Base Correlation Explained

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Since the advent of standardised single tranche CDOs on the liquid CDS indices of CDX and iTraxx, there has been a need for a commonly agreed method of quoting the implied correlation between the assets in the respective CDS index. Initially the market chose compound correlation as its quotation convention. More recently, base correlation has become more widely used in the market. In this paper we define, discuss and compare both conventions. We conclude that base correlation possesses a number of desirable properties that make it a more powerful measure of tranche implied correlation. However, we argue that base correlation does not constitute a proper model for correlation skew. [1]

INTRODUCTION

The advent of standard CDO tranches with standard CDS indices as the reference portfolio has greatly enhanced liquidity and transparency in the synthetic CDO market. We are now able to observe daily pricing on a range of tranches linked to US, European and Japanese investment grade and high yield CDS indices. An example of tranche pricing on a selection of these indices is shown in Figure 1.

Figure 1. Indicative pricing for the five standard tranches linked to the CDX Investment Grade NA Series 3 and iTraxx Europe Series 2 indices, for 13 October 2004.[2]

	CDX Investment Grade North America Series 3		iTraxx Europe Series 2	
Tranche	**Lower-Upper strike**	**Upfront / Running Spread (bp)**	**Lower-Upper Strike**	**Upfront / Running Spread (bp)**
Equity	0-3%	37.125% + 500	0-3%	24.25% + 500
Junior Mezzanine	3-7%	259.5	3-6%	137.5
Senior Mezzanine	7-10%	101.0	6-9%	47.5
Senior	10-15%	38.5	9-12%	34.5
Super Senior	15-30%	11.5	12-22%	15.5

Note: On 13 October 2004, the CDX IG NA 3 index traded at 53.5bp and the iTraxx Europe 2 traded at 37bp. Both have a maturity date of 20 March 2010.
Source: Lehman Brothers.

The price of a CDO tranche is a function of the default correlation between the assets in the reference portfolio. See O'Kane, Naldi *et al* [2003] for a discussion. An equity tranche investor can be shown to be long the default correlation between the credits in the underlying CDS index while a senior tranche investor is short this default correlation. Hence an equity tranche will increase in value and a senior tranche will fall in value if the default correlation of the underlying CDS index increases.

Before the advent of standard tranches, dealers looked to historical measures of default correlation. One widespread approach was to proxy the asset return correlation of latent variable models with the correlation of historical equity market returns. For a discussion of

[1] *We would like to thank Wenjun Ruan, Saurav Sen, Minh Trinh and Lutz Schloegl for discussions and comments.*
[2] *Note that the convention for quoting prices is different for equity tranches. From Table 1 we can see that an investor who goes long the credit risk of the 0-3% equity tranche receives an upfront payment of 37.125 percent plus a running annual spread of 500bp. An investor who buys the 3-6% tranche receives an annualised spread of 259.5 bp (paid in quarterly instalments).*

such models see O'Kane, Naldi *et al* [2003]. What has changed recently is that by observing the market prices of synthetic CDO tranches, we can begin to extract information about market-implied rather than historical default correlation.

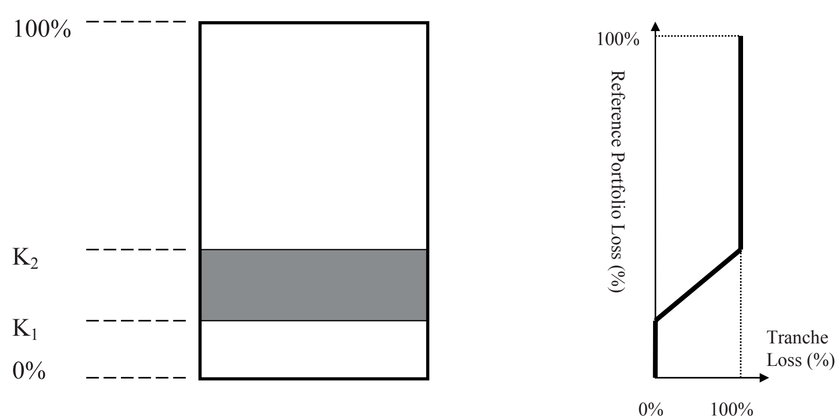
Initially, the market focused on compound correlation as the standard convention. More recently, dealers have begun to use base correlation instead. The main aim of this paper is to define, describe and compare these different measures of implied correlation. We begin with compound correlation.

COMPOUND CORRELATION

The first way to calculate an implied tranche correlation is to calculate the flat correlation that reprices each tranche to fit market prices. This method computes what is known as compound correlation.

We begin by defining a tranche simply in terms of its lower and upper "strikes", which we denote by K_1 and K_2. These are expressed as a percentage of the total notional of the reference portfolio. The lower strike is traditionally referred to as the tranche subordination or attachment point. The upper strike is referred to as the detachment point. The tranche loss is shown as a function of the percentage loss on the reference portfolio notional.

