2015-2019年AMC12A真题汇编含答案

2015 AMC12A

Problem 1

What is the value of

$$(2^0 - 1 + 5^2 - 0)^{-1} \times 5?$$

下列算式的值是多少

$$(2^0 - 1 + 5^2 - 0)^{-1} \times 5?$$

$\textbf{(A)}\ -125 \qquad \textbf{(B)}\ -120 \qquad \textbf{(C)}\ \frac{1}{5} \qquad \textbf{(D)}\ \frac{5}{24} \qquad \textbf{(E)}\ 25$

Problem 2

Two of the three sides of a triangle are 20 and 15. Which of the following numbers is not a possible perimeter of the triangle?

一个三角形的其中两条边长度是 20 和 15，下面哪个数字不可能成为三角形的周长?

$\textbf{(A)}\ 52 \qquad \textbf{(B)}\ 57 \qquad \textbf{(C)}\ 62 \qquad \textbf{(D)}\ 67 \qquad \textbf{(E)}\ 72$

Problem 3

Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

Patrick 先生给 15 个学生上数学课，他正批改考试试卷，发现当他把除了 Payton 外的所有其他学生的试卷都改好后，班级的平均分是 80，当他批改好 Payton 的试试卷后，班级的平均分变成了 81 分，问 Payton 这次考试得分是多少?

$\textbf{(A)}\ 81 \qquad \textbf{(B)}\ 85 \qquad \textbf{(C)}\ 91 \qquad \textbf{(D)}\ 94 \qquad \textbf{(E)}\ 95$

Problem 4

The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

两个正数之和是它们之差的 5 倍，那么大的数比小的数的比值是多少？

$\textbf{(A)}\ \frac{5}{4} \qquad \textbf{(B)}\ \frac{3}{2} \qquad \textbf{(C)}\ \frac{9}{5} \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ \frac{5}{2}$

Problem 5

Amelia needs to estimate the quantity $\frac{a}{b} - c$, where $a, b,$ and c are large positive integers. She rounds each of the integers so that the calculation will be easier to do mentally. In which of these situations will her answer necessarily be greater than the exact value of $\frac{a}{b} - c$?

Amelia 需要估算$\frac{a}{b} - c$的值，其中 a，b 和 C 都是很大的正整数，她需要对每个整数进行舍位（变小）或者进位（变大），这样算就更方便。在下面哪种情况她的结果会比$\frac{a}{b} - c$的准确值大？

（A） She rounds all three numbers.up. ｜ 她把三个数字都向上进位。
（B） She rounds a and b up, and she rounds c down. | 她把 a 和 b 向上进位，把 c 向下舍位。
（C） She rounds a and c up, and she rounds b down. | 她把 a 和 c 向上进位，把 b 向下舍位。
（D） She rounds a up, and she rounds b and c down. | 她把 a 向上进位，把 b 和 c 向下舍位。
（E） She rounds c up, and she rounds a and b down. | 她把 c 向上进位，把 a 和 b 向下舍位。

Problem 6

Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2 : 1$?

两年前 Pete 的年龄是他表妹年龄的 3 倍，从那时起再往前推两年，Pete 的年龄是 Claire 年龄的 4 倍，多少年后他们年龄的比值将是$2 : 1$?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 7

Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

两个正圆柱体的体积相同，第二个圆柱体的半径比第一个的半径多 10%，问这两个圆柱体的高有什么关系？

（A）The second height is 10% less than the first | 第二个的高比第一个高少 10%

（B）The first height is 10% more than the second | 第一个的高比第二个高多 10%

（C）The second height is 21% less than the first | 第二个的高比第一个高少 21%

（D）The first height is 21% more than the second | 第一个的高比第二个高多 21%

（E）The second height is 80% of the first | 第二个的高是第一个高的 80%

Problem 8

The ratio of the length to the width of a rectangle is $4:3$. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

一个长方形的长和宽之比为$4:3$，如果长方形的对角线的长度是 d，那么它的面积可以写成kd^2，这里 k 是常数，那么 k 是多少?

$\textbf{(A)}\ \frac{2}{7} \qquad \textbf{(B)}\ \frac{3}{7} \qquad \textbf{(C)}\ \frac{12}{25} \qquad \textbf{(D)}\ \frac{16}{25} \qquad \textbf{(E)}\ \frac{3}{4}$

Problem 9

A box contains 2 red marbles, 2 green marbles, and 2 yellow marbles. Carol takes 2 marbles from the box at random; then Claudia takes 2 of the remaining marbles at random; and then Cheryl takes the last 2 marbles. What is the probability that Cheryl gets 2 marbles of the same color?

