Suspensions

8.1 Introduction

The suspension system comprises the interface between the vehicle body frame and the road surface. (This statement assumes that the wheels and tires comprise part of the suspension system, which they indeed do.) Most people consider that the sole function of the suspension is to provide a comfortable ride. Although this is true, the system can be said to have three primary functions:

1． Isolate passengers and cargo from vibration and shock. It is desirable to make the Passengers as comfortable as possible; thus, the suspension system must be able to absorb shocks and dampen vibration caused by irregularities in the road surface.

2． Improve mobility. The suspension provides clearance between the road and the bottom of the vehicle. It also provides lateral and longitudinal stability and resists chassis roll.

3. Provide for vehicle control. The suspension reacts to tire forces including acceleration, braking, and steering and forces. Furthermore, the suspension system is tasked to maintain the proper steer and camber angles relative to the road surface, as well as to keep all four tires in contact with the road while maneuvering

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The analysis of vehicle suspensions and their dynamic response is an extremely complicated task. Because this is an introductory text, many simplifying assumptions will be made in the analysis of suspension systems. Although the models introduced are simple, they nevertheless illustrate several important characteristics and design requirements for suspensions.

This chapter will begin with simplified vibration analysis in both one and two degrees of freedom. Next, the primary components of the suspension will be discussed, with representative examples of current suspension systems. The chapter concludes with a discussion of the effect of suspension design on the dynamics of the vehicle.

8.2 Perception of Ride

Passenger opinion regarding what constitutes good ride quality obviously is extremely subjective. What one person considers the optimum ride may be completely unacceptable to another. The person who prefers sports cars will be appalled by the handling of a large luxury vehicle, whereas the owner of the luxury vehicle will be quite dissatisfied with the ride of a sports car.

Other factors come into play when people evaluate the ride quality of a vehicle. Certainly, the acoustic quality is a factor, and although not a direct result of the suspension，people object to noises, rattles, and squeaks in their vehicles. The of the seats is another important consideration and has an impact on the level of force or vibration transmitted to the occupant's body. The climate control system, while not at all influenced by the suspension design, influences perception of ride, too. If a person is uncomfortable because of the interior temperature, his or her subjective evaluation of ride quality will be affected. Thus, one of the challenges facing the suspension engineer is to take highly subjective evaluations and convert them into numerical standards.

However, the rate of change of acceleration, or the jerk, can produce discomfort. A parachutist feels discomfort at the moment of opening his or her parachute due to the shock, although it is usually mitigated by a profound sense of relief as the parachute inflates. But the jerk is not the only element that produces discomfort. The frequency of acceleration and its direction influence comfort. A car that pitches drastically when encountering a bump is seen as less comfortable than one that bounces in a more flat attitude, even if both motions continue for similar amounts of time.

A substantial body of literature is devoted to quantifying ride quality and human perception of ride. Studies and data have been collected by bodies such as the Society of Automotive Engineers (SAE) and the International Standards Organization (ISO), as well as by individual researchers. Gillespie (1994) provides a succinct overview of the literature. Although the sources are numerous, Gillespie concludes that there are no accepted standards for judging ride quality due to variables such as seat position, single versus multiple frequency inputs, multi-direction input, duration of exposure, and audible or ocular inputs.

The bottom line is that all of the research and comfort curves are a starting point for the suspension engineer. There is no substitute for the subjective evaluation provided by a road test. We could conclude that the suspension engineer should eliminate all vibration from the car, but this tends to be an infinite problem. As surely as one vibration is removed, the occupants become aware of another, more subtle vibration. As a result, suspension engineers appear to have solid job security for the foreseeable future.

8.3 Basic Vibrational Analysis

8.3.1 Single-Degree-of-Freedom Model (Quarter Car Model)

A vehicle consists of a multiple spring-mass-damper system that in reality has six degrees of freedom. Although the effective transverse and longitudinal stiffnesses of the suspension are much greater than the vertical stiffness, lateral and transverse compliance cannot generally be disregarded and may have a large impact on the vehicle dynamics. As a first simplifying assumption, the vehicle and suspension can be modeled in two dimensions, as shown in Fig. 8.1. The main mass consists of the vehicle itself and is comprised of all components that are supported by the springs. Hence, it is known as the sprung mass. Several components, such as axles, hubs, possibly the differential, and so forth, are not supported by the springs and are called the unsprung mass. The tires, being made of rubber, have inherent stiffness and damping and hence are modeled as a separate spring-damper system.

