Chapter 1 Overview

This chapter reviews the most important things you need to know to start learning calculus. It also introduces the use of a graphing utility as a tool to investigate mathematical ideas, to support analytic work, and to solve problems with numerical and graphical methods. The emphasis is on functions and graphs, the main building blocks of calculus.

Functions and parametric equations are the major tools for describing the real world in mathematical terms, from temperature variations to planetary motions, from brain waves to business cycles, and from heartbeat patterns to population growth. Many functions have particular importance because of the behavior they describe. Trigonometric functions describe cyclic, repetitive activity; exponential, logarithmic, and logistic functions describe growth and decay; and polynomial functions can approximate these and most other functions.

1.1 Lines

What you'll learn about

- Increments
- Slope of a Line
- Parallel and Perpendicular Lines
- Equations of Lines
- Applications

. . . and why

Linear equations are used extensively in business and economic applications.

Increments

One reason calculus has proved to be so useful is that it is the right mathematics for relating the rate of change of a quantity to the graph of the quantity. Explaining that relationship is one goal of this book. It all begins with the slopes of lines.

When a particle in the plane moves from one point to another, the net changes or *increments* in its coordinates are found by subtracting the coordinates of its starting point from the coordinates of its stopping point.

DEFINITION Increments

If a particle moves from the point (x_1, y_1) to the point (x_2, y_2), the **increments** in its coordinates are

$$x \quad x_2 \quad x_1 \quad \text{and} \quad y \quad y_2 \quad y_1.$$

The symbols x and y are read "delta x" and "delta y." The letter is a Greek capital d for "difference." Neither x nor y denotes multiplication; x is not "delta times x" nor is y "delta times y."

Increments can be positive, negative, or zero, as shown in Example 1.

EXAMPLE 1 Finding Increments

The coordinate increments from 4, 3 to (2, 5) are

$$x \quad 2 \quad 4 \quad 2, \qquad y \quad 5 \quad 3 \quad 8.$$

From (5, 6) to (5, 1), the increments are

$$x \quad 5 \quad 5 \quad 0, \qquad y \quad 1 \quad 6 \quad 5.$$

Now try Exercise 1.

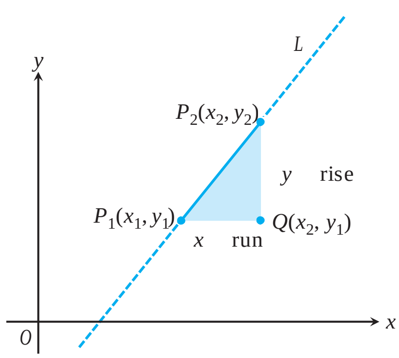


Figure 1.1 The slope of line L is $m \quad \frac{\text{rise}}{\text{run}} \quad \frac{y}{x}$.

Slope of a Line

Each nonvertical line has a *slope*, which we can calculate from increments in coordinates.

Let L be a nonvertical line in the plane and $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ two points on L (Figure 1.1). We call $y \quad y_2 \quad y_1$ the **rise** from P_1 to P_2 and $x \quad x_2 \quad x_1$ the **run** from

P_1 to P_2. Since L is not vertical, x 0 and we define the slope of L to be the amount of rise per unit of run. It is conventional to denote the slope by the letter m.

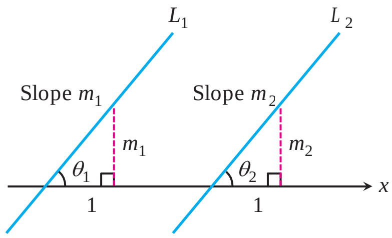


Figure 1.2 If L_1 L_2, then u$_1$ u$_2$ and m_1 m_2. Conversely, if m_1 m_2, then u$_1$ u$_2$ and L_1 L_2.

DEFINITION Slope

Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be points on a nonvertical line, L. The **slope** of L is

$$m \quad \frac{\text{rise}}{\text{run}} \quad \frac{y}{x} \quad \frac{y_2 \quad y_1}{x_2 \quad x_1}.$$

A line that goes uphill as x increases has a positive slope. A line that goes downhill as x increases has a negative slope. A horizontal line has slope zero since all of its points have the same y-coordinate, making y 0. For vertical lines, x 0 and the ratio y x is undefined. We express this by saying that vertical lines *have no slope*.

Parallel and Perpendicular Lines

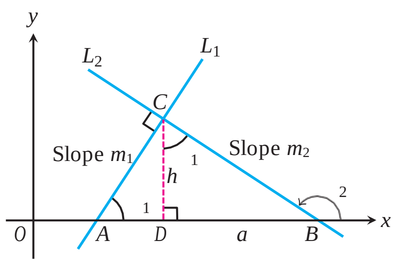


Figure 1.3 ADC is similar to CDB. Hence f$_1$ is also the upper angle in CDB, where tan f$_1$ a h.

Parallel lines form equal angles with the x-axis (Figure 1.2). Hence, nonvertical parallel lines have the same slope. Conversely, lines with equal slopes form equal angles with the x-axis and are therefore parallel.