一个盒子里装有 2 颗红色玻璃球，2 颗绿色玻璃球，2 颗黄色玻璃球。Carol 从盒子里随机挑选 2 颗玻璃球，然后 Claudia 从剩下的玻璃球中随机挑选 2 颗，最后 Cheryl 拿走最后剩下的两颗。问 Cheryl 拿走的是两颗颜色一样的玻璃球的概率是多少？

$\textbf{(A)}\ \frac{1}{10} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ \frac{1}{5} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 10

Integers x and y with $x > y > 0$ satisfy $x + y + xy = 80$. What is x?

整数 x 和 y 满足$x + y + xy = 80$且$x > y > 0$，则 x 是多少？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 26$

Problem 11

On a sheet of paper, Isabella draws a circle of radius 2, a circle of radius 3, and all possible lines simultaneously tangent to both circles. Isabella notices that she has drawn exactly $k \ge 0$ lines. How many different values of k are possible?

Isabella 在一张纸上画了一个半径为 2 的圆，一个半径为 3 的圆，以及这两个圆所有可能的公切线。Isabella 发现她恰好画了$k \ge 0$根直线。那么 k 有多少种不同的可能取值？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 12

The parabolas $y = ax^2 - 2$ and $y = 4 - bx^2$ intersect the coordinate axes in exactly four points, and these four points are the vertices of a kite of area 12. What is $a + b$?

抛物线$y = ax^2 - 2$和$y = 4 - bx^2$与坐标轴恰好相交于 4 个不同的点，且这 4 个点是一个面积为 12 的筝形的 4 个顶点。求$a + b$是多少?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 1.5 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 2.5 \qquad \textbf{(E)}\ 3$

Problem 13

A league with 12 teams holds a round-robin tournament, with each team playing every other team exactly once. Games either end with one team victorious or else end in a draw. A team scores 2 points for every game it wins and 1 point for every game it draws. Which of the following is NOT a true statement about the list of 12 scores?

12 支队伍组成的联盟举行了一场循环赛，每支队伍都要和其他每支队伍打一场比赛。比赛的结果要么一胜一负，要么平局。每支队伍若胜一局则得 2 分，若平局则得 1 分。下面关于这 12 支队伍获得的 12 个分数的说法，哪个是错误的？

（A）There must be an even number of odd scores. | 一定有偶数个分数是奇数。

（B）There must be an even number of even scores. | 一定有偶数个分数是偶数。

（C）There cannot be two scores of 0. | 不可能有 2 个 0 分。

（D）The sum of the scores must be at least 100. | 所有分数之和一定至少是 100。

（E）The highest score must be at least 12. | 最高分一定至少是 12 分。

Problem 14

What is the value of a for which $\frac{1}{\log_2 a} + \frac{1}{\log_3 a} + \frac{1}{\log_4 a} = 1$?

满足方程 $\frac{1}{\log_2 a} + \frac{1}{\log_3 a} + \frac{1}{\log_4 a} = 1$ 的 a 的值是多少？

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 36$

Problem 15

What is the minimum number of digits to the right of the decimal point needed to express the fraction $\frac{123456789}{2^{26}\cdot 5^4}$ as a decimal?

小数点的右边至少需要有多少位才能把分数 $\frac{123456789}{2^{26}\cdot 5^4}$ 表达成小数？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 22 \qquad \textbf{(C)}\ 26 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 104$

Problem 16

Tetrahedron $ABCD$ has $AB = 5, AC = 3, BC = 4, BD = 4, AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

四面体 *ABCD* 有 *AB*=5，*AC*=3，*BC*=4，*BD*=4，*AD*=3，$CD = \frac{12}{5}\sqrt{2}$，那么这个四面体的体积为多少？

$\textbf{(A)}\ 3\sqrt{2} \qquad \textbf{(B)}\ 2\sqrt{5} \qquad \textbf{(C)}\ \frac{24}{5} \qquad \textbf{(D)}\ 3\sqrt{3} \qquad \textbf{(E)}\ \frac{24}{5}\sqrt{2}$

