

Section A : Pure Mathematics [40 marks]

- 1** Given that $f(x) = \frac{2x-1}{(x+2)(x-3)}$, ($x \in \mathbb{R}, x \neq -2, x \neq 3$), express $f(x)$ in partial fractions. [2]
- (i) Expand $f(x)$ as a series in ascending powers of x , up to and including the term in x^2 . [3]
- (ii) Find the set of values of x for which the expression is valid. [2]

- 2** Prove by induction that, for all positive integers n , $\sum_{r=1}^n \frac{2r}{2^r} = 4 - 2^{1-n}(n+2)$. [3]

Hence find $\sum_{r=n}^{2n} \frac{r+2^{2r}}{2^r}$, simplifying your answer in terms of n . [3]

- 3** The functions f and g are defined as follows:

$$\begin{aligned} f : x &\mapsto x^2 + x - 6, & x &\in \mathbb{R} \\ g : x &\mapsto 6x + 5, & x &\in \mathbb{R} \end{aligned}$$

- (i) Explain why the inverse function of f does not exist. [1]
- (ii) Suppose that the domain of f is restricted to $x < a$, where a is a constant. Find the largest value of a for which the inverse function of f exists. [1]

With the new domain of f found in (ii),

- (iii) Define f^{-1} in a similar form; [3]
- (iv) Show that the composite function gf exists; [1]
- (v) Define gf in a similar form and find its range. [3]

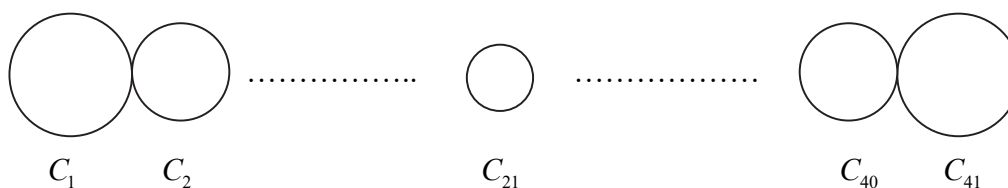
- 4(a)** Circles of different sizes are arranged in a row from the biggest to the smallest circle. Given that the diameter of the biggest circle is 200 cm and that the diameter of each

succeeding circle is $\frac{2}{5}$ that of the preceding circle.

(i) Find the first circle whose diameter is less than 2 cm. [2]

(ii) Consider an infinite number of such circles, find the total area of the circles, giving your answer correct to one decimal place. [3]

- (b) 41 circles, $C_1, C_2, \dots, C_{41}$, are arranged in a row such that the smallest circle, C_{21} , is in the middle and C_1 has the same diameter as C_{41} , C_2 has the same diameter as C_{40} , ... and C_{20} has the same diameter as C_{22} , as shown in the diagram below.



Given that the diameters of the first 21 circles, starting from the biggest circle whose diameter is 200 cm, are in arithmetic progression and the total diameter of the 41 circles is 7600 cm, find the diameter of the smallest circle in the arrangement. [4]

5(a) Given that $S_n = \frac{2^{n+1}}{n!} - 1$, find

(i) T_n , giving your answer in a single fraction. [2]

(ii) the exact value of $T_4 + T_5 + \dots + T_8$. [2]

(b) It is given that $S_n = \sum_{r=1}^n (-1)^{r+1} \left[\frac{a^r - 1}{(a-1)(a^2-1)\dots(a^r-1)} + \frac{1}{(a-1)(a^2-1)\dots(a^r-1)} \right]$.

Show that

$$S_n = 1 + \frac{(-1)^{n+1}}{(a-1)(a^2-1)\dots(a^n-1)}. \quad [3]$$

Hence find S_3 when $a = 2$. [2]

Section B : Statistics [60 marks]

- 6 At the SAJC Grad Night, there are 25 tables of 10 students seated at each table. The Student Council would like to conduct a survey and they carry out the following method to obtain a sample of 50.

