

含压电主动层复合材料夹持梁 在电场作用下的解

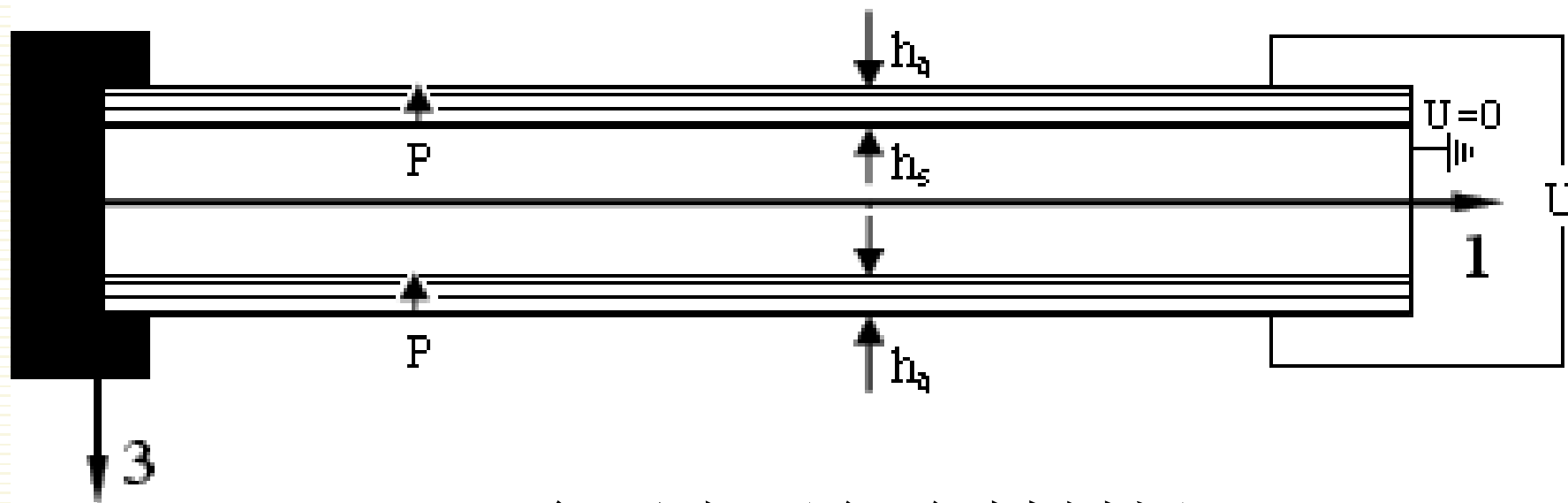


1 内容和目的

- ➔ 内容：由弹性力学基本方程和力电耦合方程出发，采用半逆解法得到了含压电主动层复合材料夹持梁在电场作用下的解析解，该解较为适合揭示复合材料梁弯曲控制方面的作动机理问题，同时电势沿压电介质的厚度方向由一高阶多项式表示更为贴近真实情况。
- ➔ 目的：为压电复合梁的使用者和智能结构的形状控制提供理论参考和借鉴。

2 挟持压电复合材料梁

➔ 概况：图所示的含压电层复合材料梁(x 方向也可称之为1方向， z 方向也可称之为3方向)。



含压电层复合材料梁

- 简化：由于 y 方向(垂直于 xz 平面，也可称之为2方向)可以自由伸展，加之电压施加的方向为3向，因而该层合压电梁在电场作用下的问题可以作为 x - z 平面内的平面应力问题来求解。



2.1 基本平衡方程

✈ (1) 平衡方程

由前面的分析可得复合材料梁的平衡方程为：

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = 0$$

(1)

➔ (2)协调方程:
其协调方程为:

$$\frac{\partial^2 \varepsilon_x}{\partial z^2} - \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} + \frac{\partial^2 \varepsilon_z}{\partial x^2} = 0 \quad (2)$$