Problem 17

Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

8 个人坐在一张圆桌旁，每个人都手持一枚硬币。8 个人一起扔硬币，扔到正面朝上的就站起来，扔到反面朝上的则仍然坐着，没有相邻的两个人同时站着的概率是多少？

$\textbf{(A)}\ \frac{47}{256} \qquad \textbf{(B)}\ \frac{3}{16} \qquad \textbf{(C)}\ \frac{49}{256} \qquad \textbf{(D)}\ \frac{25}{128} \qquad \textbf{(E)}\ \frac{51}{256}$

Problem 18

The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

函数$f(x) = x^2 - ax + 2a$的零点都是整数。问 a 的所有可能值之和是多少？

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 18$

Problem 19

For some positive integers p, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter p, right angles at B and C, $AB=2$, and $CD=AD$. How many different values of $p<2015$ are possible?

对于某些正整数 p，存在一个四边形 $ABCD$，使得它的边长都是正整数，周长是 p，直角顶点为 B 和 C，AB=2， CD=AD. 若$p<2015$，那么满足条件的不同的 p 值有多少个？

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 61 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 63$

Problem 20

Isosceles triangles T and T' are not congruent but have the same area and the same perimeter. The sides of T have lengths of $5,5,$ and 8, while those of T' have lengths of $a,a,$ and b. Which of the following numbers is closest to b?

等腰三角形 T 和 T' 不是全等三角形，但是面积和周长都相等。T 的边长是 5，5，8，而 T' 的边长是 a，a，b，下面哪个数字最接近 b 的值?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 21

A circle of radius r passes through both foci of, and exactly four points on, the ellipse with equation $x^2+16y^2=16$. The set of all possible values of r is an interval $[a,b)$. What is $a+b$?

半径为 r 的圆通过方程为$x^2+16y^2=16$的椭圆的两个焦点，与椭圆恰好交于 4 个点，所有可能的 r 的取值是一个区间$[a,b)$，求$a+b$是多少？

$\textbf{(A)}\ 5\sqrt{2}+4 \qquad \textbf{(B)}\ \sqrt{17}+7 \qquad \textbf{(C)}\ 6\sqrt{2}+3 \qquad \textbf{(D)}\ \sqrt{15}+8 \qquad \textbf{(E)}\ 12$

Problem 22

For each positive integer n, let $S(n)$ be the number of sequences of length n consisting solely of the letters A and B, with no more than three As in a row and no more than three Bs in a row. What is the remainder when $S(2015)$ is divided by 12?

n 是一个正整数，$S(n)$表示长度为 n 的满足以下要求的数列的个数：数列只由字母 A 和 B 组成，并且不允许有超过 3 个连续的 A，也不能有超过 3 个连续的 B。问$S(2015)$除以 12 所得的余数是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 10$

Problem 23

Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where $a, b,$ and c are positive integers and $\gcd(a,b,c)=1$. What is $a+b+c$?

S 是边长为 1 的正方形，从 S 的边上随机选择 2 个点，这两个点的直线距离至少是$\frac{1}{2}$的概率是$\frac{a-b\pi}{c}$，其中 a，b，c 都是正整数，且 $\gcd(a,b,c)=1$，问$a+b+c$是多少？

$\textbf{(A)}\ 59 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 61 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 63$

Problem 24

Rational numbers a and b are chosen at random among all rational numbers in the interval $[0, 2)$ that can be written as fractions $\frac{n}{d}$ where n and d are integers with $1 \leq d \leq 5$. What is the probability that

$$(\cos(a\pi) + i\sin(b\pi))^4$$

is a real number?

有理数 a 和 b 是从区间［0，2）内随机选择的两个有理数，均可以写成$\frac{n}{d}$的形式，其中 n 和 d 都是整数，且$1 \leq d \leq 5$，那么$(\cos(a\pi) + i\sin(b\pi))^4$是一个实数的概率是多少？

