} = X_{21} \sin(\omega_1 t + \alpha_1) \end{cases}$$

$$\frac{X_{11}}{X_{21}} = \frac{k_{12}}{m_1 \omega_1^2 - k_{11}}$$

按特解振动时有如下特点:

- 1) 各质点同频同步;
- 2) 任意时刻, 各质点位移的比值保持不变

$$\frac{y_1(t)}{y_2(t)} = \frac{X_{11} \sin(\omega_1 t + \alpha_1)}{X_{21} \sin(\omega_1 t + \alpha_1)} = \frac{X_{11}}{X_{21}}$$

几点说明:

- 1) 按振型作自由振动时, 各质点的速度的比值也为常数, 且与位移比值相同。

$$\frac{\dot{y}_1(t)}{\dot{y}_2(t)} = \frac{X_{11} \omega_1 \cos(\omega_1 t + \alpha_1)}{X_{21} \omega_1 \cos(\omega_1 t + \alpha_1)} = \frac{X_{11}}{X_{21}}$$

- 2) 发生按振型的自由振动是有条件的。

$$\frac{y_1(0)}{y_2(0)} = \frac{X_{11}}{X_{21}}, \quad \frac{\dot{y}_1(0)}{\dot{y}_2(0)} = \frac{X_{11}}{X_{21}}$$

- 3) 振型与频率是体系本身固有的属性, 与外界因素无关。

定义: 体系上所有质量按相同频率作自由振动时的振动形状称作体系的主振型。

4) N个自由度体系 有N个频率和N个振型

频率方程 $|[k] - \omega^2[m]| = 0$

解频率方程得 ω 的N,从小到大排列 $\omega_1, \omega_2 \cdots \omega_N$

依次称作第一频率,第二频率...

第一频率称作基本频率,
其它为高阶频率。

将频率代入振型方程

$$([k] - \omega^2[m])\{X\} = \{0\}$$

得N个振型 $\{X\}_i \quad (i = 1, 2, \cdots N)$

N个振型是线性无关的。

几点说明:

1) 按振型作自由振动时, 各质点的速度的比值也为常数, 且与位移比值相同。

$$\frac{\dot{y}_1(t)}{\dot{y}_2(t)} = \frac{X_{11}\omega_1 \cos(\omega_1 t + \alpha_1)}{X_{21}\omega_1 \cos(\omega_1 t + \alpha_1)} = \frac{X_{11}}{X_{21}}$$

2) 发生按振型的自由振动是有条件的。

$$\frac{y_1(0)}{y_2(0)} = \frac{X_{11}}{X_{21}}, \quad \frac{\dot{y}_1(0)}{\dot{y}_2(0)} = \frac{X_{11}}{X_{21}}$$

3) 振型与频率是体系本身固有的属性,与外界因素无关。

4) N自由度体系有
N个频率和N个振型

频率方程 $| [k] - \omega^2 [m] | = 0$

解频率方程得 ω 的N,从小到大排列 $\omega_1, \omega_2 \cdots \omega_N$

依次称作第一频率,第二频率...

第一频率称作基本频率,
其它为高阶频率.

将频率代入振型方程

$$([k] - \omega^2 [m]) \{X\} = \{0\}$$

得N个振型 $\{X\}_i \quad (i = 1, 2, \cdots N)$

N个振型是线性无关的.

5) 若已知柔度矩阵时 振型方程

$$([I] - \omega^2 [\delta] [m]) \{X\} = \{0\}$$

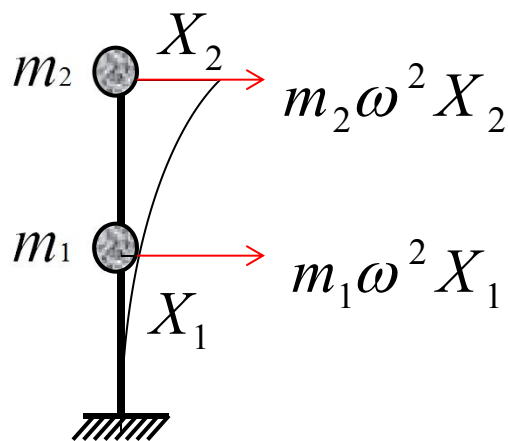
频率方程

$$| [I] - \omega^2 [\delta] [m] | = 0$$

6) 求振型、频率可列幅值方程.

按振型振动时

$$\begin{cases} y_1 = X_1 \sin(\omega t + \alpha) \\ y_2 = X_2 \sin(\omega t + \alpha) \\ \ddot{y}_1 = -X_1 \omega^2 \sin(\omega t + \alpha) \\ \ddot{y}_2 = -X_2 \omega^2 \sin(\omega t + \alpha) \\ I_1 = m_1 X_1 \omega^2 \sin(\omega t + \alpha) \\ I_2 = m_2 X_2 \omega^2 \sin(\omega t + \alpha) \end{cases}$$



$$\begin{cases} X_1 = \delta_{11} m_1 \omega^2 X_1 + \delta_{12} m_2 \omega^2 X_2 \\ X_2 = \delta_{21} m_1 \omega^2 X_1 + \delta_{22} m_2 \omega^2 X_2 \end{cases}$$

$$([I] - \omega^2 [\delta][m])\{X\} = \{0\}$$

$$| [I] - \omega^2 [\delta][m] | = 0$$

振型可看作是体系按振型振动时，**惯性力幅值**作为静荷载所引起的**静位移**

5. 若已知柔度矩阵时

振型方程

$$([I] - \omega^2 [\delta][m])\{X\} = \{0\}$$

频率方程

$$| [I] - \omega^2 [\delta][m] | = 0$$

6. 求振型、频率可列幅值方程.