If two nonvertical lines L_1 and L_2 are perpendicular, their slopes m_1 and m_2 satisfy $m_1 m_2$ 1, so each slope is the *negative reciprocal* of the other:

$$m_1 \quad \frac{1}{m_2}, \qquad m_2 \quad \frac{1}{m_1}.$$

The argument goes like this: In the notation of Figure 1.3, m_1 tan f$_1$ a h, while m_2 tan f$_2$ h a. Hence, $m_1 m_2$ $(a\ h)(\ h\ a)$ 1.

Equations of Lines

The vertical line through the point (a, b) has equation x a since every x-coordinate on the line has the value a. Similarly, the horizontal line through (a, b) has equation y b.

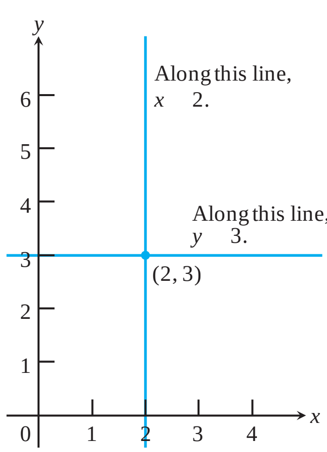


Figure 1.4 The standard equations for the vertical and horizontal lines through the point (2, 3) are x 2 and y 3. (Example 2)

EXAMPLE 2 Finding Equations of Vertical and Horizontal Lines

The vertical and horizontal lines through the point (2, 3) have equations x 2 and y 3, respectively (Figure 1.4).

Now try Exercise 9.

We can write an equation for any nonvertical line L if we know its slope m and the coordinates of one point $P_1(x_1, y_1)$ on it. If $P(x, y)$ is *any* other point on L, then

$$\frac{y \quad y_1}{x \quad x_1} \quad m,$$

so that

$$y \quad y_1 \quad m(x \quad x_1) \quad \text{or} \quad y \quad m(x \quad x_1) \quad y_1.$$

DEFINITION Point-Slope Equation

The equation

$$y \quad m(x \quad x_1) \quad y_1$$

is the **point-slope equation** of the line through the point (x_1, y_1) with slope m.

EXAMPLE 3 Using the Point-Slope Equation

Write the point-slope equation for the line through the point (2, 3) with slope 3 2.

SOLUTION

We substitute x_1 2, y_1 3, and m 3 2 into the point-slope equation and obtain

$$y \qquad \frac{3}{2}\, x \quad 2 \qquad 3 \qquad \text{or} \qquad y \qquad \frac{3}{2}\, x \qquad 6.$$

Now try Exercise 13.

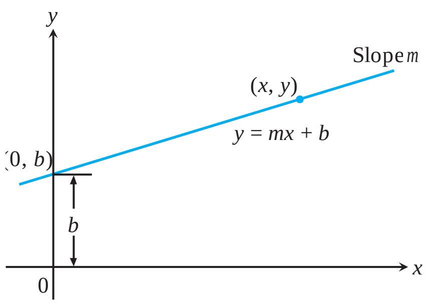


Figure 1.5 A line with slope m and y-intercept b.

The y-coordinate of the point where a nonvertical line intersects the y-axis is the ***y*-intercept** of the line. Similarly, the x-coordinate of the point where a nonhorizontal line intersects the x-axis is the ***x*-intercept** of the line. A line with slope m and y-intercept b passes through (0, b) (Figure 1.5), so

$$y \quad m\, x \quad 0 \qquad b, \qquad \text{or, more simply,} \qquad y \quad mx \quad b.$$

DEFINITION Slope-Intercept Equation

The equation

$$y \quad mx \quad b$$

is the **slope-intercept equation** of the line with slope m and y-intercept b.

EXAMPLE 4 Writing the Slope-Intercept Equation

Write the slope-intercept equation for the line through (2, 1) and (3, 4).

SOLUTION

The line's slope is

$$m \quad \frac{4}{3} \qquad \frac{1}{2} \quad \frac{5}{5} \quad 1.$$

We can use this slope with either of the two given points in the point-slope equation. For (x_1, y_1) (2, 1), we obtain

$$y \quad 1 \cdot x \qquad 2 \qquad\quad 1$$
$$y \quad x \quad 2 \qquad 1$$
$$y \quad x \quad 1.$$

Now try Exercise 17.

If A and B are not both zero, the graph of the equation Ax By C is a line. Every line has an equation in this form, even lines with undefined slopes.

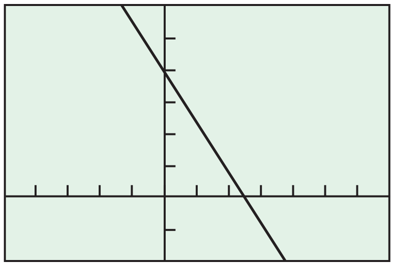
DEFINITION General Linear Equation

The equation

$$Ax \quad By \quad C \quad (A \text{ and } B \text{ not both } 0)$$

is a **general linear equation** in x and y.