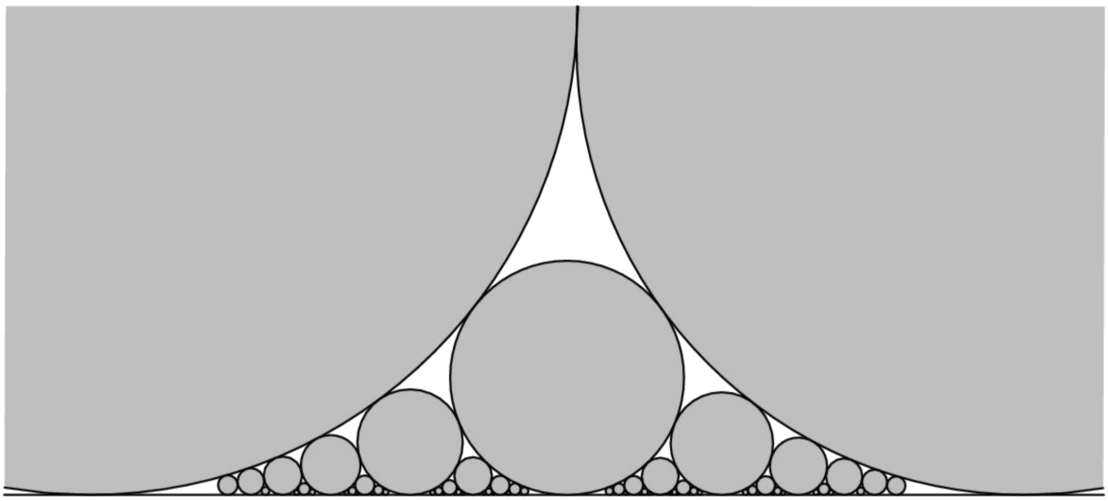
$\textbf{(A)}\ \frac{3}{50} \qquad \textbf{(B)}\ \frac{4}{25} \qquad \textbf{(C)}\ \frac{41}{200} \qquad \textbf{(D)}\ \frac{6}{25} \qquad \textbf{(E)}\ \frac{13}{50}$

Problem 25

A collection of circles in the upper half-plane, all tangent to the x-axis, is constructed in layers as follows. Layer L_0 consists of two circles of radii 70^2 and 73^2 that are externally tangent. For $k \geq 1$, the circles in $\bigcup_{j=0}^{k-1} L_j$ are ordered according to their points of tangency with the x-axis. For every pair of consecutive circles in this order, a new circle is constructed externally tangent to each of the two circles in the pair. Layer L_k consists of the 2^{k-1} circles constructed in this way. Let $S=\bigcup_{j=0}^{6} L_j$, and for every circle C denote by $r(C)$ its radius. What is

$$\sum_{C \in S} \frac{1}{\sqrt{r(C)}}?$$

在上半平面内，以如下方式画出若干层和 x 轴相切的圆，第L_0层由两个半径分别70^2和73^2且互相外切的圆组成。当$k \geq 1$，集合$\bigcup_{j=0}^{k-1} L_j$内的所有的圆依照它们和 x 轴的切点进行排序。对于这种排序下的任意连续 2 个圆，画出的新的圆和这 2 个圆都外切。第L_k层是由2^{k-1}个以这种方式画出来的圆组成。令$S=\bigcup_{j=0}^{6} L_j$，且用$r(C)$表示圆 C 的半径，$\sum_{C \in S} \frac{1}{\sqrt{r(C)}}$是多少？



(A) $\frac{286}{35}$ **(B)** $\frac{583}{70}$ **(C)** $\frac{715}{73}$ **(D)** $\frac{143}{14}$ **(E)** $\frac{1573}{146}$

2015 AMC 12A Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
C	E	E	B	D	B	D	C	C	E	D	B	E
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	C	C	A	C	B	A	D	D	A	D	D	

2016 AMC12A

Problem 1

What is the value of $\frac{11!-10!}{9!}$?

表达式$\frac{11!-10!}{9!}$的值是多少?

$\textbf{(A)}\ 99 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 110 \qquad \textbf{(D)}\ 121 \qquad \textbf{(E)}\ 132$

Problem 2

For what value of x does $10^{x}\cdot 100^{2x}=1000^{5}$?

x的值为多少时，$10^{x}\cdot 100^{2x}=1000^{5}$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 3

The remainder can be defined for all real numbers x and y with $y \neq 0$ by

$$\text{rem}(x,y)=x-y\left\lfloor \frac{x}{y} \right\rfloor$$

where $\left\lfloor \tfrac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\tfrac{x}{y}$. What is the value of $\text{rem}(\tfrac{3}{8}, -\tfrac{2}{5})$?