按振型振动时

$$\begin{cases} y_1 = X_1 \sin(\omega t + \alpha) \\ y_2 = X_2 \sin(\omega t + \alpha) \end{cases}$$

$$\begin{cases} \ddot{y}_1 = -X_1 \omega^2 \sin(\omega t + \alpha) \\ \ddot{y}_2 = -X_2 \omega^2 \sin(\omega t + \alpha) \end{cases}$$

$$\begin{cases} I_1 = m_1 X_1 \omega^2 \sin(\omega t + \alpha) \\ I_2 = m_2 X_2 \omega^2 \sin(\omega t + \alpha) \end{cases}$$

三. 求频率、振型例题

例2-1 求体系的频率、振型

解: $\delta_{11} = \delta_{22} = \frac{4}{243} \frac{l^3}{EI}$

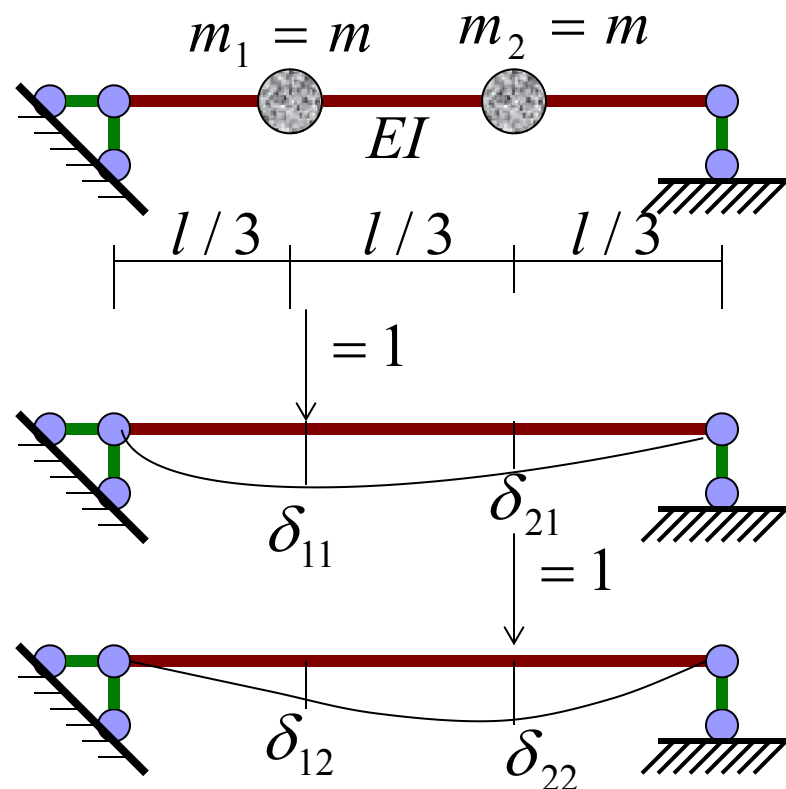
$$\delta_{12} = \delta_{21} = \frac{7}{486} \frac{l^3}{EI}$$

$$\begin{vmatrix} [I] - \omega^2 [\delta][m] \\ \delta_{11}m_1 - 1/\omega^2 & m_2\delta_{12} \\ m_1\delta_{21} & \delta_{22}m_2 - 1/\omega^2 \end{vmatrix} = 0$$

令 $\lambda = \frac{1}{\delta_{11}m_1\omega^2}$

$$\begin{vmatrix} 1 - \lambda & \delta_{12}/\delta_{11} \\ \delta_{21}/\delta_{11} & 1 - \lambda \end{vmatrix} = 0$$

$$(1 - \lambda)^2 - (7/8)^2 = 0$$



$$\lambda_1 = 15/8 \quad \lambda_2 = 1/8$$

$$\omega_1 = 5.692 \sqrt{\frac{EI}{ml^3}}$$

$$\omega_2 = 22.045 \sqrt{\frac{EI}{ml^3}}$$

$$\begin{cases} X_1 = \delta_{11}m_1\omega^2 X_1 + \delta_{12}m_2\omega^2 X_2 \\ X_2 = \delta_{21}m_1\omega^2 X_1 + \delta_{22}m_2\omega^2 X_2 \end{cases}$$

$$X_1 = \delta_{11}m_1\omega^2 X_1 + \delta_{12}m_2\omega^2 X_2$$

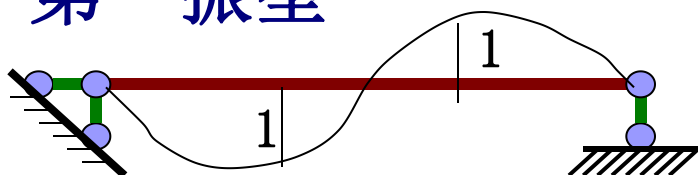
$$\frac{X_1}{X_2} = \frac{\delta_{12}m_2\omega^2}{1 - \delta_{11}m_1\omega^2}$$

$$\frac{X_{11}}{X_{21}} = \frac{\delta_{12}m_2\omega_1^2}{1 - \delta_{11}m_1\omega_1^2} = 1$$

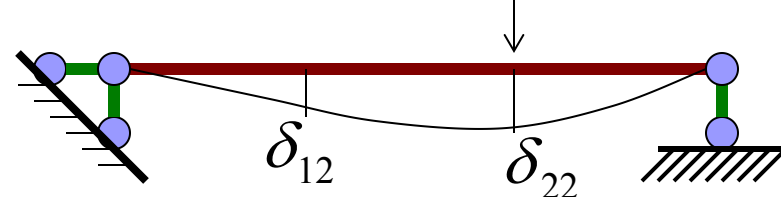
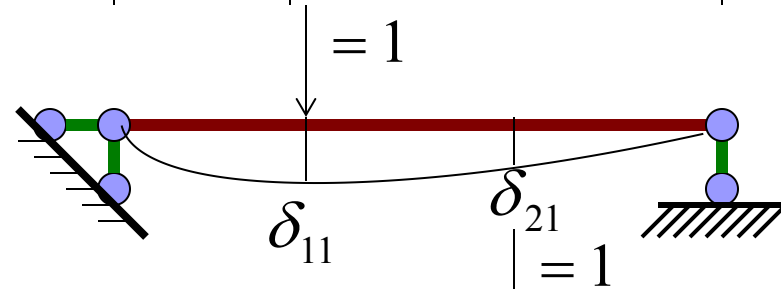
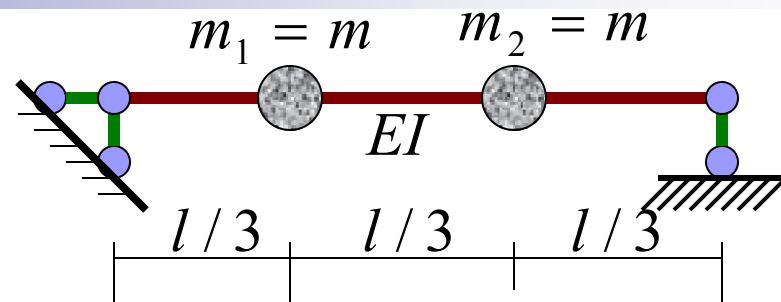
$$\frac{X_{12}}{X_{22}} = \frac{\delta_{12}m_2\omega_2^2}{1 - \delta_{11}m_1\omega_2^2} = -1$$



