组合导航系统

姿态矩阵更新

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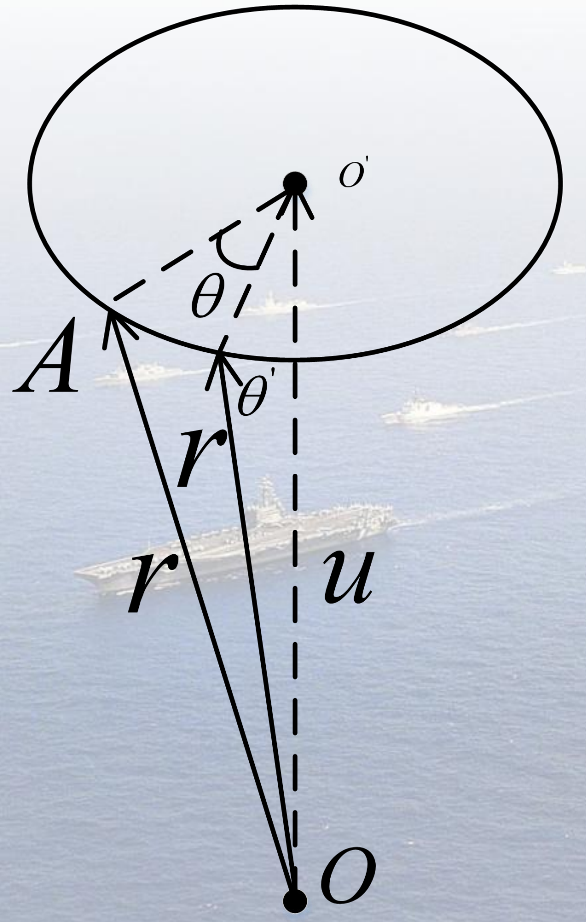
一、四元数法(Quaternions)

➢ 1. 四元数定义



物理意义：b系是由t系经过无中间过程的一次性旋转形成的，Q包含了这次旋转的全部信息。u为旋转轴，θ为转过的角度。

广义复数表示法

$$\mathbf{Q} = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$$

三角表示法

$$\mathbf{Q} = \cos\frac{\theta}{2} + \boldsymbol{u}\sin\frac{\theta}{2}$$

一、四元数法(Quaternions)

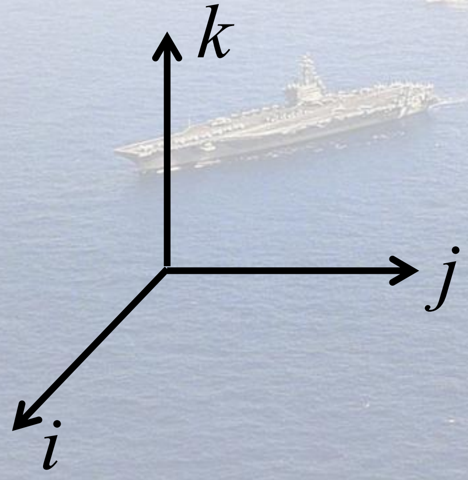
➢ 2. 姿态矩阵用四元数表示

四元数的性质

①规范四元数 $\hat{\mathbf{Q}} = \dfrac{\mathbf{Q}}{\sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}}$ $\quad \hat{\mathbf{Q}}^{-1} = \hat{\mathbf{Q}}^* = \hat{q}_0 - \hat{q}_1\mathbf{i} - \hat{q}_2\mathbf{j} - \hat{q}_3\mathbf{k}$

②四元数相乘 $\mathbf{Q} \otimes \mathbf{P} = \left(q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}\right) \otimes \left(p_0 + p_1\mathbf{i} + p_2\mathbf{j} + p_3\mathbf{k}\right)$

$$= r_0 + r_1\mathbf{i} + r_2\mathbf{j} + r_3\mathbf{k}$$



$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & p_3 & -p_2 \\ p_2 & -p_3 & p_0 & p_1 \\ p_3 & p_2 & -p_1 & p_0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

一、四元数法(Quaternions)

➢ 3. 姿态矩阵用四元数表示

③转动四元数： 存在一个四元数Q，使 $\mathbf{R}' = \mathbf{Q} \otimes \mathbf{R} \otimes \mathbf{Q}^{-1}$

其中R绕u轴转过θ到R′

$$\mathbf{R}^n = \mathbf{Q} \otimes \mathbf{R}^b \otimes \mathbf{Q}^{-1}$$

$$(q_0 + q_1 i + q_2 j + q_3 k) \otimes (R_x i + R_y j + R_z k) \otimes (q_0 - q_1 i - q_2 j - q_3 k)$$

$$\mathbf{R}^n = \mathbf{C}_b^n \mathbf{R}^b$$

$$\mathbf{C}_b^n = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_2 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