所有实数x和y的余数定义为: $\text{rem}(x,y)=x-y\left\lfloor \frac{x}{y} \right\rfloor$，其中$y \neq 0$，且$\left\lfloor \tfrac{x}{y} \right\rfloor$表示取小于或等于$\tfrac{x}{y}$的最大整数，$\text{rem}(\tfrac{3}{8}, -\tfrac{2}{5})$的值是多少?

$\textbf{(A)}\ -\frac{3}{8} \qquad \textbf{(B)}\ -\frac{1}{40} \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ \frac{3}{8} \qquad \textbf{(E)}\ \frac{31}{40}$

Problem 4

The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x?

60，100，x，40，50，200，90 这 7 个数字的平均值、中位数和众数都等子 x，那么 x 的值是？

$\textbf{(A)}\ 50 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 75 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 100$

Problem 5

Goldbach's conjecture states that every even integer greater than 2 can be written as the sum of two prime numbers (for example, $2016 = 13 + 2003$). So far, no one has been able to prove that the conjecture is true, and no one has found a counterexample to show that the conjecture is false. What would a counterexample consist of?

哥德巴赫猜想的内容是：任何大于 2 的偶数都可以写成 2 个质数之和。（例如，2016=13+2003）。到目前为止，还没人能够证明这个猜想是正确的，也没人能够找到一个反例证明这个猜想是错的。那么反例是什么？

(A)An odd integer greater than 2 that can be written as the sum of two prime numbers. ｜ 存在一个大于 2 的奇数，可以写成 2 个质数之和。

(B) An odd integer greater than 2 that cannot be written as the sum of two prime numbers. | 存在一个大于 2 的奇数，无法写成 2 个质数之和。

(C) An even integer greater than 2 that can be written as the sum of two numbers that are not prime | 存在一个大于 2 的偶数，可以写成两个不是质数的数之和。

(D) An even integer greater than 2 that can be written as the sum of two prime numbers. ｜ 存在一个大于 2 的偶数，可以写成 2 个质数之和。

(E)An even integer greater than 2 that cannot be written as the sum of two prime numbers. | 存在一个大于 2 的偶数，不能写成 2 个质数之和。

Problem 6

A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N ?

含有总共 2016 个硬币的三角形阵列的第一行有 1 个硬币，第二行有 2 个硬币，第三行有 3 个硬币，以此类推直到第 N 行有 N 个硬币。问 N 的各个位上数字之和为多少？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

Problem 7

Which of these describes the graph of $x^2(x+y+1)=y^2(x+y+1)$?

下面哪句话描述了方程$x^2(x+y+1)=y^2(x+y+1)$的图像？

（A）Two parallel lines | 两条平行线

（B）Two intersecting lines | 两条相交的线

（C）Three lines that all pass through a common point | 三条交于一点的直线

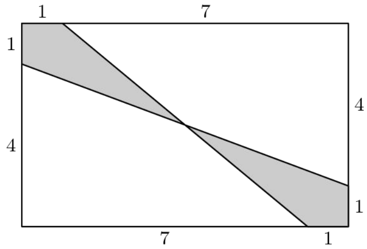
（D）Three lines that do not áll pass through a common point | 三条不交于一点的直线

（E）A line and a parabola | 一条直线和一个抛物线

Problem 8

What is the area of the shaded region of the given 8×5 rectangle?

下面给定的8×5的长方形内阴影部分的面积是多少？

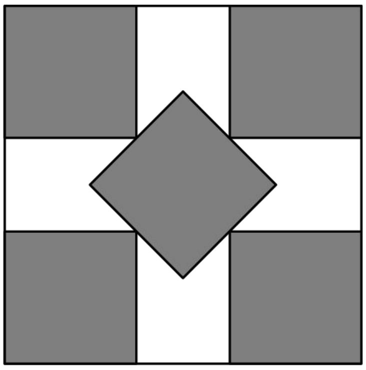


$\textbf{(A)}\ 4.75 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 5.25 \qquad \textbf{(D)}\ 6.5 \qquad \textbf{(E)}\ 8$

Problem 9

The five small shaded squares inside this unit square are congruent and have disjoint interiors. The midpoint of each side of the middle square coincides with one of the vertices of the other four small squares as shown. The common side length is $\frac{a-\sqrt{2}}{b}$, where a and b are positive integers. What is $a+b$?