(3) 物理方程
本体材料物理方程为:

$$\begin{Bmatrix} \varepsilon_x^s \\ \varepsilon_z^s \\ \gamma_{zx}^s \end{Bmatrix} = \begin{bmatrix} s_{11}^s & s_{13}^s & 0 \\ s_{13}^s & s_{33}^s & 0 \\ 0 & 0 & s_{44}^s \end{bmatrix} \begin{Bmatrix} \sigma_x^s \\ \sigma_z^s \\ \tau_{xz}^s \end{Bmatrix} \quad (3)$$

压电层材料物理方程为:

$$\begin{Bmatrix} \varepsilon_x^a \\ \varepsilon_z^a \\ \gamma_{xz}^a \end{Bmatrix} = \begin{bmatrix} \overline{s_{11}^a} & \overline{s_{13}^a} & 0 \\ \overline{s_{13}^a} & \overline{s_{33}^a} & 0 \\ 0 & 0 & \overline{s_{44}^a} \end{bmatrix} \begin{Bmatrix} \sigma_x^a \\ \sigma_z^a \\ \tau_{xz}^a \end{Bmatrix} - \begin{bmatrix} \overline{d_{31}} \\ \overline{d_{33}} \\ 0 \end{bmatrix} \{E_z\} \quad (4)$$

$$\{D_z\} = \begin{bmatrix} \overline{d_{31}} & \overline{d_{33}} & 0 \end{bmatrix} \begin{Bmatrix} \sigma_x^a \\ \sigma_z^a \\ \tau_{xz}^a \end{Bmatrix} + \begin{bmatrix} \overline{\varepsilon_{33}} \end{bmatrix} \{E_3\} \quad (5)$$

(4) 几何方程

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix} \quad (6)$$

(5) 电学方程

$$\frac{\partial D_z}{\partial z} = 0 \quad (7)$$

$$\{E_z\} = \left\{ -\frac{\partial \varphi}{\partial z} \right\} \quad (8)$$

2.2 方程求解

➔ (1) 应力函数的求取

由平衡方程式可知存在应力函数 Φ 使得：

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial^2 \Phi}{\partial z^2} \\ \frac{\partial^2 \Phi}{\partial x^2} \\ -\frac{\partial^2 \Phi}{\partial x \partial z} \end{Bmatrix} \quad (9)$$

对于下层压(+)电复合层，设其应力函数用 Φ_1^a 表示，电势用 φ_1^a 表示，将(9)和(8)代入(5)式有：

$$D_z = d_{31} \frac{\partial^2 \Phi_1^a}{\partial z^2} + d_{33} \frac{\partial^2 \Phi_1^a}{\partial x^2} - \varepsilon_{33} \frac{\partial \varphi_1^a}{\partial z} \quad (10)$$

将上式(10)代入(7)式中有：

$$d_{31} \frac{\partial^3 \Phi_1^a}{\partial z^3} + d_{33} \frac{\partial^3 \Phi_1^a}{\partial x^2 \partial z} - \varepsilon_{33} \frac{\partial^2 \varphi_1^a}{\partial z^2} = 0 \quad (11)$$

由式(11)可设下压电层的电势分布函数 为:

$$\varphi_1^a = u_1 z^2 + u_2 z + u_3 \quad (12)$$

式中 z 的取值范围为:

$$0.5h_s < z < (0.5h_s + h_a)$$

设下压电层的应力函数 Φ_1^a 表达式为:

$$\begin{aligned} \Phi_1^a = & a_1 xz + a_2 x^2 + a_3 z^2 + a_4 xz^2 \\ & + a_5 x^2 z + a_6 z^3 \end{aligned} \quad (13)$$

(12)和(13)式代入(11)式有：

$$6d_{31}a_6 + 2d_{33}a_5 - 2\varepsilon_{33}u_1 = 0 \quad (14)$$

(13)式代入(9)式结合边界条件有：

式中：

$$z_i = 0.5h_s + h_a$$

(15)