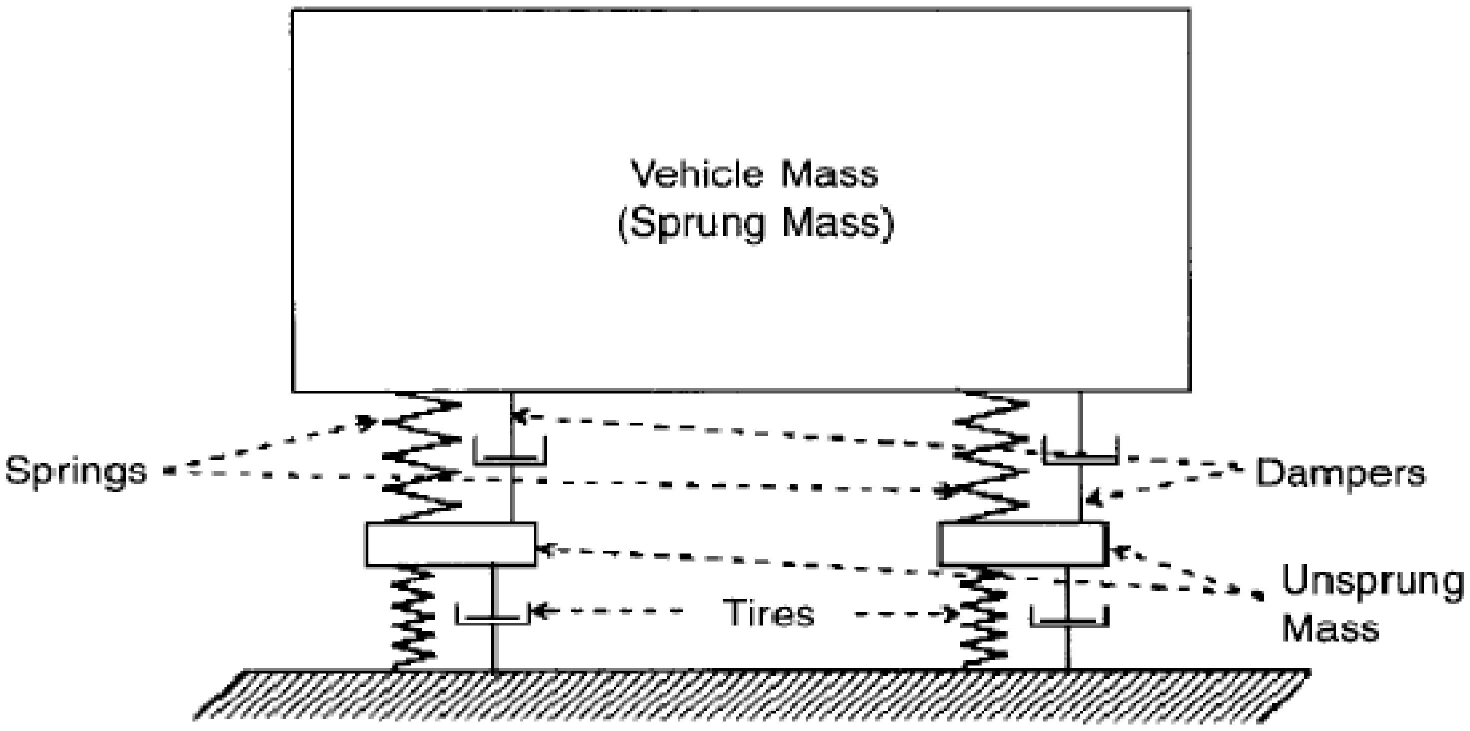

Figure 8.1. *Schematic representation of a vehicle as a spring-mass system.*

The primary motion of the vehicle mass is in the vertical direction. However, because of the separate springs and dampers at the front and rear, rotational motion usually results. At this point, it is instructive to examine a simple one-degree-of-freedom system to outline the basics of suspension analysis. In this case, it will be assumed that the tire stiffness is infinite, and the undamped motion of one spring will be examined. Figure 8.2 shows this model.

The equation of motion for the system can be obtained by applying Newton's Law and, in the case of unforced (free) vibration, is

$$\ddot{x} + \frac{k}{m} x = 0 \qquad (8.1)$$

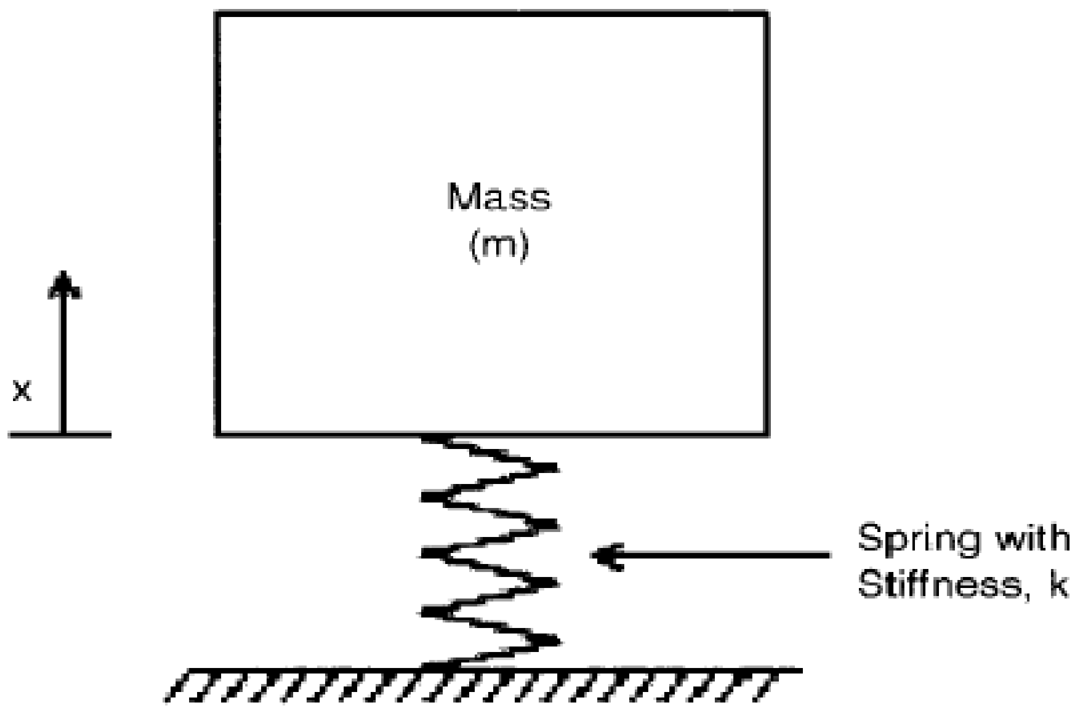

Figure 8.2. *Simple spring-mass system.*

The general solution for this linear differential equation is

$$x = A\cos(w_n t) + B\sin(w_n t) \quad (8.2)$$

Where A and B are constants that depend on initial conditions, and w_n is the natural frequency of the system and is defined as

$$w_n = \sqrt{\frac{k}{m}} \ (rad/sec) \quad (8.3)$$

Or

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \ Hz \quad (8.4)$$

Of course, this system, once disturbed from its datum position, would continue to oscillate at its natural frequency indefinitely. Although all real springs have some internal damping, a vehicle requires a more positive source of damping. Thus, the vehicle contains dampers. (In the United States, dampers are known as shock absorbers, although the name is misleading.)Figure 8.3 shows such a model.

Most automotive dampers can be modeled with sufficient accuracy by assuming they are viscous dampers. In other words, the damping force is proportional to the velocity of the displacements, or