如图所示，单位正方形里有 5 个全等的小正方形，且内部不重合，中间正方形每条边的中点和其他 4 个小正方形的一个顶点重合。这些小正方形共同的边长是$\frac{a-\sqrt{2}}{b}$，其中 a 和 b 都是正整数。问$a+b$是多少？



$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Problem 10

Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

5 个朋友在电影院里坐在一排从左向右编号为 1 到 5 的 5 个座位上（这里的“左”和“右”是指当这 5 人坐在座位上，从他们的角度看到的左和右）。在看电影的过程中，Ada 去休息室拿了一些爆米花，当她回来时，她发现 Bea 已经向右挪动了 2 个位置，Ceci 向左挪动了一个位置，Dee 和 Edie 交换了位置，空了一张最边上的位置给 Ada，那么 Ada 在她起身离开前，原先是坐在哪个座位的？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 11

Each of the 100 students in a certain summer camp can either sing, dance, or act. Some students have more than one talent, but no student has all three talents. There are 42 students who cannot sing, 65 students who cannot dance, and 29 students who cannot act. How many students have two of these talents?

某个夏令营的 100 个学生有的会唱歌，有的会跳舞，还有的会表演。有些学生具有不止一种技能，但是没有学生三种技能都会。已知有 42 名学生不会唱歌，65 个学生不会跳舞，29 个学生不会表演。问有多少个学生具备两种技能？

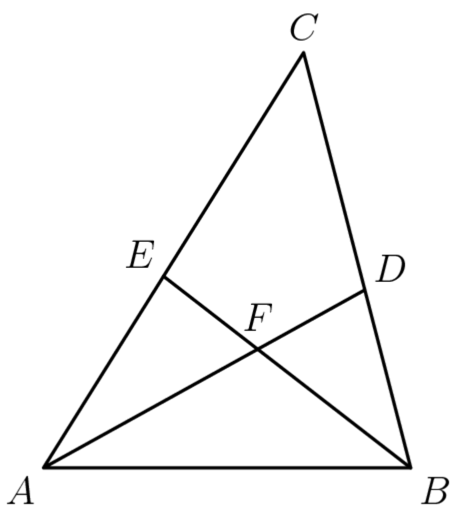
$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 49 \qquad \textbf{(E)}\ 64$

Problem 12

In $\triangle ABC$, $AB = 6$, $BC = 7$, and $CA = 8$. Point D lies on $\overline{BC}$, and $\overline{AD}$ bisects $\angle BAC$. Point E lies on $\overline{AC}$, and $\overline{BE}$ bisects $\angle ABC$. The bisectors intersect at F. What is the ratio $AF : FD$?

在$\triangle ABC$中， AB=6 ，BC=7 ，CA=8，点 D 在线段$\overline{BC}$上，$\overline{AD}$平分$\angle BAC$，.点

E 在$\overline{AC}$上，$\overline{BE}$平分$\angle ABC$，这两条角平分线交于点 F。问 $AF : FD$是多少？



$\textbf{(A)}\ 3:2 \qquad \textbf{(B)}\ 5:3 \qquad \textbf{(C)}\ 2:1 \qquad \textbf{(D)}\ 7:3 \qquad \textbf{(E)}\ 5:2$

Problem 13

Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

N 是一个正整数且是 5 的倍数，一个红球和 *N* 个绿球被随机排成一排。令$P(N)$表示至少$\frac{3}{5}$的绿球在红球的同一边的概率，注意到，$P(5) = 1$，且当 *N* 很大时，$P(N)$接近$\frac{4}{5}$，问满足$P(N) < \frac{321}{400}$的最小的 *N* 的各个位上数字之和是多少？

P（5）=1

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

Problem 14

Each vertex of a cube is to be labeled with an integer from 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

一个立方体的每个顶点被 1 到 8 这 8 个整数标记，且每个整数只能使用 1 次，满足每个面的 4 个顶点上的数字之和都相同，两种排列方法如果可以通过旋转立方体变成同一种，那么这两种排列方法被认为是同一种方法，问一共有多少种不同的排列方法？

(A) 1 **(B)** 3 **(C)** 6 **(D)** 12 **(E)** 24

Problem 15

Circles with centers P, Q and R, having radii $1, 2$ and 3, respectively, lie on the same side of line l and are tangent to l at P', Q' and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of triangle PQR?

