MATH376
Actuarial Models

CARMEN BOADO PENAS
Office 308, Maths Sci Bldg



Office hours: Monday 14.00-15.00
Wednesday 12.00-13.00



Carmen.Boado@liverpool.ac.uk

UNIVERSITY OF LIVERPOOL

LOGO



This lecture is a collection of statistical methods for

1. **testing how good is a proposed smooth function approximation (or graduation) to crude mortality rates.**
2. **Or is our data consistent with another?**

We discuss goodness of fit and other tests.



We often want to know if our data is consistent with another, known experience.
For example, if our data is the recent experience of the policyholders of a life insurance company, we might ask:

- Is it consistent with the companys own past experience, or is the experience changing?
- Is it consistent with the published life tables? This is important if the company plans to use published tables for any financial calculation.



Structure

1. Notation
2. Tests
3. Graduation
4. Statistical test: X2 test, standardise deviation test, sign test, cumulative deviation test, the serial correlation test

5. Effect of duplicate policies



Notation

Notation:

q_x^s : **mortality rate from a published standard table**

$\mu^s_{x+\frac{1}{2}}$: **force of mortality from a published standard table**

$\hat{q}_x$

$\hat{\mu}_{x+\frac{1}{2}}$

Are our estimates consistent with the published standard table?

Tests

We can derive tests of this hypothesis using the distribution assumptions

a) (Multiple-state): under the hypothesis

$$D_x \sim N(E^c_x \mu^s_{x+1/2}, E^c_x \mu^s_{x+1/2})$$

(approximately)

b) (Binomial): under the hypothesis

$$D_x \sim N(E_x q^s_x, E_x q^s_x (1 - q^s_x))$$

(approximately)

We can find test statistics comparing the actual deaths d_x with the expected deaths given by these distributions.

Graduation

The crude estimates $\hat{q}_x, \hat{\mu}_{x+\frac{1}{2}}$ will progress more or less roughly.

We prefer to work with $q_x, \mu_{x+\frac{1}{2}}$ which were smooth function of age.

Therefore we graduate or smooth the crude estimates to produce a set of graduated estimates and we denote these by $\overset{0}{q}_x, \overset{0}{\mu}_{x+\frac{1}{2}}$



Reasons for graduation

Our desire to graduate is the intuitive idea that quanties such as q_x, μ_x should be smooth functions of age. There is some evidence from large investigations to support this, but it is nevertheless an assumption.

- **A crude estimate at any age *x*, $\hat{q}_x, \hat{\mu}_x$, also carries out information about the values of q_{x-1}, q_{x+1} or $\mu_{x-1/2}, \mu_{x+1/2}$. If q_x is smooth and not changing too rapidly, then $\hat{q}_x$ should not be too far away from estimating q_{x-1}, q_{x+1}**
- **Smoothing reduces the sampling errors at each age.**
- **A purely practical reason for smoothing mortality data is that we will use the life table to compute financial quantities (premiums for insurance contracts). It is very desirable that such quantitites progress smoothly with age, since irregularities are hard to justify.**
- **What graduation cannot do is to remove any bias in the data arising ~~from faulty data collection.~~**

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