第一振型



第二振型



$$\lambda_1 = 15/8 \quad \lambda_2 = 1/8$$

$$\omega_1 = 5.692 \sqrt{\frac{EI}{ml^3}}$$

$$\omega_2 = 22.045 \sqrt{\frac{EI}{ml^3}}$$

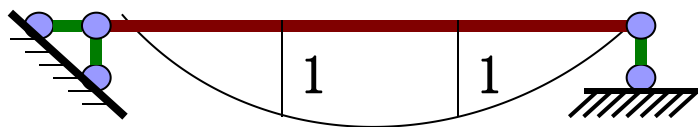
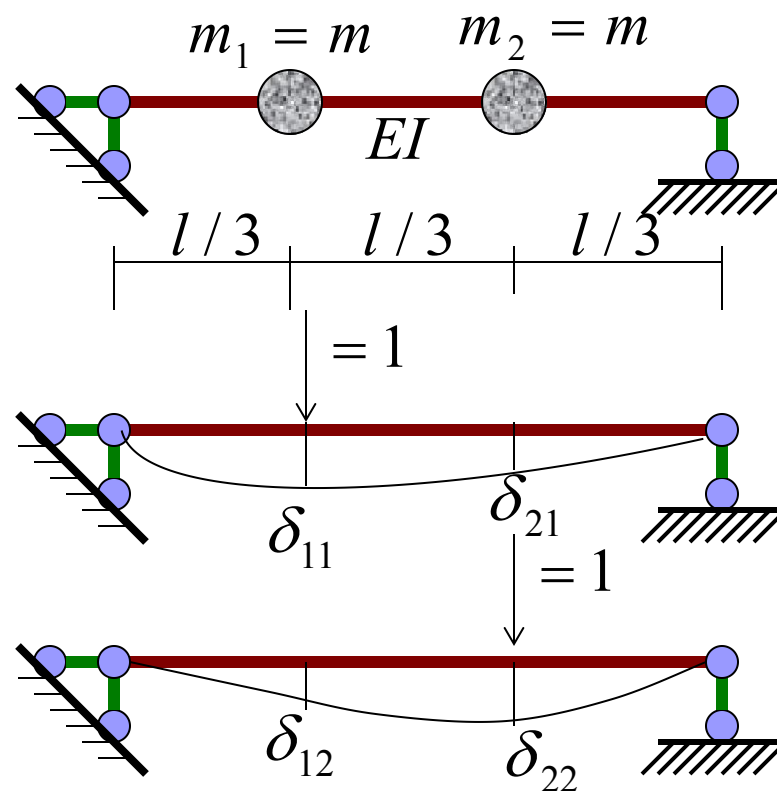
$$\begin{cases} X_1 = \delta_{11}m_1\omega^2 X_1 + \delta_{12}m_2\omega^2 X_2 \\ X_2 = \delta_{21}m_1\omega^2 X_1 + \delta_{22}m_2\omega^2 X_2 \end{cases}$$

$$X_1 = \delta_{11}m_1\omega^2 X_1 + \delta_{12}m_2\omega^2 X_2$$

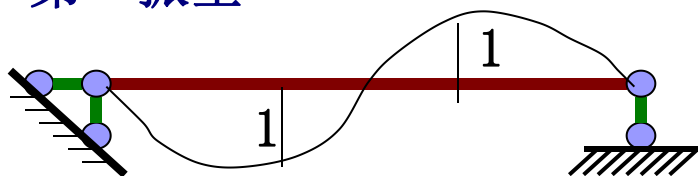
$$\frac{X_1}{X_2} = \frac{\delta_{12}m_2\omega^2}{1 - \delta_{11}m_1\omega^2}$$

$$\frac{X_{11}}{X_{21}} = \frac{\delta_{12}m_2\omega_1^2}{1 - \delta_{11}m_1\omega_1^2} = 1$$

$$\frac{X_{12}}{X_{22}} = \frac{\delta_{12}m_2\omega_2^2}{1 - \delta_{11}m_1\omega_2^2} = -1$$



第一振型



第二振型

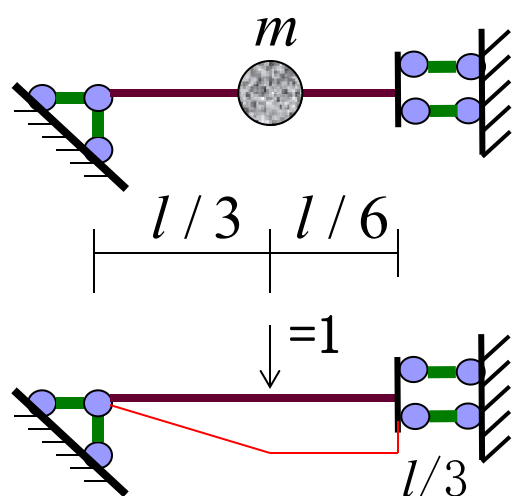
$$\{X\}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

对称体系的振型分成两组：

$$\{X\}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

一组为对称振型
一组为反对称振型

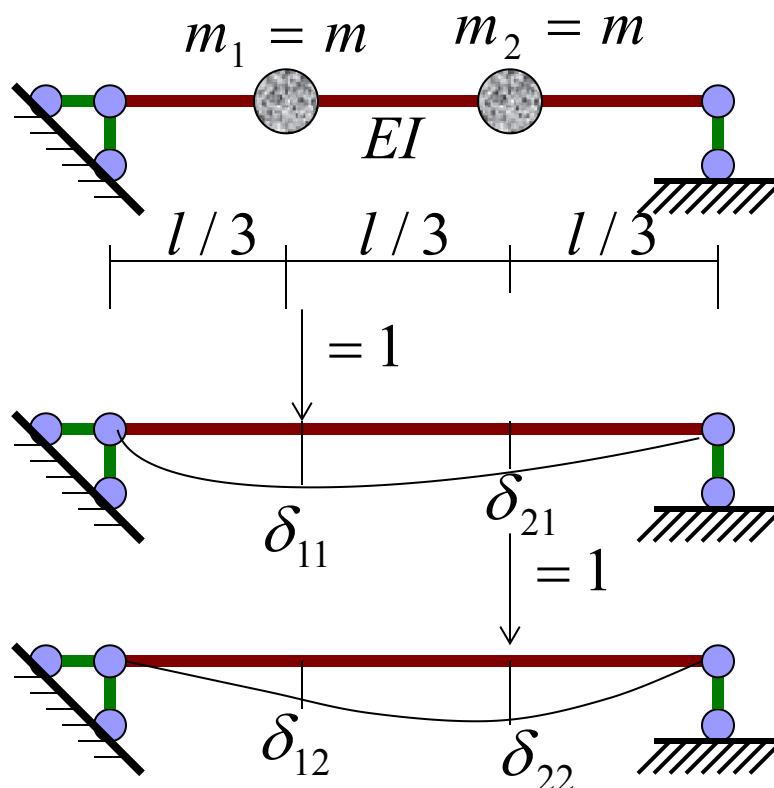
按对称振型振动



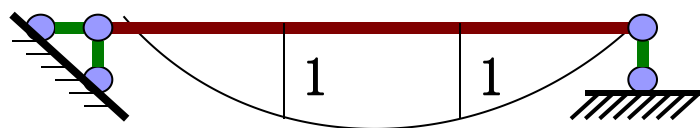
$$\delta_{11} = \frac{5}{162} \frac{l^3}{EI}$$

