

第二类换元法常见类型：三角代换、倒代换、根式代换

$$\textcircled{1} \int f(x) \sqrt{a-x^2} dx \quad \text{令 } x = a \sin t \text{ 或 } x = a \cos t$$

$$\textcircled{2} \int f(x) \sqrt{a+x^2} dx \quad \text{令 } x = a \tan t$$

$$\textcircled{3} \int f(x) \sqrt{x^2-a} dx \quad \text{令 } x = a \sec t$$

(4) 分母中因子次数较高时，可试用倒代换

$$\textcircled{5} \int f(x) \sqrt{ax+b} dx \quad \text{令 } t = \sqrt{ax+b} \quad \text{即 } x = \frac{t^2 - b}{a}$$

练习

$$1. \int \frac{2x-1}{\sqrt{x^2+2x-1}} dx$$

$$2. \int \frac{dx}{\sqrt{x^2-2x}}$$

$$3. \int \frac{1}{x} \sqrt{\frac{1+x}{x}} dx$$

$$1. \int \frac{2x-1}{\sqrt{x^2+2x-1}} dx$$

$$= \int \frac{d(x^2+2x-1)}{\sqrt{x^2+2x-1}} - \int \frac{3}{\sqrt{x^2+2x-1}} dx$$

$$= 2\sqrt{x^2+2x-1} - 3 \int \frac{1}{\sqrt{(x+1)^2-2}} d(x+1)$$

$$= 2\sqrt{x^2+2x-1} - 3 \ln |(x+1) + \sqrt{x^2+2x-1}| + c.$$



$$2 \int \frac{dx}{(x+1)\sqrt{x^2-1}}$$

$$= \int \frac{dx}{(x+1)\sqrt{(x+1)(x-1)}}$$

$$\diamond x+1 = \frac{1}{t}$$

$$= \int \frac{t^3}{\sqrt{\frac{1}{t^2}-1}} \left(\frac{1}{t^2}\right) dt = \int \frac{t^2}{\sqrt{1-t^2}} dt$$

$$= \int \frac{(1-t^2)-1}{\sqrt{1-t^2}} dt = \int \sqrt{1-t^2} dt - \int \frac{1}{\sqrt{1-t^2}} dt$$

$$= \frac{1}{2}\sqrt{1-t^2} + \frac{1}{2}\arcsin t - \arcsin t$$

$$= \frac{\sqrt{x^2-1}}{2(x+1)} - \frac{1}{2}\arcsin \frac{1}{x+1}$$

$$3 \int \frac{1}{x} \sqrt{\frac{1+x}{x}} dx$$

$$\text{令 } t = \sqrt{\frac{1+x}{x}}, \text{ 则 } x = \frac{1}{t^2 - 1}, dx = \frac{-2t dt}{(t^2 - 1)^2}$$

$$\text{原式} = \int (t^2 - 1) \cdot \frac{-2t}{(t^2 - 1)^2} dt$$

$$= -2 \int \frac{t^2}{t^2 - 1} dt = -2 \int \left(1 + \frac{1}{t^2 - 1}\right) dt$$

$$= -2t - \ln \left| \frac{t-1}{t+1} \right| + C$$

$$= -2 \sqrt{\frac{1+x}{x}} - \ln \left| \frac{\sqrt{\frac{1+x}{x}} - 1}{\sqrt{\frac{1+x}{x}} + 1} \right| + C$$

第三节

分部积分法

问题 $\int x \cos x dx = ?$

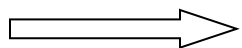
处理思绪

由导数公式

$$\frac{d}{dx}(x \sin x) = \sin x + x \cos x$$

积分得:

$$\int (\sin x + x \cos x) dx = -\cos x + x \sin x + C$$



$$\int x \cos x dx = -\cos x + x \sin x + C$$

或

$$\int x \cos x dx = x \sin x - \int \sin x dx$$

} 分部积分公式

例1 求积分 $\int x \cos x dx$.

解 (一) 令 $u = \cos x$ $x dx = \frac{1}{2} dx^2 = dv$

$$\int x \cos x dx = \int \cos x d\left(\frac{x^2}{2}\right) = \frac{x^2}{2} \cos x + \int \frac{x^2}{2} \sin x dx$$

显然, u, v' 选择不当, 积分更难进行.

解 (二) 令 $u = x$ $\cos x dx = d \sin x = dv$

$$\begin{aligned} \int x \cos x dx &= \int x d \sin x = x \sin x - \int \sin x dx \\ &= x \sin x + \cos x + C \end{aligned}$$

~~选择原则:~~

1) v 轻易求得;

2) $\int u'v dx$ 比 $\int uv' dx$ 轻易计算

例2. $\int x \arctan x dx$

$$= \frac{1}{2} \int \arctan x d(x^2)$$

$$= \frac{1}{2} \left[x^2 \arctan x - \int x^2 d \arctan x \right]$$

$$= \frac{x^2}{2} \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$

$$= \frac{x^2}{2} \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx$$

$$= \frac{x^2}{2} \arctan x - \frac{1}{2} (x - \arctan x) + c$$

$$= \frac{x^2+1}{2} \arctan x - \frac{x}{2} + c.$$

例3. $\int x \ln x dx = \frac{1}{2} \int \ln x d(x^2)$ ~~$(u = \ln x, v = x^2)$~~

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x^2 d \ln x = \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x^2 \cdot \frac{1}{x} dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + c .$$

例4. $\int x e^x dx = \frac{1}{2} \int e^x d(x^2)$ ~~$(u = x, v = x^2)$~~

$$= \frac{1}{2} e^x x^2 - \frac{1}{2} \int x^2 de^x = \frac{1}{2} e^x x^2 - \frac{1}{2} \int x^2 e^x dx = ?$$

重解: $\int x e^x dx = \int x de^x$ ~~$(u = x, v = e^x)$~~

$$= x e^x - \int e^x dx = x e^x - e^x + c .$$

解题技巧: **选取的一般**

把被积函数视为两个函数之积 按“反对幂指三”

顺序 前者为 u , 后者为 v'

,

的

反: 反三角函数

对: 对数函数

幂: 幂函数

指: 指数函数

三: 三角函数



机动



目录



上页



下页



返回



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