Chapter 8 THE LAPLACE TRANSFORM AND THE TRANSFER FUNCTION REPRESENTATION

8.1 Laplace Transform of a Signal

8.2 Properties of the Laplace Transform

8.3 Computation of the Inverse Laplace Transform

8.4 Transform of the Input/Output Differential Equation

8.5 Transfer Function Representation

8.6 Transfer Function of Block Diagrams

8.1 Laplace Transform of a Signal

The Fourier transform of a continuous-time signal *x(t)* was defined by

$$X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt,-\infty<t<\infty$$

However, some common signals have no Fourier transform in general sense, I.e., u(t). In this case, an exponential convergence factor $e^{-\sigma t}$ can be added to the integrand, where σ is a real number, and it leads

$$X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}e^{-\sigma t}dt,-\infty<t<\infty$$

8.1 Laplace Transform of a Signal

The previous formula can be rewritten as

$$X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t)e^{-(\sigma + j\omega)t}dt, -\infty < t < \infty$$

Let $s=\sigma+j\omega$, one gets

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt, -\infty < t < \infty$$

The above formula is called *Laplace Transform*

8.1 Laplace Transform of a Signal

$$X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}dt, -\infty<t<\infty$$

$$x(t)=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}X(s)e^{st}ds$$

c is any real number for which the path *s=c+j ω* lies in the region of convergence of *X(s)*, and the inverse Laplace Transform is evaluated along the path *s=c+jω*

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