$$\omega^2 = \frac{1}{m\delta_{11}}$$

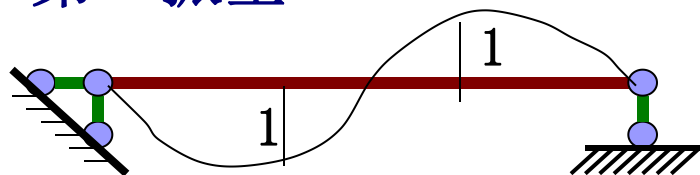
$$\omega = 5.692 \sqrt{EI / ml^3}$$



按反对称振型振动



第一振型



第二振型

$$\{X\}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

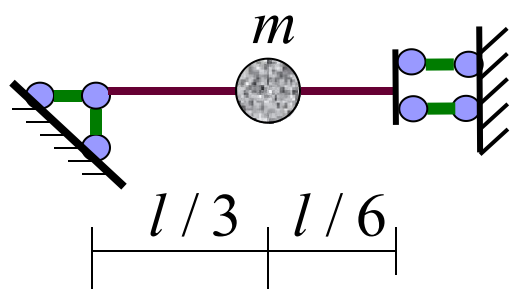
$$\{X\}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

对称系的振型分成两组：

一组为对称振型

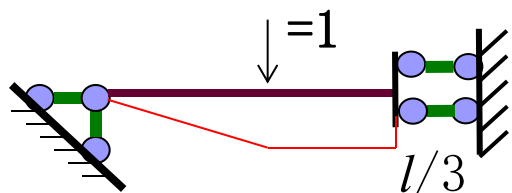
一组为反对称振型

按对称振型振动

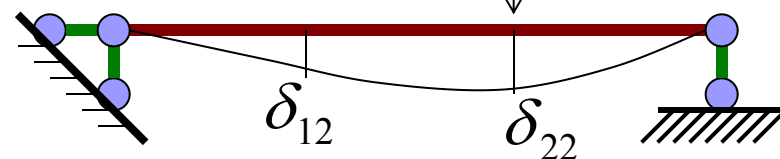
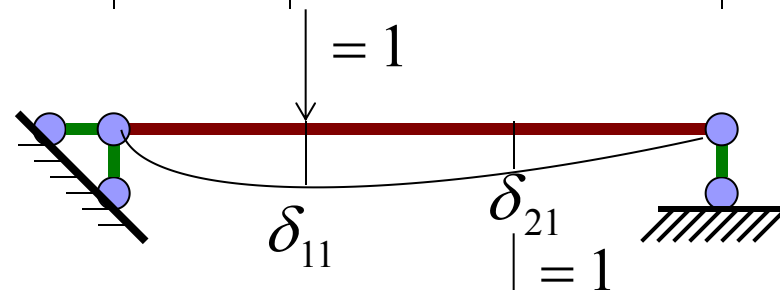
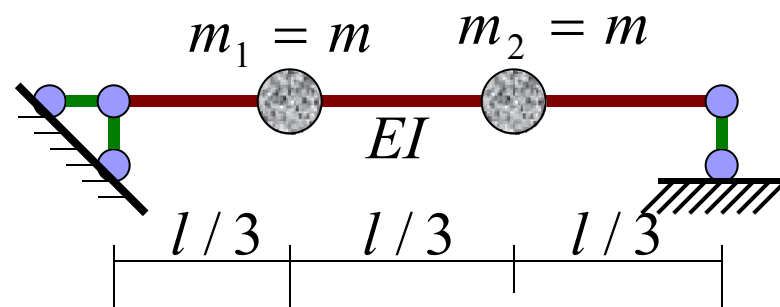


$$\delta_{11} = \frac{5}{162} \frac{l^3}{EI}$$

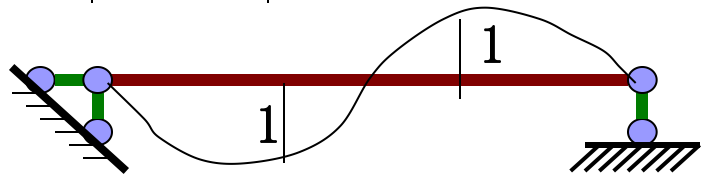
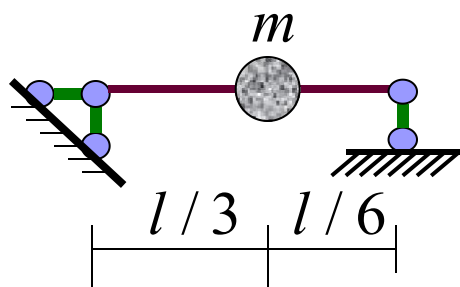
$$\omega^2 = \frac{1}{m\delta_{11}}$$



$$\omega = 5.692 \sqrt{EI / ml^3}$$



按反对称振型振动



$$\{X\}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\{X\}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