$$F_d = c\dot{x} \quad (8.5)$$

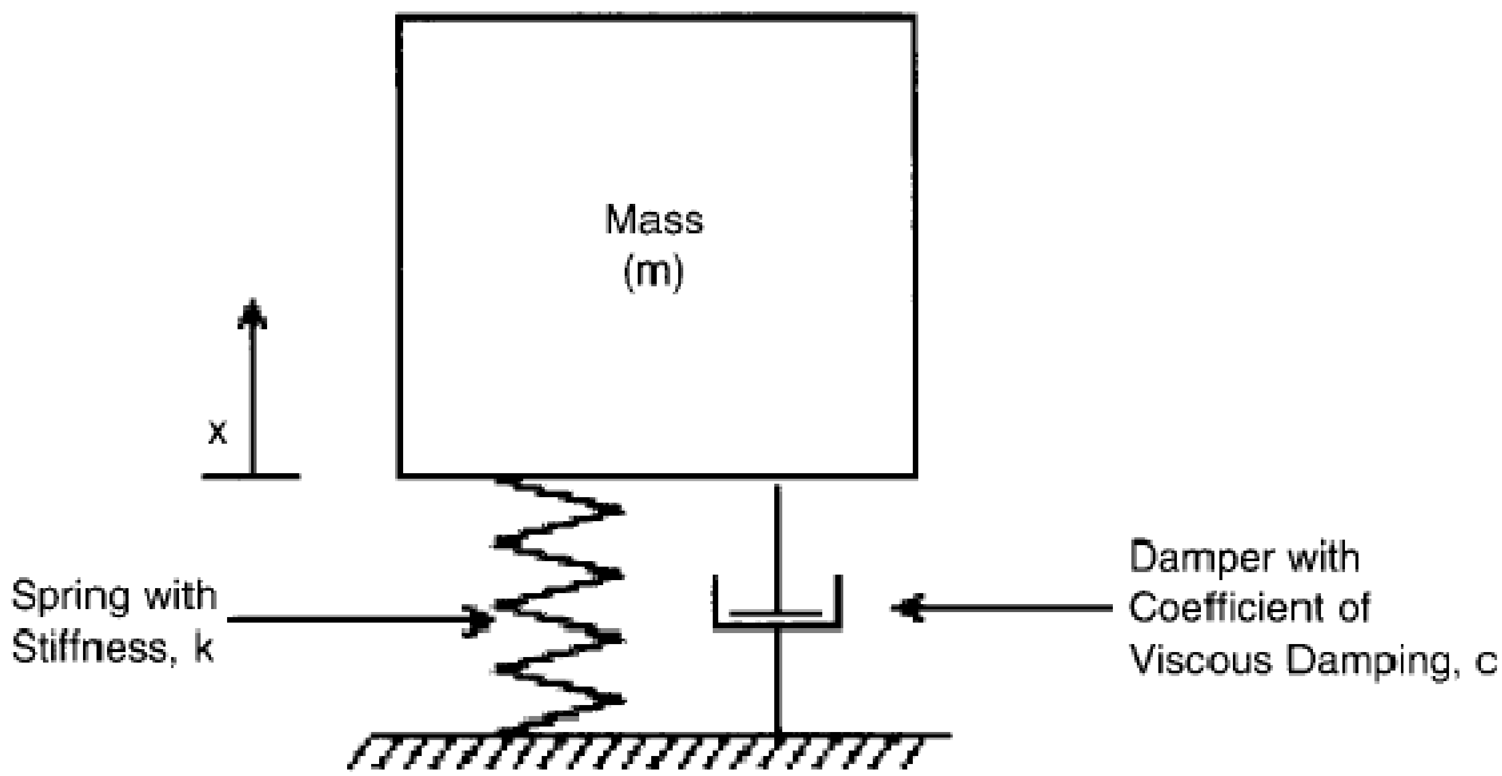

Figure 8.3 . Simple spring-mass-damper system.

where c is the coefficient of viscous damping. In this case, the unforced (homogenous) equation of motion becomes

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0 \qquad (8.6)$$

and the general solution to Eq. 8.6 is

$$x = e^{-(c/2m)t}\left(Ae^{\sqrt{(c/2m)^2 - k/m}\,t} + Be^{-\sqrt{(c/2m)^2 - k/m}\,t}\right) \qquad (8.7)$$

The first term in Eq. 8.7 is an exponentially decaying function of time. The terms in parentheses are dependent on whether the term under the radical is greater than, less than, or equal to zero. If the damping term $(c/2m)^2$ is greater than k/m, the terms in the radical are real numbers, and no oscillation is possible. If the damping term is less than k/m, the exponent becomes an imaginary number indicating oscillatory motion. The limiting case is when the damping term equals k/m. This case is known as critical damping, and the value of the critical damping coefficient is then

$$c_c = 2m\sqrt{\frac{k}{m}} = 2mw_n \qquad (8.8)$$

Now, any damping condition can be expressed in relation to the critical damping. Thus, the damping ratio, ζ is defined as

$$\zeta = \frac{c}{c_c} \qquad (8.9)$$

A vehicle is normally underdamped (ζ < 1.0); thus, Eq. 8.7 can be written as

$$x = Xe^{-\zeta\omega_n t}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \phi\right) \qquad (8.10)$$

where X and ϕ are arbitrary constants determined from the initial conditions. This equation indicates that the damped frequency of oscillation is modified in the presence of damping, and the damped natural frequency is given by

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n \qquad (8.11)$$

The response to excitation is an exponentially decreasing sine wave, as depicted in Fig. 8.4. In the case of forced vibration, the amplitude of the displacement is dependent on both the damping ratio and the excitation frequency. If harmonic forcing is assumed, Eq. 8.6 becomes