1. For each table, every student is assigned a number from 1 to 10.
 2. Two distinct random numbers are generated from 1 to 10.
 3. From each table, the students corresponding to the 2 distinct numbers are selected for the survey.
- (i) State, giving a reason, whether this method would results in an unbiased sample. [1]
- (ii) The Student Council Teacher suggested another method: to wait outside the door and select the first student, and then every 5th student that walks through the door. State, giving a reason, whether this method would result in a random sample. [1]
- Name another method that would produce a random sample, and state what might be needed to carry out the sampling method. [2]

- 7 Find the number of ways in which the letters of the word **RESILIENCE** can be arranged
- (i) if there are no restrictions, [1]
- (ii) if all Es must be separated, [2]
- (iii) if between any two Es, there must be exactly two other letters and the letters are not Is, [3]
- (iv) in two rows of 5 letters each such that all Es are in one row and all Is are in the other row. [3]

- 8 The latest study indicates that $p\%$ of the total population in Singapore consists of females. The study also shows that 55% of female Singaporeans are employed, while 76% of male Singaporeans are employed. It is given that the probability of a randomly chosen Singaporean being employed is 0.6571.

By drawing a tree diagram, show that $p = 49$. [2]

Using this value of p , find the probability that

- (i) a randomly chosen Singaporean is a male and unemployed; [1]
- (ii) a randomly chosen unemployed Singaporean is a female. [2]

Two Singaporeans are chosen at random.

- (iii) Using the same value of p , find the probability that exactly one of them is a male and exactly one of them is employed. [3]

- 9 An agriculturist carries out a research looking into how the yield of eggplant is affected by the amount of fertilizer applied to eggplant plants. Ten eggplant plants of the same variety were selected at random and treated, weekly, with a solution in which x grams of fertilizer was dissolved in a fixed quantity of water. The yield, y kilograms, of eggplant was recorded. The results are given in the table.

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
y	2.0	4.3	5.7	6.5	6.9	7.0	7.2	7.6	7.8	7.9

- (i) Calculate the product moment correlation coefficient between x and y , and explain whether your answer suggests that a linear model is appropriate. [2]
- (ii) Draw a scatter diagram for the data and explain why it may be consistent with a model of the form $y = a + b \ln x$. [2]
- (iii) Calculate the product moment correlation coefficient between $\ln x$ and y and comment on the value. [2]
- (iv) Calculate least squares estimates of a and b for the model $y = a + b \ln x$. [2]

Use a regression line to give the best estimate that you can of the amount of fertilizer used on a plant of the same variety when the yield of eggplants is 5 kilograms. [2]

- 10** When Henry travels from his home in England to visit his relatives in India, his journey is completed in four stages. The times taken (in hours) for the stages have independent normal distributions as follows.

Time taken (hours)	Mean	Variance
Bus journey from his house to the airport in England, X	3.75	1.45
Waiting time at the airport, T	3.1	0.785
Flight from England to India, U	11	1.3
Car journey from the airport in India to his relatives' house, V	3.2	0.81

- (i) Find the probability that the duration of the flight is shorter than the total time for the other three stages. [2]
- (ii) Find the probability that, for 6 journeys from England to India, the mean waiting time at the airport is less than 4 hours. [2]
- (iii) The cost of the bus trip, in dollars, from his house to the airport in England, Y is related to the duration of the bus journey, X by the formula $Y = a + bX$, where a and b are constants. Given that $P(Y < 7.5) = P(Y > 13.5) = 0.10644$, show that $E(Y) = 10.5$ and $Var(Y) = 5.80$. Hence find the values of a and b . [6]

- 11(a)** An engineer has developed a new, rapid-charging battery. He claims that a fully discharged battery will take approximately one hour to be completely charged with standard home outlets. A random sample of 15 such batteries is tested and the mean charge time is found to be 62 minutes with a standard deviation of 5 minutes.

Test, at the 5% level of significance, whether the engineer has underestimated the mean charge time. [4]

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