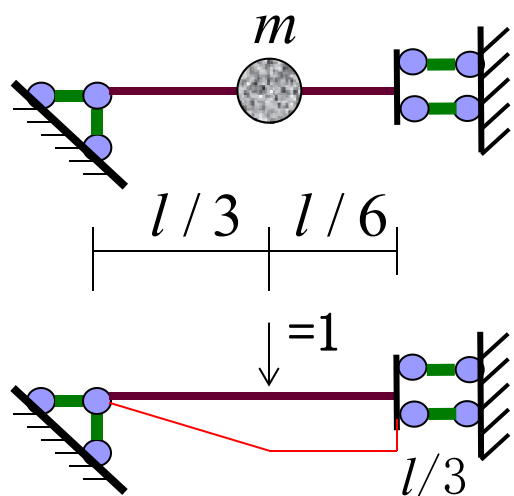
对称系的振型分成两组：

一组为对称振型

一组为反对称振型

第二振型

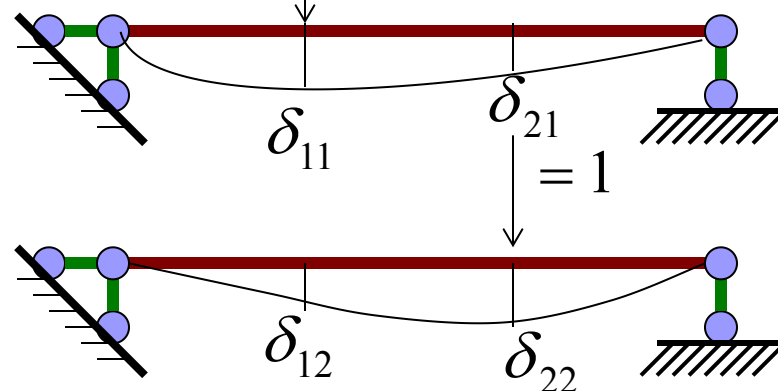
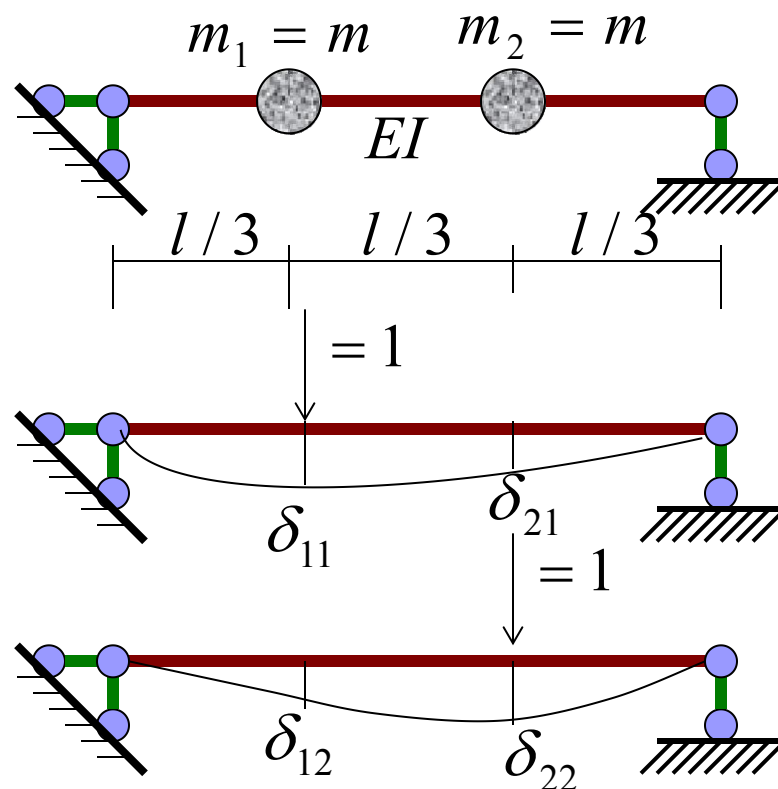
按对称振型振动



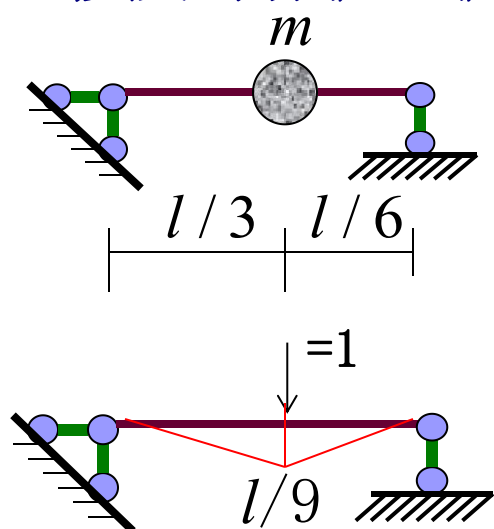
$$\delta_{11} = \frac{5}{162} \frac{l^3}{EI}$$

$$\omega^2 = \frac{1}{m\delta_{11}}$$

$$\omega = 5.692\sqrt{EI/ml^3}$$



按反对称振型振动



$$\delta_{11} = \frac{1}{486} \frac{l^3}{EI}$$

$$\omega = 22.045\sqrt{EI/ml^3}$$

$$\omega_1 = 5.692\sqrt{EI/ml^3}$$

$$\omega_2 = 22.045\sqrt{EI/ml^3}$$

对称系的振型分成两组：
 一组为对称振型
 一组为反对称振型

例2-2 求图示体系的频率、振型。

已知: $k_1 = k_2 = k; m_1 = m_2 = m.$

解:
$$\begin{cases} k_{11}X_1 + k_{12}X_2 - m_1\omega^2 X_1 = 0 \\ k_{21}X_1 + k_{22}X_2 - m_2\omega^2 X_2 = 0 \end{cases}$$

$$\begin{vmatrix} k_{11} - m_1\omega^2 & k_{12} \\ k_{21} & k_{22} - m_2\omega^2 \end{vmatrix} = 0$$

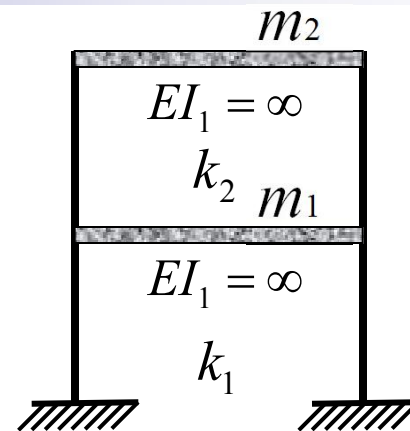
$$k_{11} = k_1 + k_2 = 2k \quad k_{12} = k_{21} = -k \quad k_{22} = k$$

$$\begin{vmatrix} 2k - m\omega^2 & -k \\ -k & k - m\omega^2 \end{vmatrix} = 0$$

$$(2k - \omega^2 m)(k - \omega^2 m) - k^2 = 0$$

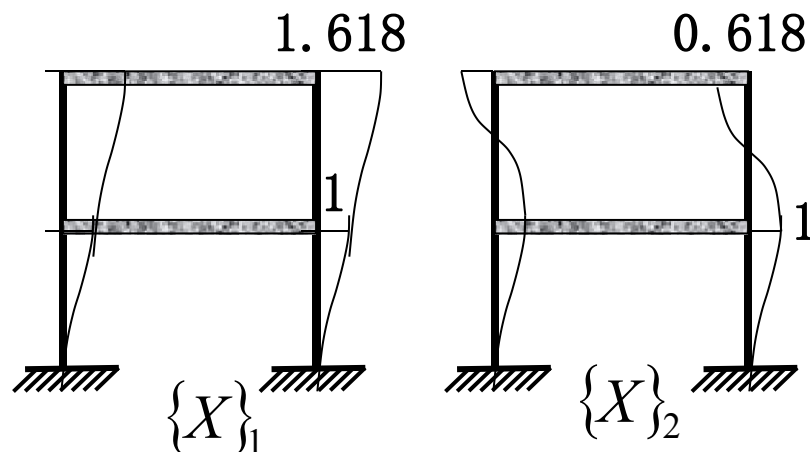
$$\omega_1 = 0.618\sqrt{k/m} \quad \omega_2 = 1.618\sqrt{k/m}$$