$$m\ddot{x} + c\dot{x} + kx = F_0\sin(\omega t) \qquad (8.12)$$

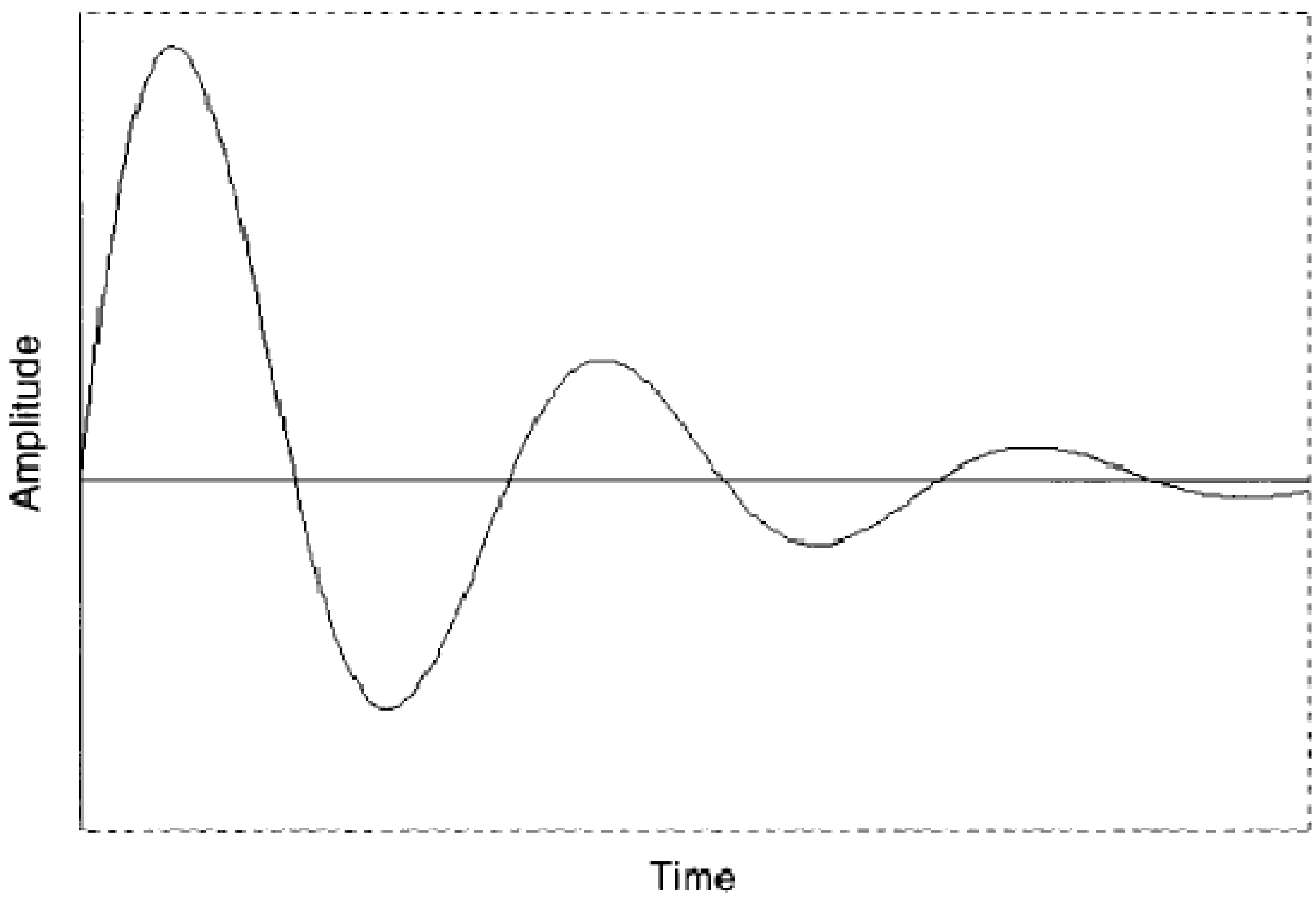

Figure 8.4. Underdamped oscillation, $\zeta < 1.0$.

where ω is the excitation frequency. The solution to Eq. 8.12 consists of a complementary function, which is the solution to the homogenous equation, and a particular solution (Thomson,1988). The particular solution reduces to a steady-state oscillation at the excitation frequency, ω. The displacement can be nondimensionalized with respect to the static displacement (Folk), so that

$$\frac{Xk}{F_0} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2}} \qquad (8.13)$$

Figure 8.5 shows a plot of Eq. 8.13 and illustrates the effect of damping on the nondimensional amplitude. As expected, displacements are largest when the damping is light and the system is excited near (or at) its natural frequency

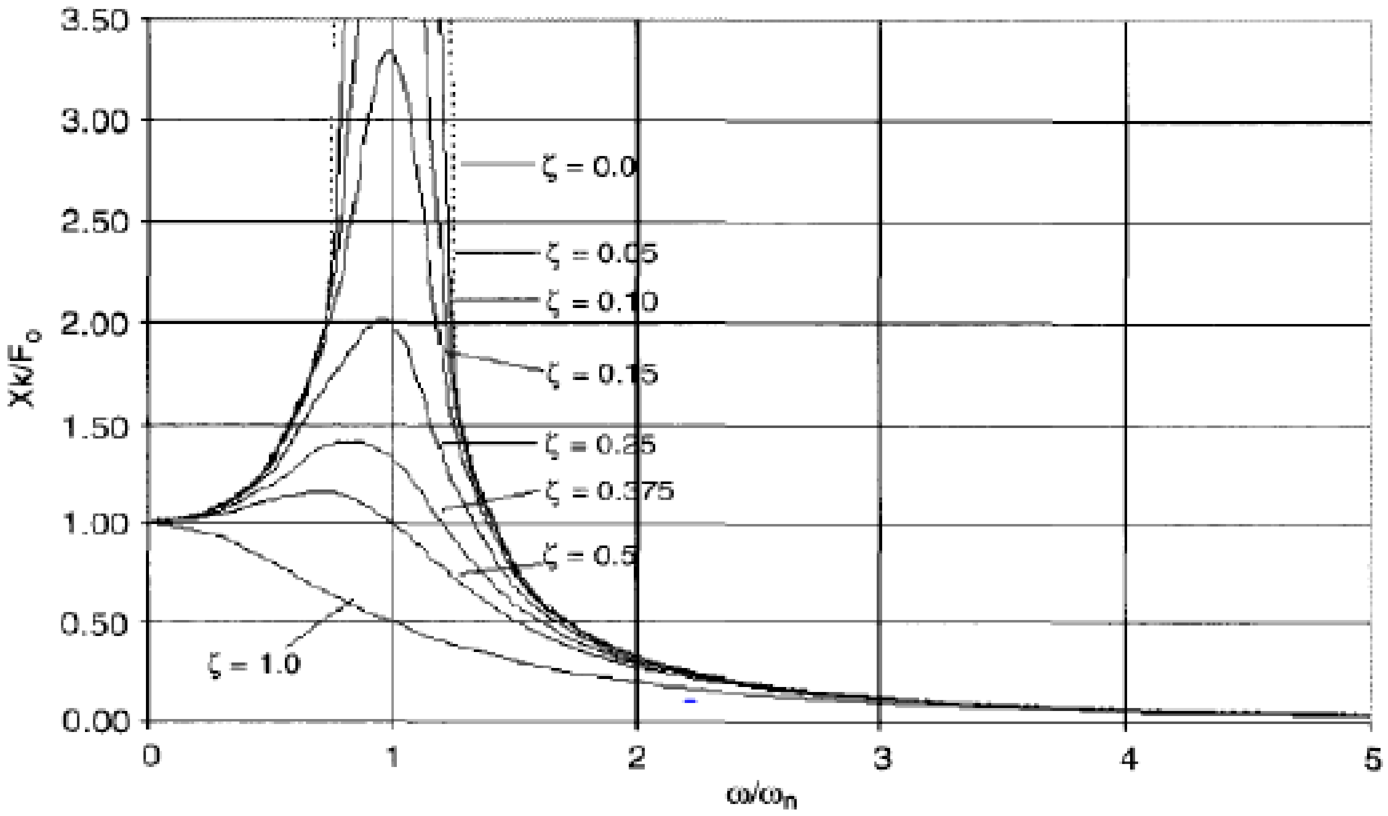

Figure 8.5. Plot of Eq. 8.13.

