Exponential Growth and Decay

Section 6.7

Problem:

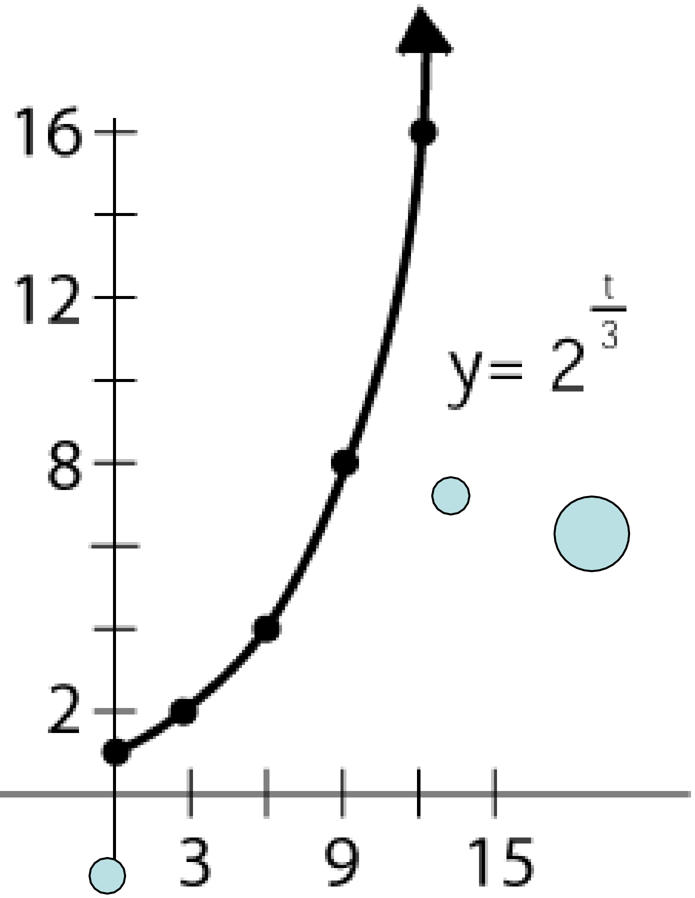
A single bacterium is in a Petri dish.

Every 3 seconds the bacteria doubles.

Find the relationship between *t,* the number of seconds, and *N*(*t*), the number of bacteria.

t (seconds)	*N*(*t*) (# of bacteria)
0	$1 = 1 * 2^0$
3	$2 = 1 * 2^1$
6	$4 = 1 * 2^2$
9	$8 = 1 * 2^3$
12	$16 = 1 * 2^4$
…	
t	$1 * 2^{t/3}$

$$N(t) = 2^{t/3}$$



Since one times a number is that number, we can ignore the 1 in front and just write $y = 2^{t/3}$.

The graph has the shloopy shape of an exponential function.

Why does the graph not go to the left of the *y*-axis?

When will bacteria population reach 1000?

$$N(t) = 2^{t/3}$$

$$1000 = 2^{t/3}$$

Variable in exponent

$$\log_2 1000 = \log_2 2^{t/3}$$

Take log (base 2) of both sides to undo the exponential function.

$$\log_2 1000 = t/3 \left(\log_2 2\right)$$

$$\log_b M^r = r * \log_b M$$

$$\log_2 2 = 1$$

$$\frac{\ln 1000}{\ln 2} = \frac{t}{3}$$

Change of base

$$9.9658 = \frac{t}{3}$$

$$29.90 = t$$

The bacteria will increase its population from 1 to 1000 in 29.90 seconds.

We say the bacteria obey the **law of uninhibited growth**.

This means the number of bacteria grows exponentially, the relationship between the number of bacteria and time is given by an exponential function.

Formula for uninhibited growth / decay

$$A(t) = A_0 e^{kt}$$

A_0 = initial amount (at time 0)

$A(t)$ = amount after t years, days, etc

t = time (years, days, etc)

k = growth / decay constant

k is specific to substance
if $k < 0$, decay
if $k > 0$, growth

You do not need this formula. You can derive a formula like we did using the table.

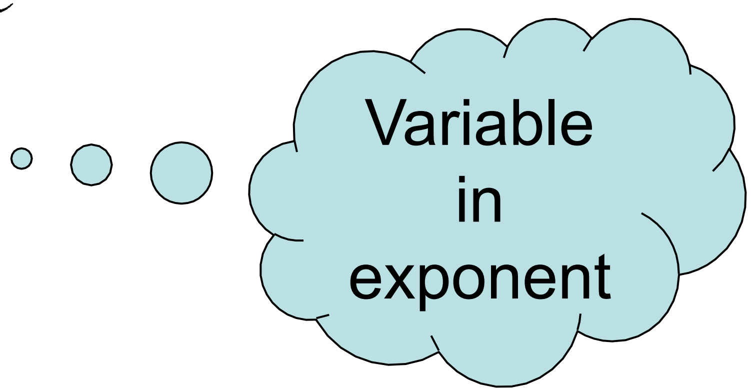
To use the formula, you need to find *k* first. Then you can use the formula to answer any questions, like when will the population reach 1000. Let's redo the problem this way.

$$A(t) = A_0 e^{kt}$$

We know A_0, the initial amount, is 1. We also know the number of bacteria will be 2 at t = 3; meaning A(3) = 2.

$$A(3) = 1 * e^{k*3}$$

$$2 = e^{3k}$$



Take log (base e) of both sides to undo the exponential function

$$\ln 2 = \ln e^{3k}$$

$$\log_b M^r = r * \log_b M$$

$$\ln 2 = 3k \ln e$$

$$.6931 = 3k$$

$$\ln e = 1$$

$$.2310 = k$$

Substituting this value of *k* and A_0 = 1 into the equation $A(t) = A_0 e^{kt}$, we get $A(t) = e^{.2310t}$. This is the equation that relates time *t* to the number of bacteria present *A*(*t*).

When will the bacteria population reach 1000?

$$1000 = e^{.2310t}$$

Variable in exponent

$$\ln 1000 = \ln e^{.2310t}$$

Take log (base e) of both sides to undo the exponential function.

$$\ln 1000 = .2310t \ln e$$

$$\log_b M^r = r * \log_b M$$

$$\ln e = 1$$

$$6.9078 = .2310t$$

$$29.90 = t$$

(in seconds)

Half life of radioactive substances

the amount of time it takes for one half of the substance present to decay

expl: half life of radium is 1690 years

t (years)	amt of radium (grams)
0	100
1690	50
3380	25
5070	12.5
6760	6.25

Every 1690 years, the amount present decreases by half.

Recall the formula below. We used it for exponential decay and growth. It can also be used in half life problems.

$$A(t) = A_0 e^{kt}$$

A_0 = initial amount (at time 0)
$A(t)$ = amount after t years, days, etc
t = time in years, days, etc
k = growth / decay constant

expl: #4, pg 491

Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-.087t}$ where A_0 is the initial amount present and $A(t)$ is the amount present at time t (in days). Assume a scientist has a sample of 100 grams of iodine 131.

a.) (not doing)

b.) Graph the function.

c.) How much is left after 9 days?

d.) When will there be 70 grams left?

e.) What is the half life of iodine 131?

以上内容仅为本文档的试下载部分，为可阅读页数的一半内容。如要下载或阅读全文，请访问：https://d.book118.com/748007041020006135