$$\frac{X_{11}}{X_{21}} = \frac{1}{1.618}; \frac{X_{12}}{X_{22}} = -\frac{1}{0.618}$$



$$\{X\}_1 = \begin{Bmatrix} 1 \\ 1.618 \end{Bmatrix}$$

$$\{X\}_2 = \begin{Bmatrix} 1 \\ -0.618 \end{Bmatrix}$$



例2-3 求体系的频率、振型

解：

$$\delta_{11}m\omega^2 X_1 + \delta_{12}2m\omega^2 X_2 = X_1$$

$$\delta_{21}m\omega^2 X_1 + \delta_{22}2m\omega^2 X_2 = X_2$$

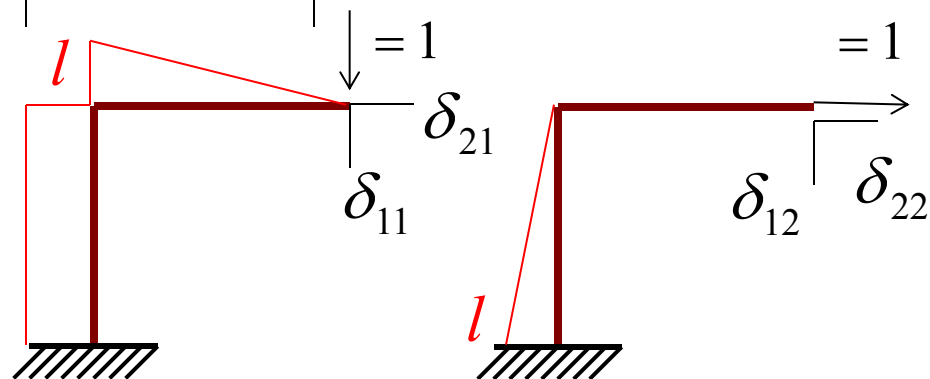
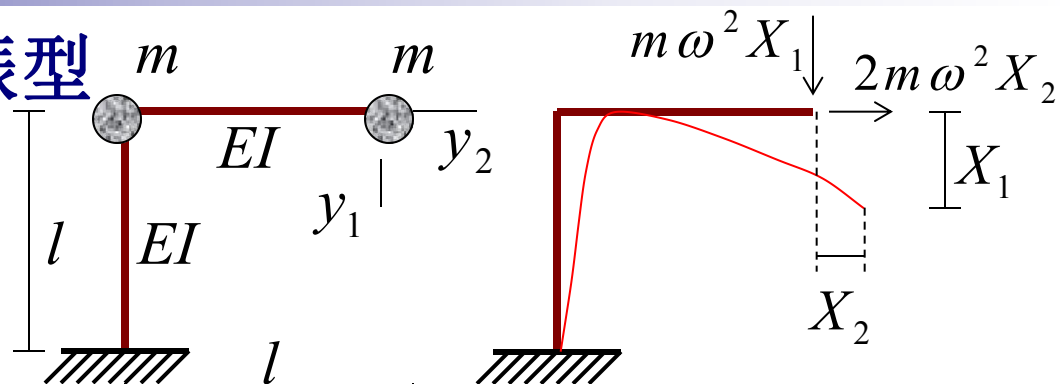
$$\text{令 } \lambda = \frac{1}{\delta_{11}m_1\omega^2}$$

$$(1-\lambda)X_1 + \frac{2\delta_{12}}{\delta_{11}}X_2 = 0$$

$$\frac{\delta_{21}}{\delta_{11}}X_1 + (2\frac{\delta_{22}}{\delta_{11}} - \lambda)X_2 = 0$$

$$\delta_{11} = \frac{4}{3} \frac{l^3}{EI} \quad \delta_{12} = \delta_{21} = \frac{1}{2} \frac{l^3}{EI}$$

$$\delta_{22} = \frac{1}{3} \frac{l^3}{EI}$$



$$\begin{vmatrix} 1-\lambda & 3/4 \\ 3/8 & 1/2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1/2-\lambda) - 9/32 = 0$$

$$\lambda_1 = 1.336 \quad \lambda_2 = 0.1637$$

$$\omega_1 = 0.749 \sqrt{\frac{EI}{ml^3}}; \omega_2 = 2.140 \sqrt{\frac{EI}{ml^3}}$$

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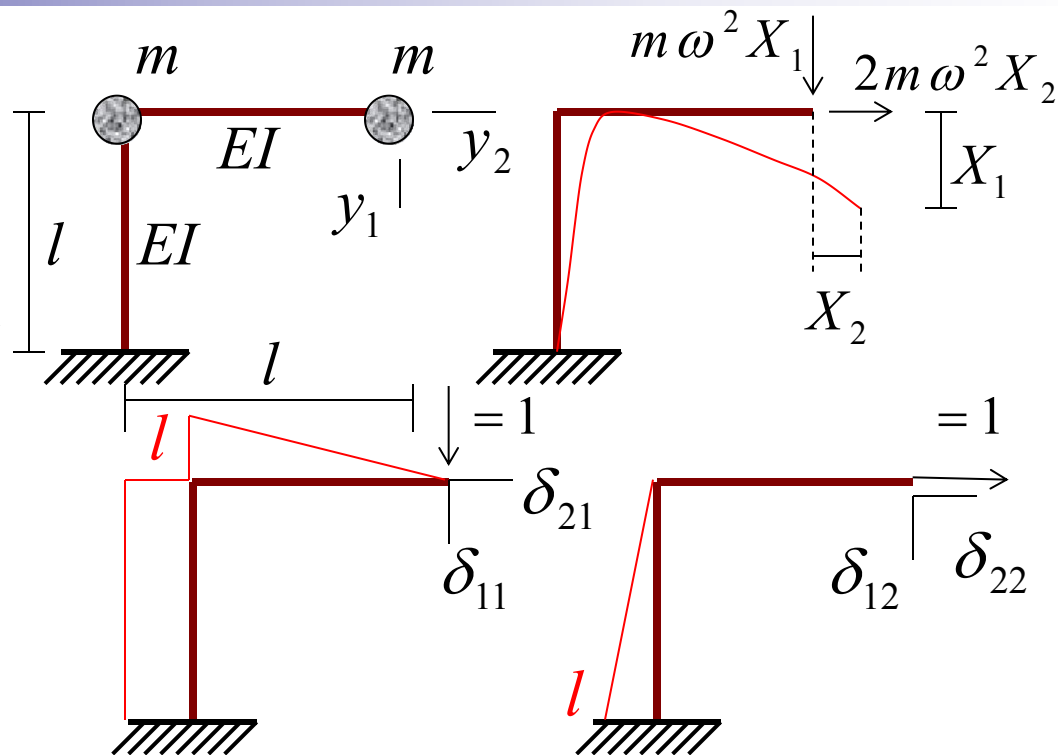
$$\text{令 } \lambda = \frac{1}{\delta_{11}m_1\omega^2}$$