8.3.2 Two-Degrees-of-Freedom Model (Quarter Car Model)

In a vehicle, the excitation of the vehicle spring-mass system is provided by the motion of the tire/ unsprung mass and can be analyzed by the same techniques used to analyze support motion. The details of such an analysis are contained in vibration texts (Thomson, 1988), and only the highlights will be discussed here. Referring to Fig. 8.1, and isolating one spring-mass-damper system, the displacement of the vehicle body will be defined by x, whereas the displacement of the unsprung mass will be designated as y, as shown in Fig. 8.6.

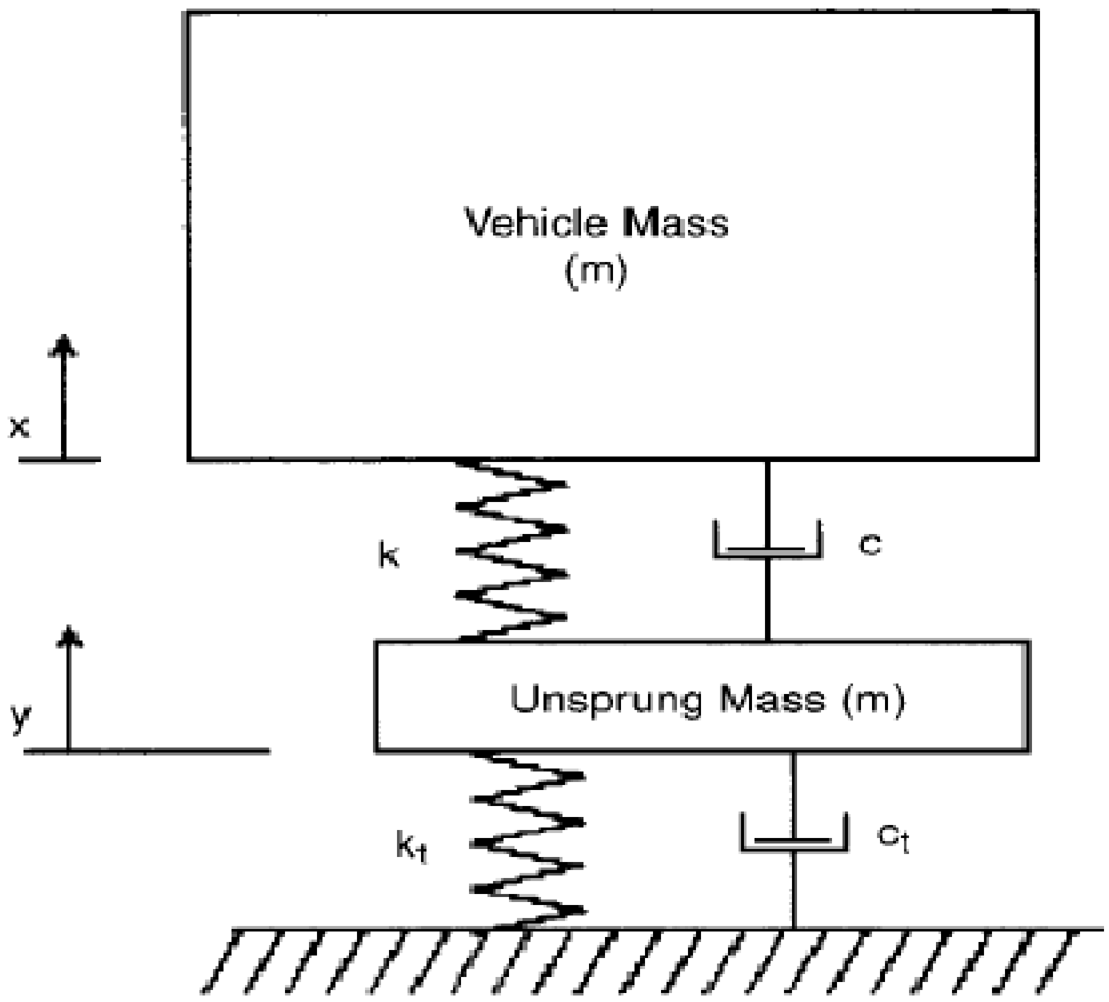

Figure 8.6. Vehicle excited by the motion of the unsprung mass.

The equation of motion for the vehicle mass is now

$$M\ddot{x} = -k(x-y) - c(\dot{x}-\dot{y}) \qquad (8.14)$$

Letting z = x - y, Eq. 8.14 can be written as

$$M\ddot{z} + c\dot{z} + kz = -m\ddot{y} \qquad (8.15)$$

At this point, it is most illustrative to assume that the motion of the unsprung mass is harmonic, which is not a bad assumption given that the tire and unsprung mass constitute a damped vibratory system. Before proceeding, the concept of transmissibility must be introduced. Transmissibility is defined as the ratio of the transmitted force to the ratio of the exciting force. Because in this case the exciting force is provided by the unsprung mass and tire, and as such is proportional to the displacement of the unsprung mass, the transmissibility is given by (Thomson, 1988)、

$$TR = \left|\frac{F_t}{F_0}\right| = \left|\frac{X}{Y}\right| = \sqrt{\frac{1+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}} \qquad (8.16)$$

where

ω = natural frequency of the unsprung mass system

ω_n = natural frequency of the vehicle mass system

Figure 8.7 shows a plot of Eq. 8.16

As mentioned in the introduction to this chapter, one of the functions of the suspension is to isolate passengers and cargo from vibration and shock. As shown in Fig. 8. 7, as long as the frequency ratio $\left(\frac{\omega}{\omega_n}\right)$ is above $\sqrt{2}$, any displacement of the vehicle mass will be less than that of the unsprung mass. In fact, the natural frequency of the unsprung mass system should be much greater than that of the vehicle mass/suspension system. There are, of course, two ways to do this, recalling that the natural frequency of a spring-mass system is given by Eq. 8.3. First, ensure that the stiffness of the tire is higher than that of the suspension springs. This usually is not an issue. The second way, and one of great importance to the vehicle designer, is to keep the unsprung mass as small as possible. As will be shown later, some suspension systems have a significant sprung