Although the general linear form helps in the quick identification of lines, the slope-intercept form is the one to enter into a calculator for graphing.



[–5, 7] by [–2, 6]

Figure 1.6 The line $8x \quad 5y \quad 20$. (Example 5)

EXAMPLE 5 Analyzing and Graphing a General Linear Equation

Find the slope and y-intercept of the line $8x \quad 5y \quad 20$. Graph the line.

SOLUTION

Solve the equation for y to put the equation in slope-intercept form:

$$8x \quad 5y \quad 20$$

$$5y \quad 8x \quad 20$$

$$y \quad \frac{8}{5}x \quad 4$$

This form reveals the slope ($m \quad 8 \; 5$) and y-intercept ($b \quad 4$), and puts the equation in a form suitable for graphing (Figure 1.6).

Now try Exercise 27.

EXAMPLE 6 Writing Equations for Lines

Write an equation for the line through the point (1, 2) that is **(a)** parallel, and **(b)** perpendicular to the line L: $y \quad 3x \quad 4$.

SOLUTION

The line L, $y \quad 3x \quad 4$, has slope 3.

(a) The line $y \quad 3(x \quad 1) \quad 2$, or $y \quad 3x \quad 5$, passes through the point (1, 2), and is parallel to L because it has slope 3.

(b) The line $y \quad (\; 1 \; 3)(x \quad 1) \quad 2$, or $y \quad (\; 1 \; 3)x \quad 5 \; 3$, passes through the point (1, 2), and is perpendicular to L because it has slope $1 \; 3$.

Now try Exercise 31.

EXAMPLE 7 Determining a Function

The following table gives values for the linear function $f(x) \quad mx \quad b$. Determine m and b.

x	$f(x)$
1	14 3
1	4 3
2	13 3

SOLUTION

The graph of f is a line. From the table we know that the following points are on the line: (1, 14 3), (1, 4 3), (2, 13 3).

Using the first two points, the slope m is

$$m \quad \frac{4\;3 \quad (14\;3)}{1 \quad (\quad 1)} \quad \frac{6}{2} \quad 3.$$

So $f(x) \quad 3x \quad b$. Because $f(\; 1) \quad 14 \; 3$, we have

$$f(\; 1) \quad 3(\; 1) \quad b$$

$$14\;3 \quad 3 \quad b$$

$$b \quad 5\;3.$$

continued

Thus, $m \quad 3, b \quad 5 \; 3$, and $f(x) \quad 3x \quad 5 \; 3$.

We can use either of the other two points determined by the table to check our work.

Now try Exercise 35.

Applications

Many important variables are related by linear equations. For example, the relationship between Fahrenheit temperature and Celsius temperature is linear, a fact we use to advantage in the next example.

EXAMPLE 8 Temperature Conversion

Find the relationship between Fahrenheit and Celsius temperature. Then find the Celsius equivalent of 90ºF and the Fahrenheit equivalent of 5ºC.

SOLUTION

Because the relationship between the two temperature scales is linear, it has the form $F \quad mC \quad b$. The freezing point of water is $F \quad 32°$ or $C \quad 0°$, while the boiling point is $F \quad 212°$ or $C \quad 100°$. Thus,

$$32 \quad m \cdot 0 \quad b \quad \text{and} \quad 212 \quad m \cdot 100 \quad b,$$

so $b \quad 32$ and $m \quad (212 \quad 32) \; 100 \quad 9 \; 5$. Therefore,

$$F \quad \frac{9}{5}C \quad 32, \quad \text{or} \quad C \quad \frac{5}{9} \; F \quad 32 \; .$$

These relationships let us find equivalent temperatures. The Celsius equivalent of 90ºF is

$$C \quad \frac{5}{9} \; 90 \quad 32 \quad 32.2°.$$

The Fahrenheit equivalent of 5ºC is

$$F \quad \frac{9}{5} \quad 5 \quad 32 \quad 23°.$$

Now try Exercise 43.

Some graphing utilities have a feature that enables them to approximate the relationship between variables with a linear equation. We use this feature in Example 9.

It can be difficult to see patterns or trends in lists of paired numbers. For this reason, we sometimes begin by plotting the pairs (such a plot is called a **scatter plot**) to see whether the corresponding points lie close to a curve of some kind. If they do, and if we can find an equation $y \quad f(x)$ for the curve, then we have a formula that

1. summarizes the data with a simple expression, and

2. lets us predict values of y for other values of x.

The process of finding a curve to fit data is called **regression analysis** and the curve is called a **regression curve.**

There are many useful types of regression curves—power, exponential, logarithmic, sinusoidal, and so on. In the next example, we use the calculator's linear regression feature to fit the data in Table 1.1 with a line.