Figure 2. A mezzanine tranche with subordination (lower strike) K_1 and upper strike K_2



Source: Lehman Brothers.

To calculate compound correlation we have to assume a mathematical framework for linking the defaults of all of the different assets in the reference collateral. The standard way of doing this is to use the Large Homogeneous Portfolio (LHP) model. For a derivation see Appendix A. The main modelling assumptions are:

1. The reference portfolio is homogeneous so that all assets share the same pairwise correlation, default probability and recovery rate.
2. The number of assets in the reference portfolio tends to infinity (see discussion below).
3. The default dependency structure is based on a Gaussian copula model.
4. Each tranche is priced off a single flat correlation (the compound correlation of the tranche).

Assumption (1) means that we model the actual reference portfolio as a portfolio of homogeneous assets each with the average spread and recovery rate of the actual reference portfolio. For standard tranches, it means that we assume that the spread and recovery rate of the individual names in the portfolio are the same as the index. This has the advantage that we do not need to exchange information about the individual CDS spreads and recovery rates of each name in the reference portfolio.

Assumption (2) means that the portfolio is infinitely granular so that all idiosyncratic risk has been diversified away. This has the advantage that it enables us to write a simple analytical expression for the tranche price and makes the calculation of the implied correlation very fast.

Assumptions (1), (3) and (4) taken together mean that the dependency structure for each tranche is characterised by a single correlation number. We can therefore solve for the compound correlation from one observed price.

Calculating Compound Correlation

Given a tranche denoted by its lower and upper strikes K_1 and K_2, its present value today, time *t,* is given by:

$$PV_{tranche}\left(K_1, K_2, S_{K_1,K_2}, \rho_{K_1,K_2}\right) = U_{K_1,K_2} + S_{K_1,K_2}\sum_{n=1}^{N} Q_{K_1,K_2}(t_n)\Delta_n Z(t_n) - \sum_{m=1}^{M}\left(Q_{K_1,K_2}(t_{m-1}) - Q_{K_1,K_2}(t_m)\right)Z(t_m) \quad (1)$$

where

U_{K_1,K_2} is the tranche upfront payment,

S_{K_1,K_2} is the tranche contractual spread at issuance,

Δ_n is the accrual period between times t_{n-1} and t_n, usually paid quarterly, Actual 360,

$Z(t)$ is the LIBOR discount factor to time t.

The third term of equation (1) is the present value of the protection leg. Calculation of this involves an integration over time, which is usually discretised to quarterly time steps.

We define the tranche "survival probability" as follows:

$$Q_{K_1,K_2}(t) = 1 - \frac{\mathbf{E}^{LHP}_{\rho(K_1,K_2)}[Min(L(t),K_2)] - \mathbf{E}^{LHP}_{\rho(K_1,K_2)}[Min(L(t),K_1)]}{K_2 - K_1} \quad (2)$$

This survival probability is a measure of the expected percentage notional of the tranche remaining at some time t. The expectation is done using the Gaussian copula LHP model assuming a flat correlation as follows:

$$\mathbf{E}^{LHP}_{\rho}[Min(L(T),K)] = K\Phi(A) + (1-R)N\Phi_{2,-\sqrt{\rho}}(C,-A) \quad (3)$$

where

N is the portfolio notional,

R is the average recovery rate of the reference portfolio,

$C = \Phi^{-1}(p(t))$ is the default threshold for the underlying reference portfolio,

$p(t)$ is the average cumulative default probability to time t of the issuers in the underlying reference portfolio ,

$\Phi(x)$ is the cumulative normal function,

$$A = \frac{1}{\sqrt{\rho}}\left(C - \sqrt{1-\rho}.\Phi^{-1}\left(\frac{K}{N(1-R)}\right)\right)$$

ρ is the average pairwise asset correlation of the issuers in the reference portfolio,

$\Phi_{2,\rho}(x, y)$ is the cumulative bivariate normal with correlation coefficient ρ .

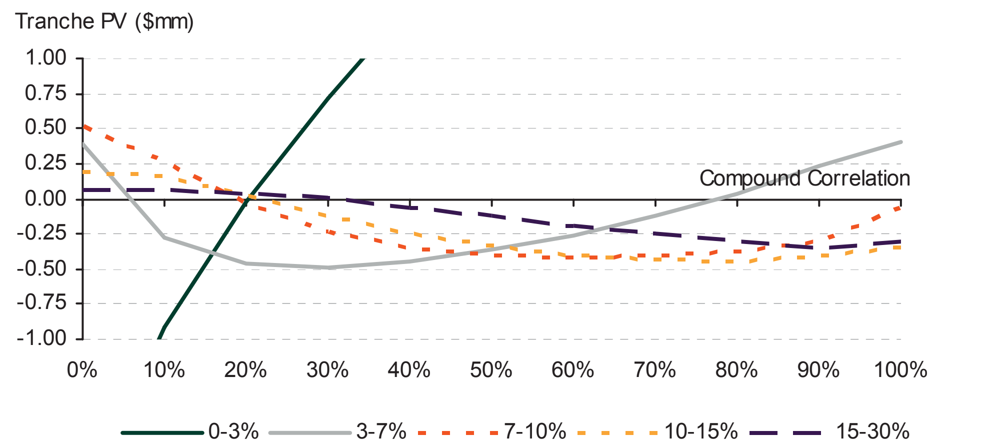
To calculate the compound correlation of a market tranche, we set the contractual spread equal to the observed market quote and, by definition, the present value of the tranche should be equal to zero:

$$PV_{tranche}\left(K_1, K_2, S_{K_1,K_2}, \rho_{K_1,K_2}\right) = 0$$

and we solve for $\rho = \rho_{K_1,K_2}$.