一、四元数法(Quaternions)

➢ 3. 姿态矩阵用四元数表示

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一、四元数法(Quaternions)

➢ 4. 四元数微分方程

$$\mathbf{Q}(t+\Delta t)=\mathbf{Q}(t)\otimes\Delta\mathbf{Q}(t)$$

$$\Delta\mathbf{Q}(t)=\cos\frac{\Delta\theta}{2}+\boldsymbol{u}\sin\frac{\Delta\theta}{2}\approx 1+\boldsymbol{u}\frac{\Delta\theta}{2}=1+\frac{1}{2}\boldsymbol{\omega}_{nb}^{b}\Delta t$$

$$\mathbf{Q}(t+\Delta t)=\mathbf{Q}(t)\otimes\left(1+\frac{1}{2}\boldsymbol{\omega}_{nb}^{b}\Delta t\right)=\mathbf{Q}(t)+\frac{1}{2}\mathbf{Q}(t)\otimes\boldsymbol{\omega}_{nb}^{b}\Delta t$$

$$\dot{\mathbf{Q}}(t)=\lim_{\Delta t\to 0}\frac{\mathbf{Q}(t+\Delta t)-\mathbf{Q}(t)}{\Delta t}=\frac{1}{2}\mathbf{Q}(t)\otimes\boldsymbol{\omega}_{nb}^{b}\Rightarrow\begin{bmatrix}\dot{q}_0\\ \dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix}=\frac{1}{2}\begin{bmatrix}0 & -\omega_{nbx}^{b} & -\omega_{nby}^{b} & -\omega_{nbz}^{b}\\ \omega_{nbx}^{b} & 0 & \omega_{nbz}^{b} & -\omega_{nby}^{b}\\ \omega_{nby}^{b} & -\omega_{nbz}^{b} & 0 & \omega_{nbx}^{b}\\ \omega_{nbz}^{b} & \omega_{nby}^{b} & -\omega_{nbx}^{b} & 0\end{bmatrix}\begin{bmatrix}q_0\\ q_1\\ q_2\\ q_3\end{bmatrix}$$

一、四元数法(Quaternions)

➢ 4. 四元数微分方程

$$
\begin{aligned}
\boldsymbol{\omega}_{nb}^{b} &= \boldsymbol{\omega}_{ib}^{b} - \boldsymbol{\omega}_{in}^{b} \\
&= \boldsymbol{\omega}_{ib}^{b} - \left(\mathbf{C}_b^n\right)^{-1} \boldsymbol{\omega}_{in}^{n} \\
&= \underbrace{\boldsymbol{\omega}_{ib}^{b}}_{\text{陀螺仪测量}} - \left(\underbrace{\mathbf{C}_b^n}_{\text{迭代过程中上一步已知值}}\right)^{-1} \left(\underbrace{\boldsymbol{\omega}_{ie}^{n}}_{\text{地球自转角速度}} + \boldsymbol{\omega}_{en}^{n}\right)
\end{aligned}
$$

陀螺仪测量

迭代过程中
上一步已知值

地球自转角速度

$$\mathbf{f}^b \to \mathbf{f}^n \to \mathbf{V}_{en}^n \to \boldsymbol{\omega}_{en}^n$$

一、四元数法(Quaternions)

➢ 4. 四元数微分方程

$$\boldsymbol{\omega}_{nb}^{b} = \boldsymbol{\omega}_{ib}^{b} - \left(\mathbf{C}_{b}^{n}\right)^{-1}\left(\boldsymbol{\omega}_{ie}^{n} + \boldsymbol{\omega}_{en}^{n}\right)$$

⇓

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -\omega_{nbx}^{b} & -\omega_{nby}^{b} & -\omega_{nbz}^{b} \\ \omega_{nbx}^{b} & 0 & \omega_{nbz}^{b} & -\omega_{nby}^{b} \\ \omega_{nby}^{b} & -\omega_{nbz}^{b} & 0 & \omega_{nbx}^{b} \\ \omega_{nbz}^{b} & \omega_{nby}^{b} & -\omega_{nbx}^{b} & 0 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

⇓

$$\mathbf{C}_{b}^{n} = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2\left(q_1q_2-q_0q_3\right) & 2\left(q_1q_3+q_0q_2\right) \\ 2\left(q_1q_2+q_0q_3\right) & q_0^2-q_1^2+q_2^2-q_3^2 & 2\left(q_2q_3-q_0q_1\right) \\ 2\left(q_1q_3-q_0q_2\right) & 2\left(q_2q_3+q_0q_1\right) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}$$

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