Table 1.1 World Population

Year	Population (millions)
1980	4454
1985	4853
1990	5285
1995	5696
2003	6305
2004	6378
2005	6450

Source: U.S. Bureau of the Census, *Statistical of the United States, 2004-2005.*

EXAMPLE 9 Regression Analysis——Predicting World Population

Starting with the data in Table 1.1, build a linear model for the growth of the world population. Use the model to predict the world population in the year 2010, and compare this prediction with the Statistical prediction of 6812 million.

continued

Why Not Round the Decimals in Equation 1 Even More?

If we do, our final calculation will be way off. Using $y \quad 80x \quad 153,849$, for instance, gives $y \quad 6951$ when $x \quad 2010$, as compared to $y \quad 6865$, an increase of 86 million. The rule is: *Retain all decimal places while working a problem. Round only at the end.* We rounded the coefficients in Equation 1 enough to make it readable, but not enough to hurt the outcome. However, we knew how much we could safely round *only from first having done the entire calculation with numbers unrounded.*

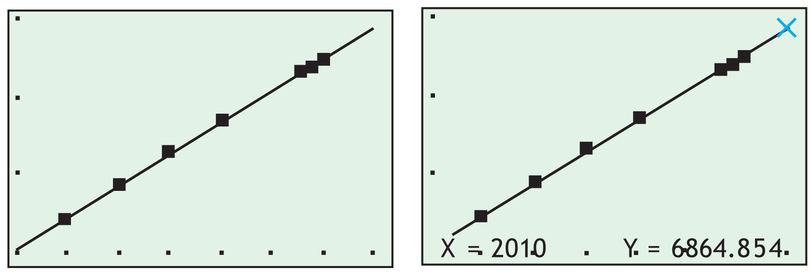
SOLUTION

Model Upon entering the data into the grapher, we find the regression equation to be approximately

$$y \quad 79.957x \quad 153848.716, \tag{1}$$

where x represents the year and y the population *in millions.*

Figure 1.7a shows the scatter plot for Table 1.1 together with a graph of the regression line just found. You can see how well the line fits the data.



[1975, 2010] by [4000, 7000]
(a)

[1975, 2010] by [4000, 7000]
(b)

Figure 1.7 (Example 9)

Solve Graphically Our goal is to predict the population in the year 2010. Reading from the graph in Figure 1.7b, we conclude that when x is 2010, y is approximately 6865.

Rounding Rule

Round your answer as appropriate, but do not round the numbers in the calculations that lead to it.

Confirm Algebraically Evaluating Equation 1 for $x \quad 2010$ gives

$$y \quad 79.957(2010) \quad 153848.716$$
$$6865.$$

Interpret The linear regression equation suggests that the world population in the year 2010 will be about 6865 million, or approximately 53 million more than the Statistical prediction of 6812 million. *Now try Exercise 45.*

Regression Analysis

Regression analysis has four steps:

1. Plot the data (scatter plot).
2. Find the regression equation. For a line, it has the form $y \quad mx \quad b$.
3. Superimpose the graph of the regression equation on the scatter plot to see the fit.
4. Use the regression equation to predict y-values for particular values of x.

Quick Review 1.1 *(For help, go to Section 1.1.)*

1. Find the value of y that corresponds to x 3 in y 2 4(x 3). 2
2. Find the value of x that corresponds to y 3 in y 3 2(x 1). 1

In Exercises 3 and 4, find the value of m that corresponds to the values of x and y.

3. x 5, y 2, m $\frac{y \quad 3}{x \quad 4}$ 1
4. x 1, y 3, m $\frac{2}{3}$ $\frac{y}{x}$ $\frac{5}{4}$

In Exercises 5 and 6, determine whether the ordered pair is a solution to the equation.

5. 3x 4y 5
   (a) (2, 1 4) Yes **(b)** (3, 1) No
6. y 2x 5
   (a) (1, 7) Yes **(b)** (2, 1) No

In Exercises 7 and 8, find the distance between the points.

7. (1, 0), (0, 1) 2
8. (2, 1), (1, 1 3) $\frac{5}{3}$

In Exercises 9 and 10, solve for y in terms of x.

9. 4x 3y 7 y $\frac{4}{3}x$ $\frac{7}{3}$
10. 2x 5y 3 y $\frac{2}{5}x$ $\frac{3}{5}$

Section 1.1 Exercises

In Exercises 1–4, find the coordinate increments from A to B.

1. A(1, 2), B(1, 1) x 2, y 3
2. A(3, 2), B(1, 2) x 2, y 4
3. A(3, 1), B(8, 1) x 5, y 0
4. A(0, 4), B(0, 2) x 0, y 6

In Exercises 5–8, let L be the line determined by points A and B.
(a) Plot A and B. **(b)** Find the slope of L.
(c) Draw the graph of L.

5. A(1, 2), B(2, 1) **(b)** 3
6. A(2, 1), B(1, 2) **(b)** $\frac{1}{3}$
7. A(2, 3), B(1, 3) **(b)** 0
8. A(1, 2), B(1, 3) **(b)** Has no slope (undefined)

In Exercise 9–12, write an equation for **(a)** the vertical line and **(b)** the horizontal line through the point P.

