Electrostatically Actuated Cantilever

Introduction

The elastic cantilever beam is an elementary structure in MEMS design. This example shows the bending of a beam due to electrostatic forces. The model uses the electromechanics interface to solve the coupled equations for the structural deformation and the electric field. Such structures are frequently tested by means of a low frequency capacitance voltage sweep. The model predicts the results of such a test.

Model Definition

Figure 1 shows the model geometry. The beam has the following dimensions:

- Length: 300 µm
- Width: 20 µm
- Thickness 2 µm

Because the geometry is symmetric only half of the beam needs to modeled. The beam is made of polysilicon with a Young's modulus, E, of 153 GPa, and a Poisson's ratio, ν, of 0.23. It is fixed at one end but is otherwise free to move. The polysilicon is assumed to be heavily doped, so that electric field penetration into the structure can be neglected. The beam resides in an air-filled chamber that is electrically insulated. The lower side of the chamber has a grounded electrode.

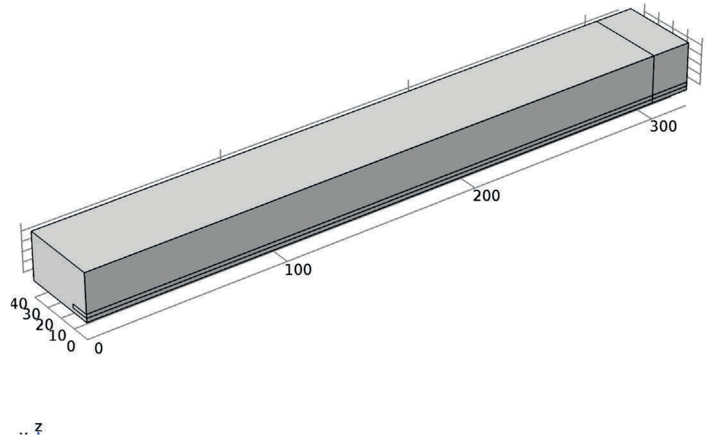

Figure 1: Model Geometry. The beam is 300 μm long and 2 μm thick, and it is fixed at x = 0. The model uses symmetry on the zx-plane at y = 0. The lower boundary of the surrounding air domain represents the grounded substrate. The model has 20 μm of free air above and to the sides of the beam, while the gap below the beam is 2 μm.

An electrostatic force caused by an applied potential difference between the two electrodes bends the beam toward the grounded plane beneath it. To compute the electrostatic force, this example calculates the electric field in the surrounding air. The model considers a layer of air 20 μm thick both above and to the sides of the beam, and the air gap between the bottom of the beam and the grounded layer is initially 2 μm. As the beam bends, the geometry of the air gap changes continuously, resulting in a change in the electric field between the electrodes. The coupled physics is handled automatically by the Electromechanics interface.

The electrostatic field in the air and in the beam is governed by Poisson's equation:

$$-\nabla \cdot (\varepsilon \nabla V) = 0$$

where derivatives are taken with respect to the spatial coordinates. The numerical model represents the electric potential and its derivatives on a mesh which is moving with respect to the spatial frame. The necessary transformations are taken care of by the Electromechanics interface, which also contains smoothing equations governing the movement of the mesh in the air domain.

The cantilever connects to a voltage terminal with a specified bias potential, V_{in}. The bottom of the chamber is grounded, while all other boundaries are electrically insulated. The terminal boundary condition automatically computes the capacitance of the system.

The force density that acts on the electrode of the beam results from Maxwell's stress tensor:

$$\mathbf{F}_{\mathrm{es}} = -\frac{1}{2}(\mathbf{E}\cdot\mathbf{D})\mathbf{n} + (\mathbf{n}\cdot\mathbf{E})\mathbf{D}$$

where **E** and **D** are the electric field and electric displacement vectors, respectively, and **n** is the outward normal vector of the boundary. This force is always oriented along the normal of the boundary.

Navier's equations, which govern the deformation of a solid, are more conveniently written in a coordinate system that follows and deforms with the material. In this case, these reference or material coordinates are identical to the actual mesh coordinates.

Results and Discussion

There is positive feedback between the electrostatic forces and the deformation of the cantilever beam. The forces bend the beam and thereby reduce the gap to the grounded substrate. This action, in turn, increases the forces. At a certain voltage the electrostatic forces overcome the stress forces, the system becomes unstable, and the gap collapses. This critical voltage is called the *pull-in voltage*.