结合(14)和(15)式有：

$$\begin{aligned}\begin{Bmatrix} a_2 \\ a_5 \end{Bmatrix} &= \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \\ a_1 &= -2a_4z = -(h_s + 2h_a)a_4 \\ u_1 &= \frac{3d_{31}}{\varepsilon_{33}}a_6\end{aligned}\tag{16}$$

从而得上层压电复合层的应力函数 Φ_1^a 表达式为：

$$\begin{aligned}\Phi_1^a &= -(h_s + 2h_a)a_4xz + a_3z^2 \\ &\quad + a_4xz^2 + a_6z^3\end{aligned}\tag{17}$$

采用类似于分析下层压电复合层的方法可得上层压电复合层的电势分布函数 φ_2^a 和应力函数 Φ_2^a 分别为:

$$\varphi_2^a = v_1 z^2 + v_2 z + v_3 \quad (18)$$

式中 z 的取值范围为:

$$-(0.5h_s + h_a) < z < -0.5h_s \quad (19)$$

$$\begin{aligned} \Phi_2^a = & (h_s + 2h_a)b_4xz + b_3z^2 \\ & + b_4xz^2 + b_6z^3 \end{aligned}$$

将(18)和(19)代入协调方程有：

$$v_1 = \frac{3d_{31}}{\varepsilon_{33}} b_6 \quad (20)$$

从而由本体结构的相容方程、 Φ_1^a 和 Φ_2^a 可设本体结构的应力函数为：

$$\Phi_3^s = c_1 xz + c_2 z^2 + c_3 xz^2 + c_4 z^3 \quad (21)$$

•(2) 应力表达式的求取

将式(17)、(19)和(21)代入(9)式有：

(a) 下压电层中的应力和电场

$$\begin{Bmatrix} \sigma_x^a \\ \sigma_z^a \\ \tau_{xz}^a \end{Bmatrix} = \begin{Bmatrix} 2a_3 + 2a_4x + 6a_6z \\ 0 \\ (h_s + 2h_a)a_4 - 2a_4z \end{Bmatrix} \quad (22)$$

$$E_z = - \left(\frac{6d_{31}a_6}{\epsilon_{33}} z + u_2 \right)$$

(b) 上压电层中的应力和电场

$$\begin{Bmatrix} \sigma_x^a \\ \sigma_z^a \\ \tau_{xz}^a \end{Bmatrix} = \begin{Bmatrix} 2b_3 + 2b_4x + 6b_4z \\ 0 \\ -(h_s + 2h_a)b_4 - 2b_4z \end{Bmatrix}$$

$$E_z = - \left(\frac{6d_{31}b_6}{\varepsilon_{33}} z + u_2 \right) \quad (23)$$

(b) 本体结构中的应力

$$\begin{Bmatrix} \sigma_x^s \\ \sigma_z^s \\ \tau_{xz}^s \end{Bmatrix} = \begin{Bmatrix} 2c_2 + 2c_3x + 6c_4z \\ 0 \\ -c_1 - 2c_3z \end{Bmatrix} \quad (24)$$

•(3) 位移表达式的求取

将(22)~(24)代入各自的材料本构方程和变形几何方程可得：

(a) 下压电复合层的位移场

$$\left\{ \begin{array}{c} \frac{\partial u_1^a}{\partial x} \\ \frac{\partial w_1^a}{\partial z} \\ \frac{\partial u_1^a}{\partial z} + \frac{\partial w_1^a}{\partial x} \end{array} \right\} = \left\{ \begin{array}{c} s_{11}^a (2a_3 + 2a_4 x + 6a_6 z) \dots \\ -d_{31} \left(\frac{6d_{31}a_6}{\varepsilon_{33}} z + u_2 \right) \\ s_{13}^a (2a_3 + 2a_4 x + 6a_6 z) \dots \\ -d_{33} \left(\frac{6d_{31}a_6}{\varepsilon_{33}} z + u_2 \right) \\ s_{44}^a (h_s + 2h_a)a_4 - 2s_{44}^a a_4 z \end{array} \right\} \quad (25)$$

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