9. P(3, 2) x 3; y 2
10. P(1, 4 3) x 1; y $\frac{4}{3}$
11. P(0, 2) x 0; y 2
12. P(p, 0) x ; y 0

In Exercises 13–16, write the point-slope equation for the line through the point P with slope m.

13. P(1, 1), m 1
14. P(1, 1), m 1
15. P(0, 3), m 2
16. P(4, 0), m 2

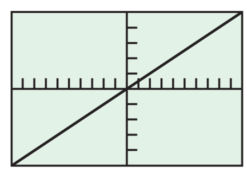
In Exercises 17–20, write the slope-intercept equation for the line with slope m and y-intercept b.

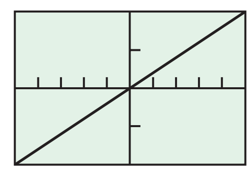
17. m 3, b 2 y 3x 2
18. m 1, b 2 y x 2
19. m 1 2, b 3
20. m 1 3, b 1 y $\frac{1}{3}x$ 1

In Exercises 21–24, write a general linear equation for the line through the two points.

21. (0, 0), (2, 3) 3x – 2y 0
22. (1, 1), (2, 1) y 1
23. (2, 0), (2, 2) x 2
24. (2, 1), (2, 2) 3x 4y 2

In Exercises 25 and 26, the line contains the origin and the point in the upper right corner of the grapher screen. Write an equation for the line.

25. 
[–10, 10] by [–25, 25]
y $\frac{5}{2}x$

26. 
[–5, 5] by [–2, 2]
y $\frac{2}{5}x$

In Exercises 27–30, find the **(a)** slope and **(b)** y-intercept, and **(c)** graph the line.

27. 3x 4y 12 **(a)** $\frac{3}{4}$ **(b)** 3
28. x y 2 **(a)** –1 **(b)** 2
29. $\frac{x}{3}$ $\frac{y}{4}$ 1 **(a)** $\frac{4}{3}$ **(b)** 4
30. y 2x 4 **(a)** 2 **(b)** 4

In Exercises 31–34, write an equation for the line through P that is **(a)** parallel to L, and **(b)** perpendicular to L.

31. P(0, 0), L: y –x 2
32. P(2, 2), L: 2x y 4
33. P(2, 4), L: x 5
34. P(1, 1 2), L: y 3

In Exercises 35 and 36, a table of values is given for the linear function $f(x)$ mx b. Determine m and b.

35.

x	$f(x)$
1	2
3	9
5	16

m $\frac{7}{2}$, b $\frac{3}{2}$

36.

x	$f(x)$
2	1
4	4
6	7

m $\frac{3}{2}$, b 2

13. y 1(x – 1) 1 14. y 1(x 1) 1
15. y 2(x – 0) 3 16. y 2(x 4) 0 19. y $\frac{1}{2}x$ 3

31. **(a)** y x **(b)** y x 32. **(a)** y 2x – 2 **(b)** y $\frac{1}{2}x$ 3
33. **(a)** x 2 **(b)** y 4 34. **(a)** y $\frac{1}{2}$ **(b)** x 1

In Exercises 37 and 38, find the value of x or y for which the line through A and B has the given slope m.

37. $A($ 2, 3), $B(4, y)$, m 2 3 y 1

38. $A($ 8, 2), $B(x, 2)$, m 2 x 6

39. ***Revisiting Example 4*** Show that you get the same equation in Example 4 if you use the point (3, 4) to write the equation.

40. **Writing to Learn** ***x- and y-intercepts***

(a) Explain why c and d are the x-intercept and y-intercept, respectively, of the line

$$\frac{x}{c} \quad \frac{y}{d} \quad 1.$$

(b) How are the x-intercept and y-intercept related to c and d in the line

$$\frac{x}{c} \quad \frac{y}{d} \quad 2?$$

41. ***Parallel and Perpendicular Lines*** For what value of k are the two lines $2x$ ky 3 and x y 1 **(a)** parallel? k 2 **(b)** perpendicular? k 2

Group Activity In Exercises 42–44, work in groups of two or three to solve the problem.

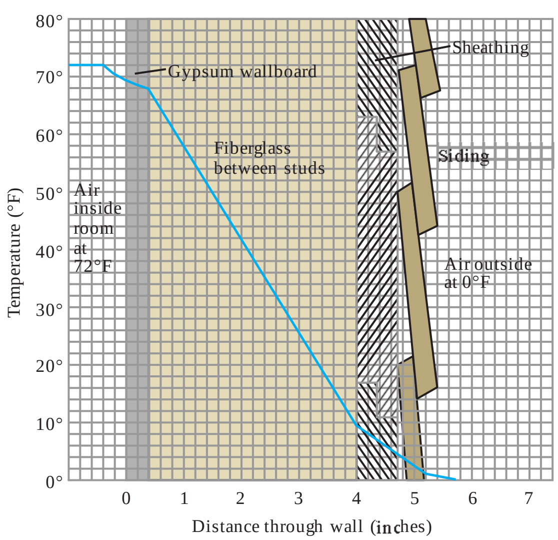
42. ***Insulation*** By measuring slopes in the figure below, find the temperature change in degrees per inch for the following materials.