$$(1-\lambda)X_1 + \frac{2\delta_{12}}{\delta_{11}}X_2 = 0$$

$$\frac{X_{11}}{X_{21}} = -\frac{3/4}{1-\lambda_1} = 2.23$$

$$\frac{X_{12}}{X_{22}} = -\frac{3/4}{1-\lambda_2} = -0.897$$

$$\{X\}_1 = \begin{Bmatrix} 2.23 \\ 1 \end{Bmatrix}; \{X\}_2 = \begin{Bmatrix} -0.897 \\ 1 \end{Bmatrix}$$



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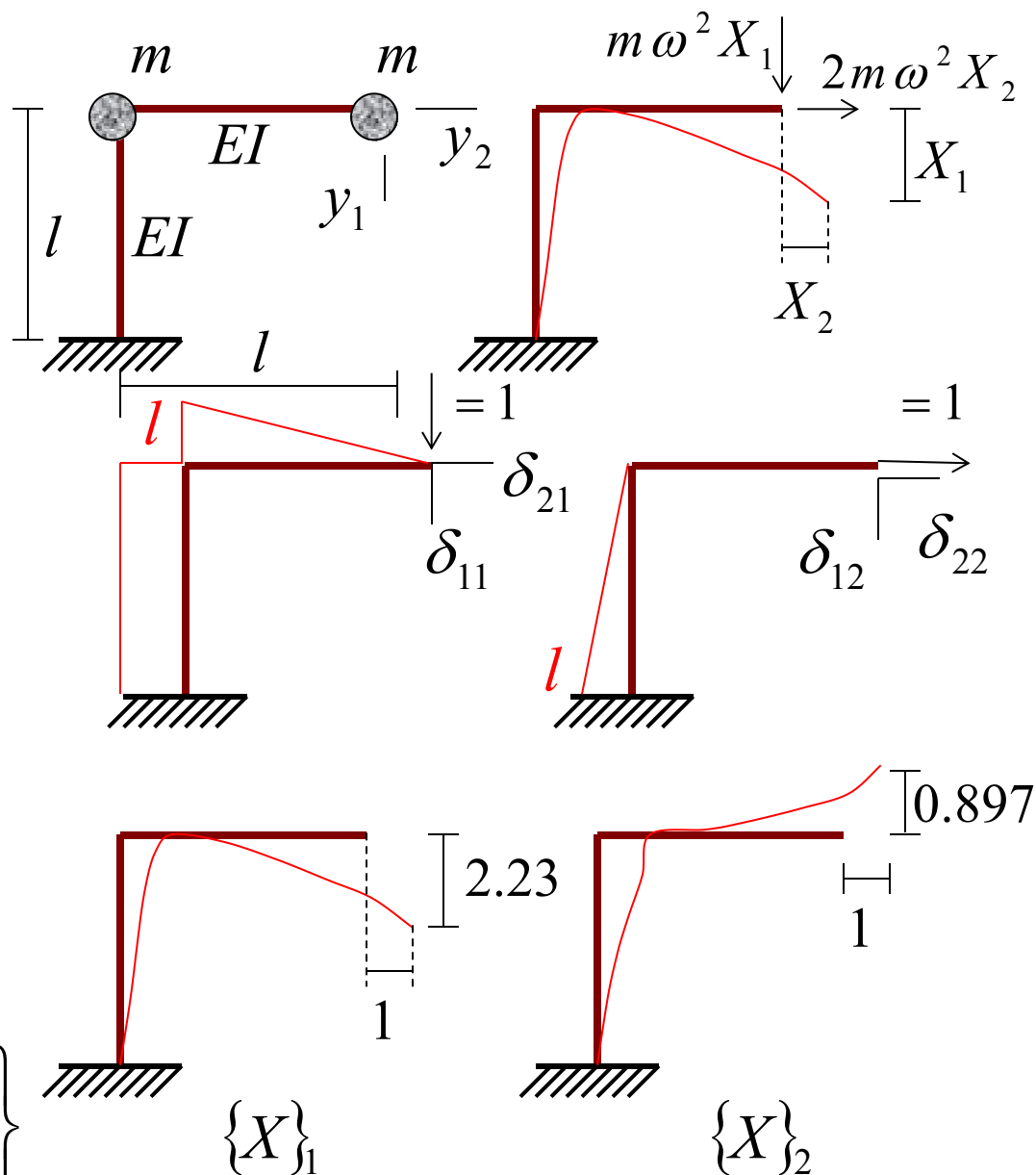
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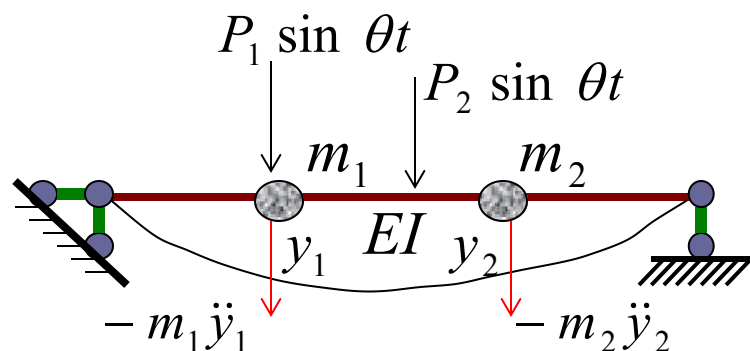
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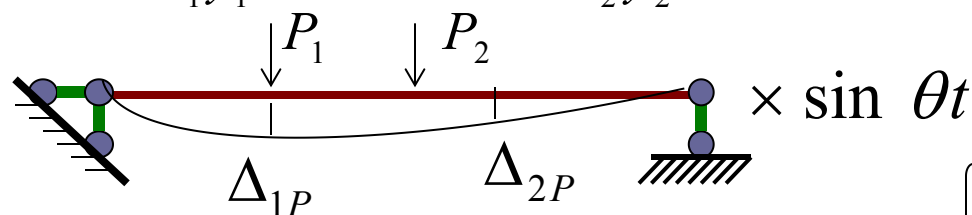
§3.2 简谐荷载作用下的受迫振动分析