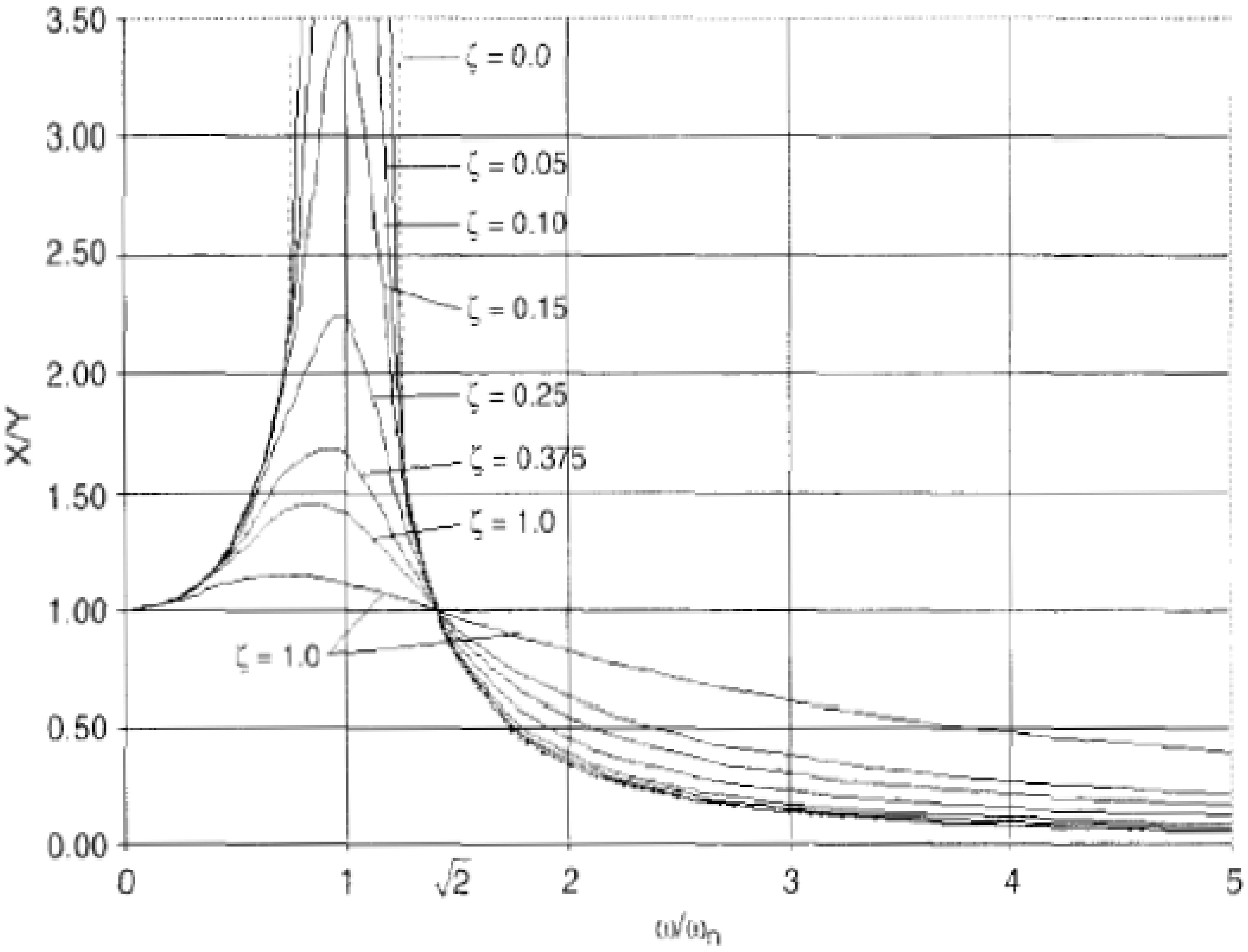

Figure 8.7. Plot of transmissibility Eq8. 16

8.4 Suspension System Components

The primary components in the suspension system are the springs and the dampers (or shocks). Although there are only two primary components, there are several variations on the theme, and these will be discussed in the following sections.

8.4.1 Springs

The spring is the main component of the suspension system, and four types are primarily in use today: (1) leaf springs, (2) torsion bars, （3）coil springs, and (4) pneumatic (air) springs.

8.4.1.1 Leaf Springs

Figure 8.8 shows a typical leaf spring. Most early cars used this type of spring because leaf springs were used extensively on horse-drawn carriages, and early designers had some experience with them. The leaf spring shown in Fig. 8.8 is a multi-leaf type. This type of spring is made of a single elliptical spring with several smaller leaves attached to it with clamps. The leaves also are fixed rigidly by the center bolt, which prevents individual leaves from moving off-center during deflection. The additional leaves make the spring stiffer, allowing it to support greater loads. Furthermore, as the spring deflects, friction is generated between the leaves, resulting in some damping capability. Leaf springs also provide fore-and-aft location, as well as some lateral location, for the axle. Although leaf springs are simple and cheap, they tend to be heavy. Leaf springs also weaken with age and are susceptible to sag