圆心为 P，Q 和 R 的圆半径分别为 1，2，3，它们在直线 l 的同一边，且和 l 相切，切点分别为P', Q'，R'，其中Q'位于P'和R'之间，圆心为 Q 的圆和另外两个圆外切，三角形 PQR 的面积是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \sqrt{6}/3 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \sqrt{6}-\sqrt{2} \qquad \textbf{(E)}\ \sqrt{6}/2$

Problem 16

The graphs of $y=\log_3 x, y=\log_x 3, y=\log_{\frac{1}{3}} x,$ and $y=\log_x \dfrac{1}{3}$ are plotted on the same set of axis. How many points in the plane with positive x-coordinates lie on two or more of the graphs?

将函数$y=\log_3 x, y=\log_x 3, y=\log_{\frac{1}{3}} x,$ $y=\log_x \dfrac{1}{3}$的图像画在同一坐标系内，那么平面内有多少个横坐标为正的点位于这里所画的两条或者多条曲线上？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 17

Let $ABCD$ be a unit square. Let E, F, G and H be the centers, respectively, of equilateral triangles with bases $\overline{AB}, \overline{BC}, \overline{CD},$ and $\overline{DA},$ each exterior to the square. What is the area of square $EFGH$?

$ABCD$ 是一个单位正方形（边长为 1 的正方形）。分别以$\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA},$为底，向外作 4 个等边三角形，它们的中心分别是点 E，F，G 和 H。问正方形 $EFGH$ 的面积是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \dfrac{2+\sqrt{3}}{3} \qquad \textbf{(C)}\ \sqrt{2} \qquad \textbf{(D)}\ \dfrac{\sqrt{2}+\sqrt{3}}{2} \qquad \textbf{(E)}\ \sqrt{3}$

Problem 18

For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

n 是个正整数，数字$110n^3$有 110 个正整数因子，包括 1 和$110n^3$，那么数字$81n^4$前有多少个正整数因子？

$\textbf{(A) }110\qquad\textbf{(B) }191\qquad\textbf{(C) }261\qquad\textbf{(D) }325\qquad\textbf{(E) }425$

Problem 19

Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{a}{b},$ where a and b are relatively prime positive integers. What is $a + b?$ (For example, he succeeds if his sequence of tosses is $HTHHHHHH$.)

Jerry 从数轴的 0 点开始移动。他将一枚标准的硬币抛了 8 次。当硬币正面朝上时，他就沿着数轴正方向移动 1 个单位；当反面朝上时，他就朝负方向移动 1 个单位。他在移动过程中的某个时间点到达 4 的概率是$\frac{a}{b}$,其中 a 和 b 是互质的正整敛。那么 $a+b$ 等于多少？（例如，若用 H 表示正面朝上，T 表示反面朝上，那么如果某 8 次抛币结果为 *HTHHHHHH*，他就成功了）

$\textbf{(A) }69\qquad\textbf{(B) }151\qquad\textbf{(C) }257\qquad\textbf{(D) }293\qquad\textbf{(E) }313$

Problem 20

A binary operation $\diamondsuit$ has the properties that $a\,\diamondsuit\, (b\,\diamondsuit \,c) = (a\,\diamondsuit \,b)\cdot c$ and that $a\,\diamondsuit \,a=1$ for all nonzero real numbers a, b and c. (Here the dot . represents the usual multiplication operation.) The solution to the equation $2016 \,\diamondsuit\, (6\,\diamondsuit\, x)=100$ can be written as $\frac{p}{q},$ where p and q are relatively prime positive integers. What is $p+q?$

二元运算符$\diamondsuit$有这样的性质：对于所有的非零实数 a，b，c，有$a\,\diamondsuit\, (b\,\diamondsuit \,c) = (a\,\diamondsuit \,b)\cdot c$，且$a\,\diamondsuit \,a=1$，（这里·表示乘法）。方程$2016 \,\diamondsuit\, (6\,\diamondsuit\, x)=100$的解可写成$\frac{p}{q}$，其中$p$和$q$是互质的正整数，则 $p+q$ 等于多少？

$\textbf{(A) }109\qquad\textbf{(B) }201\qquad\textbf{(C) }301\qquad\textbf{(D) }3049\qquad\textbf{(E) }33,601$

Problem 21

A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of its fourth side?