Solving this equation is straightforward, requiring a simple one-dimensional root searching algorithm. This works fine in almost all cases. However, there is sometimes a problem in that either we cannot find a root or that we get two roots. Why this is the case is shown in Figure 3 where we have plotted the present value of the five CDX tranches as a function of the compound correlation.

Figure 3. The present value of the five standard CDX tranches with different compound correlations – from the perspective of a protection seller (investor)



Source: Lehman Brothers.

As expected, we see in Figure 3 that the equity tranche investor is long compound correlation while the senior tranche investor is short compound correlation.

We see that for all tranches, there is a single solution at which the PV is zero, except the 3-7% mezzanine which has two solutions at 5% and 78% compound correlation.

For mezzanine investors, the relationship between changes in the tranche PV and changes in correlation itself changes with correlation. At low correlations, mezzanine tranche investors are short correlation while at high correlations, mezzanine tranche investors are long correlation. Clearly, the two compound correlation solutions of 5% and 78%, while producing the same tranche PV, imply radically different risk profiles. Typically, we choose the lower correlation as it is closer to the other compound correlation solutions for adjacent tranches and because it better fits historical observations of equity return correlation which are widely used as a proxy for asset return correlation. See O'Kane, Naldi *et al* [2003] for a discussion.

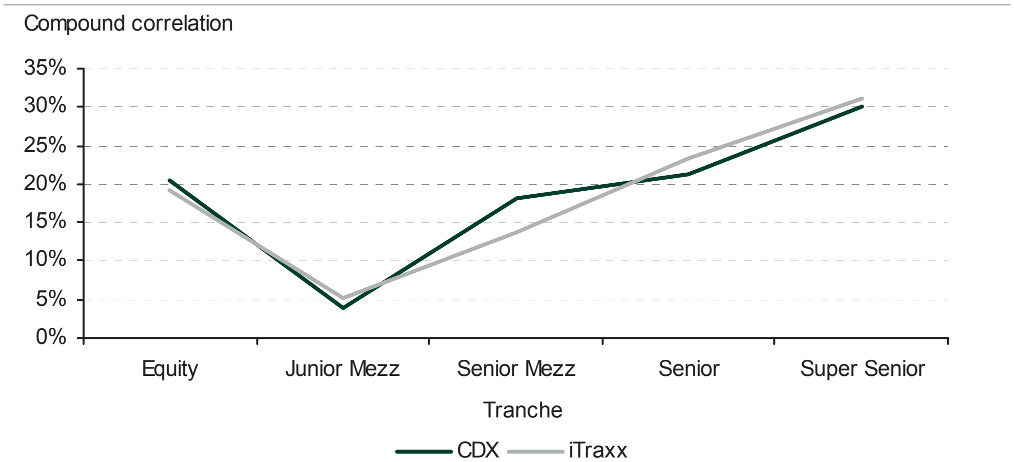
If we keep the reference portfolio spreads and recovery rates fixed and increase observed contractual tranche spreads, all of the curves in Figure 3 are shifted upwards. This causes the compound correlation of the equity tranche to decrease and the compound correlation of the other tranches to increase. If the tranche spreads are sufficiently large, there may not be a solution for the compound correlation of the mezzanine tranche. Equally, if tranche spreads fall, all of the curves in Figure 3 are shifted downwards. This can cause the mezzanine tranche to lose one, and ultimately both, of its solutions for compound correlation.

Explaining the Smile

The compound correlation curve is shown in Figure 4 for both the CDX and iTraxx tranches. The shape of the compound correlation has become known as the correlation "smile". This is because the compound correlation is higher for the equity and senior tranches than it is for the mezzanine tranches.

What is interesting is that this smile shape is common to both CDX and iTraxx tranches and has persisted through the period of the last year during which these tranche prices have been available. Although the similarity in the actual values and the shapes is apparent, care must be taken when comparing CDX and iTraxx since the standard tranches have different attachment points and widths.

Figure 4. The compound correlation curve for CDX Investment Grade NA Series 3 and iTraxx Europe Series 2 indices, for 13 October 2004



Source: Lehman Brothers.

Compound correlation is clearly not the same for all tranches. This simply tells us that a Gaussian copula does not capture the dependency structure implied by market CDO tranche prices. This is not a surprise – indeed, it would be amazing if we could exactly fit the market-

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