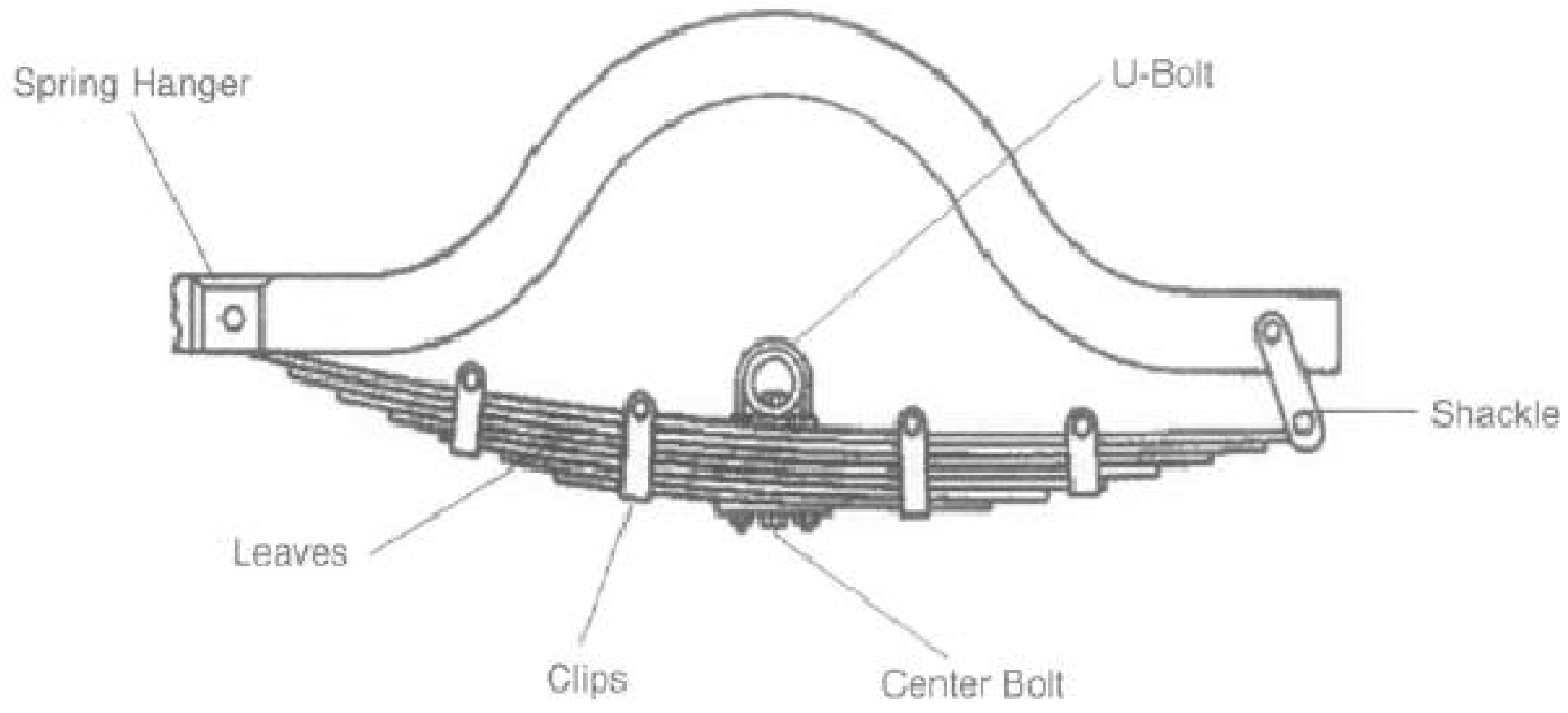

Figure 8.8 . *A multi-leaf spring. Adapted from TM 9-8000 (1985).*

8.4.1.2 Torsion Bars

frame, and loading is pure shear due to torsion. Figure 8.18 shows an example of a torsion bar. The torsion bar has very little inherent damping and therefore must be used in conjunction with dampers. As long as the bar remains in the elastic region, torque resistance will return the bar to its normal position upon unloading. The primary disadvantage of torsion bars is the axial space required for installation.

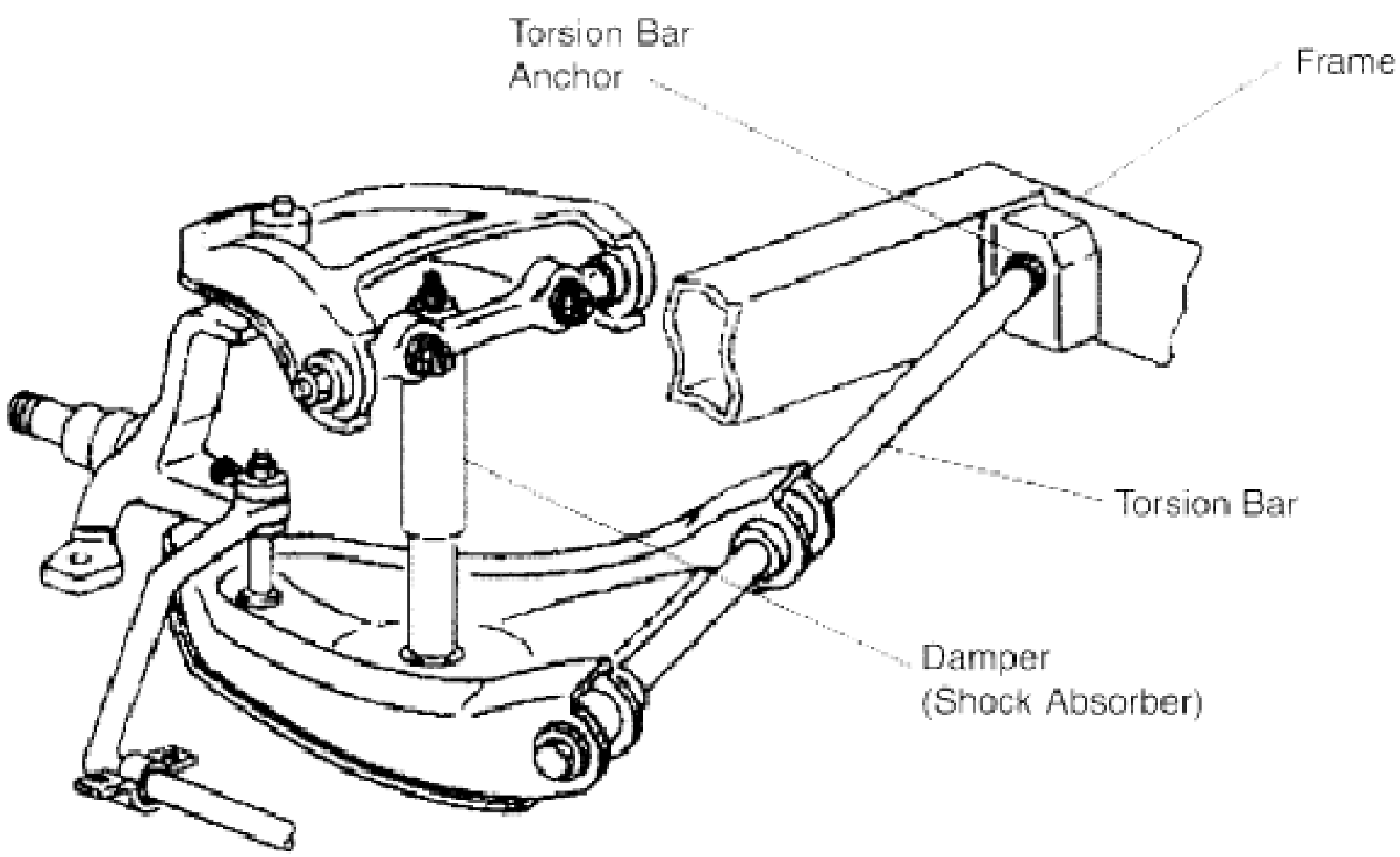

Figure 8.9 a torsion bar suspension

example of a coil spring suspension. Similar to torsion bars, coil springs have little to no inherent damping and require the use of dampers. Coil springs are used widely in automotive applications due to their compact size. However, coil springs are not capable of providing any location of the axle; thus, they require control arms to limit longitudinal and lateral suspension motion.