At applied voltages lower than the pull-in voltage, the beam stays in an equilibrium position where the stress forces balance the electrostatic forces. Figure 2 shows the beam displacement and the corresponding displacement of the mesh surrounding it. Figure 3 shows the electric potential and electric field that generates these displacements. In Figure 4 the shape of the cantilever's deflection is illustrated for each applied voltage, by plotting the z-displacement of the underside of the beam at the symmetry boundary. The tip deflection as a function of applied voltage is shown in Figure 5. Note that for applied voltages higher than the pull-in voltage, the solution does not converge because no stable stationary solution exists. This situation occurs if an applied voltage of 6.2 V is tried. The pull-in voltage is therefore between 6.1 V and 6.2 V. For comparison, computations in Ref. 1 predict a pull-in voltage of

$$V_{\mathrm{PI}} = \sqrt{\frac{4c_1B}{\varepsilon_0 L^4 c_2^2\left(1 + c_3\frac{g_0}{W}\right)}}$$

where $c_1 = 0.07$, $c_2 = 1.00$, and $c_3 = 0.42$; g_0 is the initial gap between the beam and the ground plane; and

$$B = \hat{E}H^3 g_0^3$$

If the beam has a narrow width (W) relative to its thickness (H) and length (L), $\hat{E}$ is Young's modulus, E. Otherwise, E and $\hat{E}$, the plate modulus, are related by

$$\frac{E}{\hat{E}} \approx 1 - \nu^2 \left(\frac{(W/L)^{1,37}}{0,5 + (W/L)^{1,37}} \right)^{0,98(L/H)^{-0,056}}$$

where ν is Poisson's ratio. Because the calculation in Ref. 1 uses a parallel-plate approximation for calculating the electrostatic force and because it corrects for fringing fields, these results are not directly comparable with those from the simulation. However the agreement is still reasonable: setting $W = 20$ µm results in $V_{PI} = 6.07$ V.

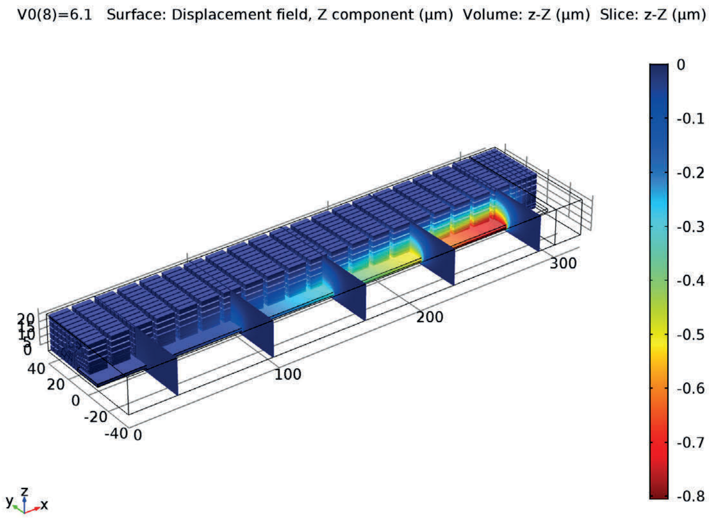

Figure 2: z-displacement for the beam and the moving mesh as a function of position. Each mesh element is depicted as a separate block in the back half of the geometry.

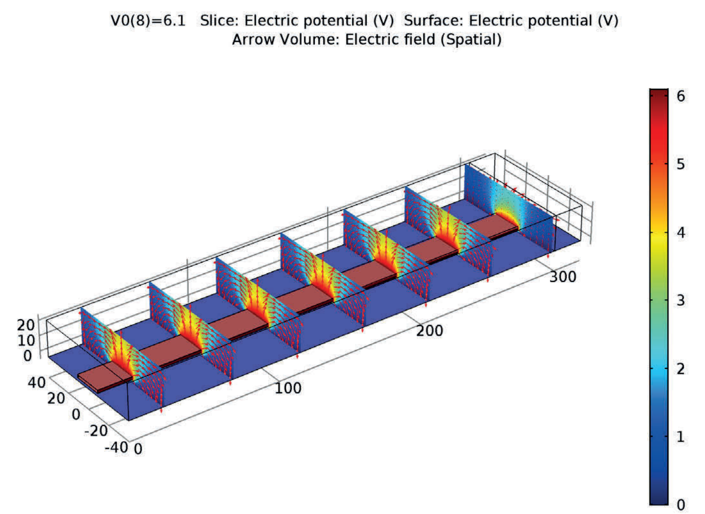

Figure 3: Electric Potential (color) and Electric Field (arrows) at various cross sections through the beam.