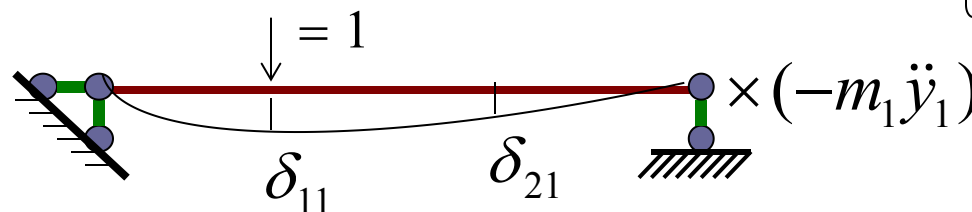
运动方程

$$\begin{cases} y_1 = \delta_{11}(-m_1 \ddot{y}_1) + \delta_{12}(-m_2 \ddot{y}_2) + \Delta_{1P} \sin \theta t \\ y_2 = \delta_{21}(-m_1 \ddot{y}_1) + \delta_{22}(-m_2 \ddot{y}_2) + \Delta_{2P} \sin \theta t \end{cases}$$

设特解为
$$\begin{cases} y_1 = A_1 \sin \theta t \\ y_2 = A_2 \sin \theta t \end{cases}$$

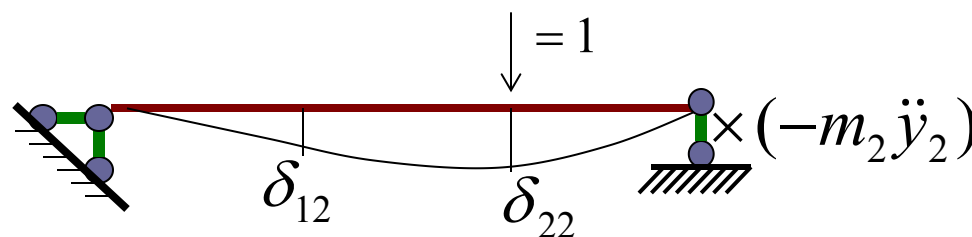


$$\begin{cases} (m_1 \delta_{11} - 1/\theta^2) A_1 + m_2 \delta_{12} A_2 = -\Delta_{1P} / \theta^2 \\ m_1 \delta_{21} A_1 + (m_2 \delta_{22} - 1/\theta^2) A_2 = -\Delta_{2P} / \theta^2 \end{cases}$$



解方程, 得 $A_1 = \frac{D_1}{D} \quad A_2 = \frac{D_2}{D}$

其中



$$D_1 = \begin{vmatrix} m_1 \delta_{11} - 1/\theta^2 & m_2 \delta_{12} \\ m_1 \delta_{21} & m_2 \delta_{22} - 1/\theta^2 \end{vmatrix}$$

§3.2 简谐荷载作用下的受迫振动分析

运动方程

$$D_2 = - \begin{vmatrix} m_1 \delta_{11} - 1/\theta^2 & \Delta_{1P}/\theta^2 \\ m_1 \delta_{21} & \Delta_{2P}/\theta^2 \end{vmatrix} \begin{cases} y_1 = \delta_{11}(-m_1 \ddot{y}_1) + \delta_{12}(-m_2 \ddot{y}_2) + \Delta_{1P} \sin \theta t \\ y_2 = \delta_{21}(-m_1 \ddot{y}_1) + \delta_{22}(-m_2 \ddot{y}_2) + \Delta_{2P} \sin \theta t \end{cases}$$

$$D = \begin{vmatrix} m_1 \delta_{11} - 1/\theta^2 & m_2 \delta_{12} \\ m_1 \delta_{21} & m_2 \delta_{22} - 1/\theta^2 \end{vmatrix}$$

设特解为
$$\begin{cases} y_1 = A_1 \sin \theta t \\ y_2 = A_2 \sin \theta t \end{cases}$$

$$\begin{cases} (m_1 \delta_{11} - 1/\theta^2)A_1 + m_2 \delta_{12}A_2 = -\Delta_{1P}/\theta^2 \\ m_1 \delta_{21}A_1 + (m_2 \delta_{22} - 1/\theta^2)A_2 = -\Delta_{2P}/\theta^2 \end{cases}$$

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其中

$$D_1 = - \begin{vmatrix} \Delta_{1P}/\theta^2 & m_2 \delta_{12} \\ \Delta_{2P}/\theta^2 & m_2 \delta_{22} - 1/\theta^2 \end{vmatrix}$$

1. 在平稳阶段, 作简谐振动, 振动频率与荷载同。

2. 当 $\theta \rightarrow 0$ 时

$$A_1 \rightarrow \Delta_{1P} \quad A_2 \rightarrow \Delta_{2P}$$

$$\begin{cases} (m_1 \delta_{11} \theta^2 - 1)A_1 + m_2 \delta_{12} A_2 \theta^2 = -\Delta_{1P} \\ \theta^2 m_1 \delta_{21} A_1 + (\theta^2 m_2 \delta_{22} - 1)A_2 = -\Delta_{2P} \end{cases}$$

3. 当 $\theta \rightarrow \infty$ 时 $A_1 \rightarrow 0 \quad A_2 \rightarrow 0$

§3.2 简谐荷载作用下的受迫振动分析

$$D_2 = - \begin{vmatrix} m_1 \delta_{11} - 1/\theta^2 & \Delta_{1P} / \theta^2 \\ m_1 \delta_{21} & \Delta_{2P} / \theta^2 \end{vmatrix}$$

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3. 当 $\theta \rightarrow \infty$ 时 $A_1 \rightarrow 0 \quad A_2 \rightarrow 0$

4. 当 $\theta \rightarrow \omega_1$ 或 $\theta \rightarrow \omega_2$ 时

$$A_1 \rightarrow \infty \quad A_2 \rightarrow \infty$$

n 自由度体系有 n 个共振区。

$$\begin{cases} (m_1 \delta_{11} - 1/\theta^2) A_1 + m_2 \delta_{12} A_2 = -\Delta_{1P} / \theta^2 \\ m_1 \delta_{21} A_1 + (m_2 \delta_{22} - 1/\theta^2) A_2 = -\Delta_{2P} / \theta^2 \end{cases}$$

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