(a) gypsum wallboard

(b) fiberglass insulation

(c) wood sheathing

(d) **Writing to Learn** Which of the materials in (a)–(c) is the best insulator? the poorest? Explain.



43. ***Pressure under Water*** The pressure p experienced by a diver under water is related to the diver's depth d by an equation of the form p kd 1(k a constant). When d 0 meters, the pressure is 1 atmosphere. The pressure at 100 meters is 10.94 atmospheres. Find the pressure at 50 meters. 5.97 atmospheres (k 0.0994)

44. ***Modeling Distance Traveled*** A car starts from point P at time t 0 and travels at 45 mph.

(a) Write an expression $d(t)$ for the distance the car travels from P. $d(t)$ $45t$

(b) Graph y $d(t)$.

(c) What is the slope of the graph in (b)? What does it have to do with the car? Slope is 45, which is the speed in miles per hour.

(d) **Writing to Learn** Create a scenario in which t could have negative values.

(e) **Writing to Learn** Create a scenario in which the y-intercept of y $d(t)$ could be 30.

In Exercises 45 and 46, use linear regression analysis.

45. Table 1.2 shows the mean annual compensation of construction workers.

Table 1.2 Construction Workers' Average Annual Compensation

Year	Annual Total Compensation (dollars)
1999	42,598
2000	44,764
2001	47,822
2002	48,966

Source: U.S. Bureau of the Census, *Statistical of the United States, 2004-2005.*

y 2216.2x 4387470.6

(a) Find the linear regression equation for the data.

(b) Find the slope of the regression line. What does the slope represent?

(c) Superimpose the graph of the linear regression equation on a scatter plot of the data.

(d) Use the regression equation to predict the construction workers' average annual compensation in the year 2008. about $62,659

46. Table 1.3 lists the ages and weights of nine girls.

Table 1.3 Girls' Ages and Weights

Age (months)	Weight (pounds)
19	22
21	23
24	25
27	28
29	31
31	28
34	32
38	34
43	39

y 0.680x 9.013

(a) Find the linear regression equation for the data.

(b) Find the slope of the regression line. What does the slope represent?

(c) Superimpose the graph of the linear regression equation on a scatter plot of the data.

(d) Use the regression equation to predict the approximate weight of a 30-month-old girl. 29 pounds

Standardized Test Questions

You should solve the following problems without using a graphing calculator.

47. True or False The slope of a vertical line is zero. Justify your answer. False. A vertical line has no slope.

48. True or False The slope of a line perpendicular to the line $y \quad mx \quad b$ is 1 m. Justify your answer. False. The slope is $1/m$.

49. Multiple Choice Which of the following is an equation of the line through (3, 4) with slope 1 2? A

(A) $y \quad 4 \quad \frac{1}{2}(x \quad 3)$ **(B)** $y \quad 3 \quad \frac{1}{2}(x \quad 4)$

(C) $y \quad 4 \quad 2(x \quad 3)$ **(D)** $y \quad 4 \quad 2(x \quad 3)$

(E) $y \quad 3 \quad 2(x \quad 4)$

50. Multiple Choice Which of the following is an equation of the vertical line through (–2, 4)? E

(A) $y \quad 4$ **(B)** $x \quad 2$ **(C)** $y \quad 4$

(D) $x \quad 0$ **(E)** $x \quad 2$

51. Multiple Choice Which of the following is the x-intercept of the line $y \quad 2x \quad 5$? D

(A) $x \quad 5$ **(B)** $x \quad 5$ **(C)** $x \quad 0$

(D) $x \quad 5 \; 2$ **(E)** $x \quad 5 \; 2$

52. Multiple Choice Which of the following is an equation of the line through (2, 1) parallel to the line $y \quad 3x \quad 1$? B

(A) $y \quad 3x \quad 5$ **(B)** $y \quad 3x \quad 7$ **(C)** $y \quad \frac{1}{3}x \quad \frac{1}{3}$

(D) $y \quad 3x \quad 1$ **(E)** $y \quad 3x \quad 4$

Extending the Ideas

53. The median price of existing single-family homes has increased consistently during the past few years. However, the data in Table 1.4 show that there have been differences in various parts of the country.

Table 1.4 Median Price of Single-Family Homes

Year	South (dollars)	West (dollars)
1999	145,900	173,700
2000	148,000	196,400
2001	155,400	213,600
2002	163,400	238,500
2003	168,100	260,900

Source: U.S. Bureau of the Census, *Statistical of the United States, 2004-2005.*

(a) Find the linear regression equation for home cost in the South.

(b) What does the slope of the regression line represent?

(c) Find the linear regression equation for home cost in the West.

(d) Where is the median price increasing more rapidly, in the South or the West?

54. *Fahrenheit versus Celsius* We found a relationship between Fahrenheit temperature and Celsius temperature in Example 8.