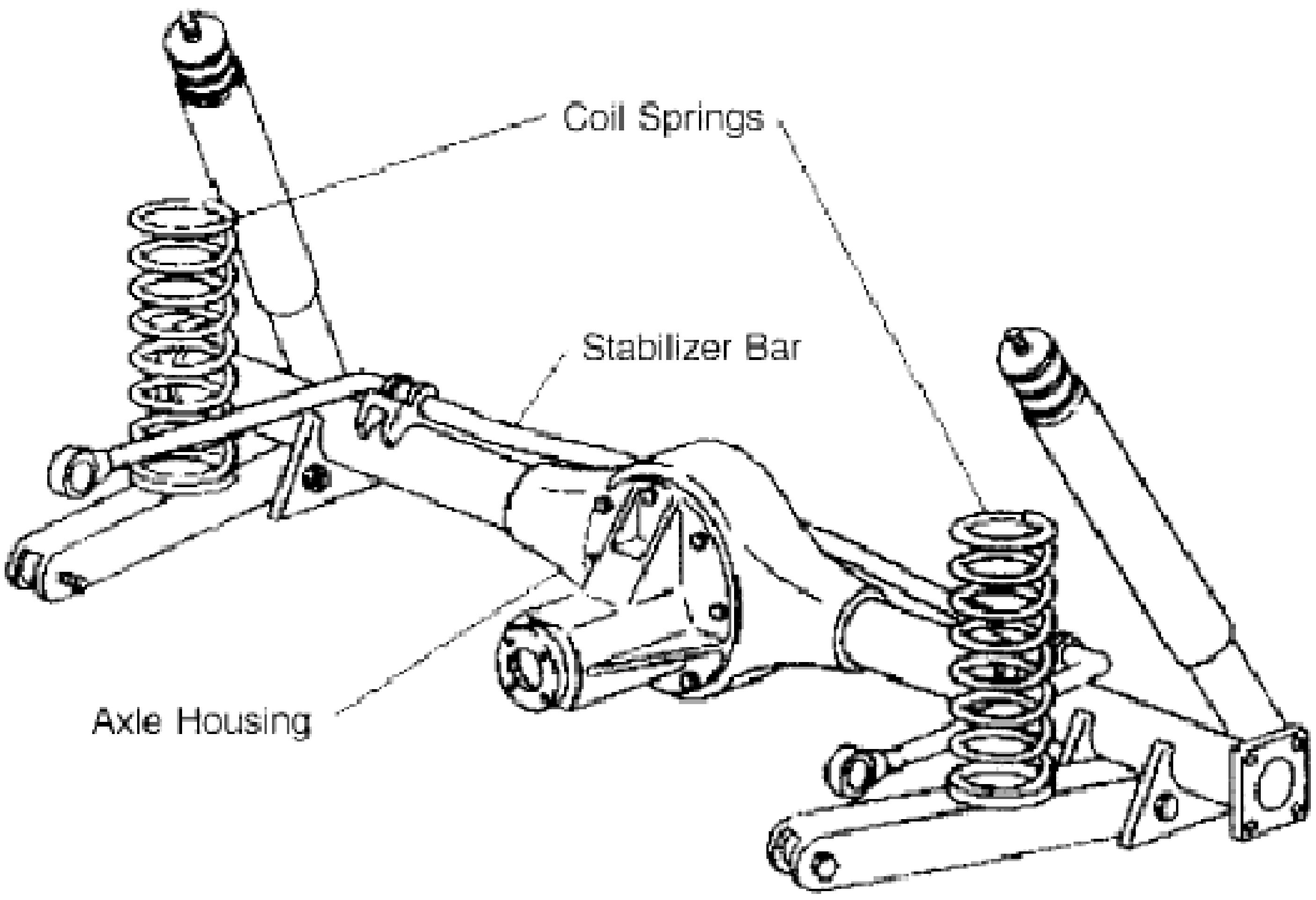

Figure 8.10 *A coil spring suspension. Adapted from TM 9-8000 (1985).*

Before analyzing coil springs, several terms must be defined. These terms are (reference Fig. 8.11):

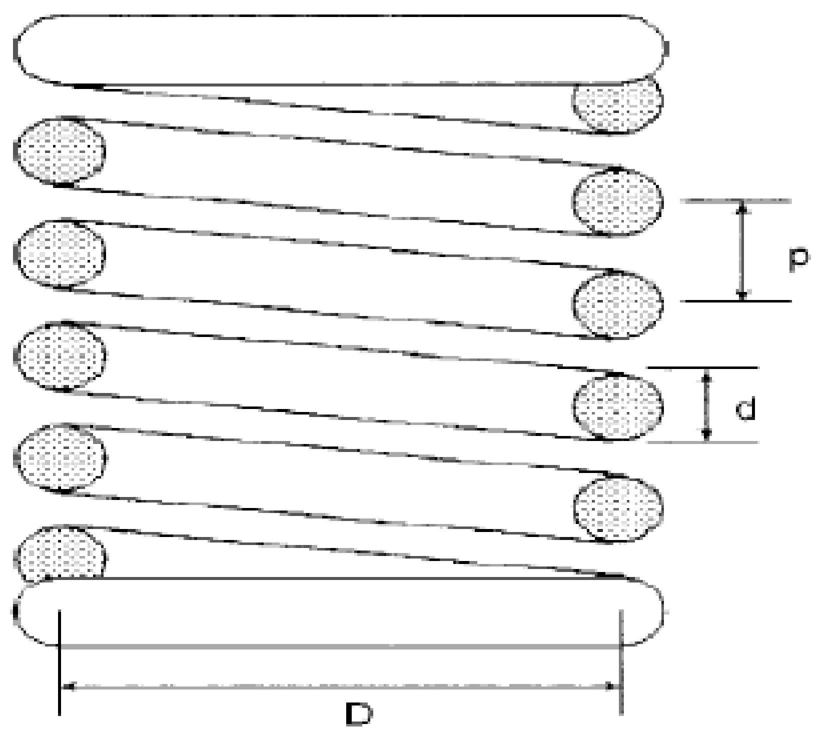

Figure 8. 11 *Coil spring dimensions.*

1、Mean coil diameter, D: The center-to-center distance of the wire across the coil
2、 Wire diameter, d