(a) Is there a temperature at which a Fahrenheit thermometer and a Celsius thermometer give the same reading? If so, what is it?

(b) Writing to Learn Graph $y_1 \quad (9 \; 5)x \quad 32$, y_2 $(5 \; 9)(x \quad 32)$, and $y_3 \quad x$ in the same viewing window. Explain how this figure is related to the question in part (a).

55. *Parallelogram* Three different parallelograms have vertices at (1, 1), (2, 0), and (2, 3). Draw the three and give the coordinates of the missing vertices.

56. *Parallelogram* Show that if the midpoints of consecutive sides of any quadrilateral are connected, the result is a parallelogram.

57. *Tangent Line* Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4).

58. Group Activity *Distance From a Point to a Line* This activity investigates how to find the distance from a point $P(a, b)$ to a line $L\colon Ax \quad By \quad C$.

(a) Write an equation for the line M through P perpendicular to L.

(b) Find the coordinates of the point Q in which M and L intersect.

(c) Find the distance from P to Q. Distance $\frac{Aa \quad Bb \quad C}{^{2}\; {}^{2}A \quad B}$

Answers

39. $y \quad 1(x - 3) \quad 4$
$y \quad x - 3 \quad 4$
$y \quad x \quad 1$, which is the same equation.

40. (a) When $y \quad 0$, $x \quad c$; when $x \quad 0$, $y \quad d$.
(b) The x-intercept is $2c$ and the y-intercept is $2d$.

42. (a) –3.75 degrees/inch **(b)** –16.1 degrees/inch
(c) –7.1 degrees/inch
(d) Best: fiberglass; poorest: gypsum wallboard. The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

44. (d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t \quad 0$.
(e) The car starts at time $t \quad 0$ at a point 30 miles past P.

53. (a) $y \quad 5980x \quad 11{,}810{,}220$
(b) The rate at which the median price is increasing in dollars per year.
(c) $y \quad 21650x \quad 43{,}105{,}030$
(d) South: \$5,980 per year, West: \$21,650 per year; more rapidly in the West

56. Suppose that the vertices of the original quadrilateral are (a, b), (c, d), (e, f), and (g, h). When the midpoints are connected, the pairs of opposite sides of the resulting figure have slopes $\frac{f \quad b}{e \quad a}$ or $\frac{h \quad d}{g \quad c}$, and opposite sides are parallel.

57. $y \quad \frac{3}{4}(x \quad 3) \quad 4$ or $y \quad \frac{3}{4}x \quad \frac{25}{4}$

58. (a) $y \quad \frac{B}{A}(x \quad a) \quad b$
(b) The coordinates are
$\frac{B^2a \quad AC \quad ABb}{^{2}\; {}^{2}A \quad B}, \frac{A^2b \quad BC \quad ABa}{^{2}\; {}^{2}A \quad B}$

1.2 Functions and Graphs

What you'll learn about

- Functions
- Domains and Ranges
- Viewing and Interpreting Graphs
- Even Functions and Odd Functions—Symmetry
- Functions Defined in Pieces
- Absolute Value Function
- Composite Functions

. . . and why

Functions and graphs form the basis for understanding mathematics and applications.

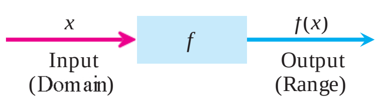


Figure 1.8 A "machine" diagram for a function.

Leonhard Euler

(1707–1783)

Leonhard Euler, the dominant mathematical figure of his century and the most prolific mathematician ever, was also an astronomer, physicist, botanist, and chemist, and an expert in oriental languages. His work was the first to give the function concept the prominence that it has in mathematics today. Euler's collected books and papers fill 72 volumes. This does not count his enormous correspondence to approximately 300 addresses. His introductory algebra text, written originally in German (Euler was Swiss), is still available in English translation.

Functions

The values of one variable often depend on the values for another:

- The temperature at which water boils depends on elevation (the boiling point drops as you go up).
- The amount by which your savings will grow in a year depends on the interest rate offered by the bank.
- The area of a circle depends on the circle's radius.

In each of these examples, the value of one variable quantity depends on the value of another. For example, the boiling temperature of water, b, depends on the elevation, e; the amount of interest, I, depends on the interest rate, r. We call b and I **dependent variables** because they are determined by the values of the variables e and r on which they depend. The variables e and r are **independent variables.**

A rule that assigns to each element in one set a unique element in another set is called a *function*. The sets may be sets of any kind and do not have to be the same. A function is like a machine that assigns a unique output to every allowable input. The inputs make up the **domain** of the function; the outputs make up the **range** (Figure 1.8).

DEFINITION Function

A **function** from a set D to a set R is a rule that assigns a unique element in R to each element in D.

In this definition, D is the domain of the function and R is a set *containing* the range (Figure 1.9).

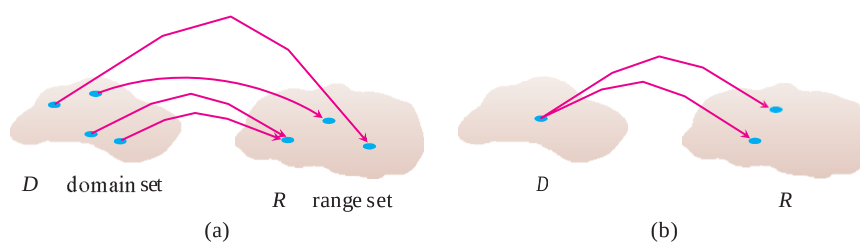


Figure 1.9 (a) A function from a set D to a set R. (b) *Not* a function. The assignment is not unique.

Euler invented a symbolic way to say "y is a function of x":

$$y \quad f(x),$$

which we read as "y equals f of x." This notation enables us to give different functions different names by changing the letters we use. To say that the boiling point of water is a function of elevation, we can write $b \quad f(e)$. To say that the area of a circle is a function of the circle's radius, we can write $A \quad A(r)$, giving the function the same name as the dependent variable.

The notation y $f(x)$ gives a way to denote specific values of a function. The value of f at a can be written as $f(a)$, read "f of a."

EXAMPLE 1 The Circle-Area Function

Write a formula that expresses the area of a circle as a function of its radius. Use the formula to find the area of a circle of radius 2 in.

SOLUTION

If the radius of the circle is r, then the area $A(r)$ of the circle can be expressed as $A(r)$ pr^2. The area of a circle of radius 2 can be found by evaluating the function $A(r)$ at r 2.

$$A(2) \quad p(2)^2 \quad 4p$$

The area of a circle of radius 2 is $4p$ in^2. *Now try Exercise 3.*

Domains and Ranges

In Example 1, the domain of the function is restricted by context: the independent variable is a radius and must be positive. When we define a function y $f(x)$ with a formula and the domain is not stated explicitly or restricted by context, the domain is assumed to be the largest set of x-values for which the formula gives real y-values—the so-called **natural domain.** If we want to restrict the domain, we must say so. The domain of y x^2 is understood to be the entire set of real numbers. We must write "y x^2, x 0" if we want to restrict the function to positive values of x.

The domains and ranges of many real-valued functions of a real variable are intervals or combinations of intervals. The intervals may be open, closed, or half-open (Figures 1.10 and 1.11) and finite or infinite (Figure 1.12).

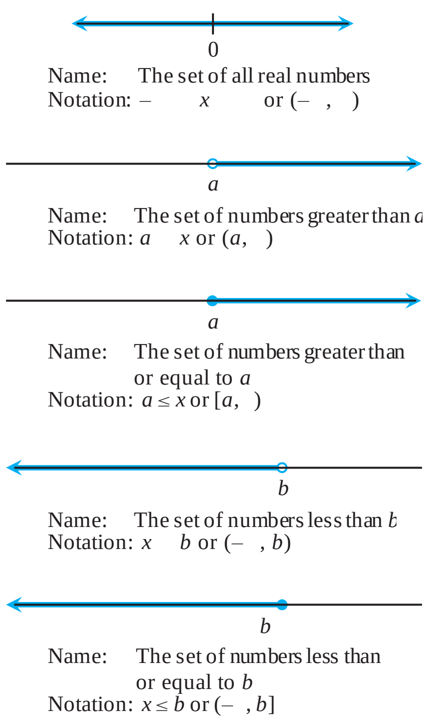


Figure 1.12 Infinite intervals—rays on the number line and the number line itself. The symbol (infinity) is used merely for convenience; it does not mean there is a number .

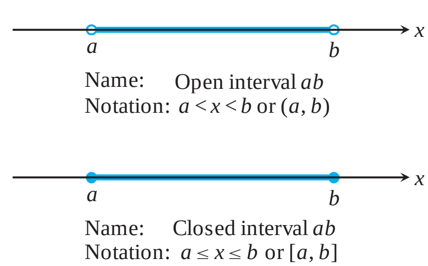


Figure 1.10 Open and closed finite intervals.

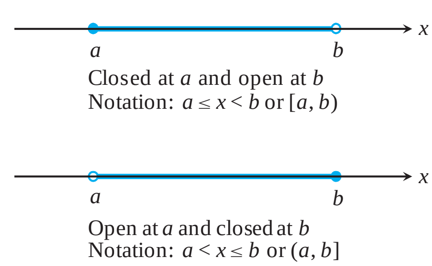


Figure 1.11 Half-open finite intervals.

The endpoints of an interval make up the interval's **boundary** and are called **boundary points.** The remaining points make up the interval's **interior** and are called **interior points.** **Closed intervals** contain their boundary points. **Open intervals** contain no boundary points. Every point of an open interval is an interior point of the interval.

Viewing and Interpreting Graphs

The points (x, y) in the plane whose coordinates are the input-output pairs of a function y $f(x)$ make up the function's **graph.** The graph of the function y x 2, for example, is the set of points with coordinates (x, y) for which y equals x 2.