一个四边形内接在半径为$200\sqrt{2}$.的圆内，四边形的三条边长度都是 200，那么第四条边长度是多少？

$\textbf{(A) }200 \qquad \textbf{(B) }200\sqrt{2} \qquad \textbf{(C) }200\sqrt{3} \qquad \textbf{(D) }300\sqrt{2} \qquad \textbf{(E) }500$

Problem 22

How many ordered triples (x,y,z) of positive integers

satisfy $\text{lcm}(x,y) = 72, \text{lcm}(x,z) = 600$ and $\text{lcm}(y,z) = 900$?

有多少个这样的有序正整数三元组(x,y,z)，满足$\text{lcm}(x,y) = 72, \text{lcm}(x,z) = 600$，且 $\text{lcm}(y,z) = 900$？

$\textbf{(A) }15 \qquad \textbf{(B) }16 \qquad \textbf{(C) }24 \qquad \textbf{(D) }27 \qquad \textbf{(E) }64$

Problem 23

Three numbers in the interval $[0,1]$ are chosen independently and at random. What is the probability that the chosen numbers are the side lengths of a triangle with positive area?

从区间［0，1］中独立随机地选择 3 个数。问选择的这 3 个数是一个面积为正的三角形的三条边的边长的概率是多少？

$\textbf{(A) }\frac{1}{6} \qquad \textbf{(B) }\frac{1}{3} \qquad \textbf{(C) }\frac{1}{2} \qquad \textbf{(D) }\frac{2}{3} \qquad \textbf{(E) }\frac{5}{6}$

Problem 24

There is a smallest positive real number a such that there exists a positive real number b such that all the roots of the polynomial $x^3 - ax^2 + bx - a$ are real. In fact, for this value of a the value of b is unique. What is the value of b?

存在一个最小的正实数 a，对应这个 a 值，存在一个正实数 b，使得多项式$x^3 - ax^2 + bx - a$ 的所有根都是实根。实际上，对于这个 a 值，正实数 b 是唯一的。那么 b 的值是多少？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

Problem 25

Let k be a positive integer. Bernardo and Silvia take turns writing and erasing numbers on a blackboard as follows: Bernardo starts by writing the smallest perfect square with $k+1$ digits. Every time Bernardo writes a number, Silvia erases the last k digits of it. Bernardo then writes the next perfect square, Silvia erases the last k digits of it, and this process continues until the last two numbers that remain on the board differ by at least 2. Let $f(k)$ be the smallest positive integer not written on the board. For example, if $k=1$, then the numbers that Bernardo writes are $16, 25, 36, 49, 64$, and the numbers showing on the board after Silvia erases are $1, 2, 3, 4,$ and 6, and thus $f(1)=5$. What is the sum of the digits of $f(2)+f(4)+f(6)+...+f(2016)$?

k 是个正整数。Bernardo 和 Silvia 以如下方式轮流在黑板上写数字和擦数字：Bemardo 一开始写下的数字是最小的 k+1 位的平方数。每次 Bernardo 写下一个数字，Silvia 就把这个数字的最后 k 位擦除。然后 Bernardo 继续写出下一个平方数，Silvia 又把这个数字的最后 k 位擦除。这个过程一直持续下去，直到在黑板上留下的最后两个数字的差至少为 2，令$f(k)$表示在黑板上没有写的最小正整数。例如，若 k=1，那么 Bernardo 写下的数字分别是,16，25，36，49，64，当 Silvia 擦除后，黑板上留下的数字是 1,2,3,4 和 6, 因此$f(1)=5$。问

$f(2)+f(4)+f(6)+...+f(2016)$的结果的各个位上的数字之和是多少

$\textbf{(A)}\ 7986 \qquad \textbf{(B)}\ 8002 \qquad \textbf{(C)}\ 8030 \qquad \textbf{(D)}\ 8048 \qquad \textbf{(E)}\ 8064$

2016 AMC 12A Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
B	C	B	D	E	D	D	D	E	B	E	C	A
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	D	D	B	D	B	A	E	A	C	B	E	

2017 AMC12A

Problem 1

Pablo buys popsicles for his friends. The store sells single popsicles for $1 each, 3-popsicle boxes for $2, and 5-popsicle boxes for $3. What is the greatest number of popsicles that Pablo can buy with $8?

Pablo 给他的朋友买棒冰，商店单根棒冰卖 1 美元一根，1 盒 3 根棒冰每盒卖 2 美元，1 盒 5 根棒冰每盒卖 3 美元，Pablo 有 8 美元，最多可以买多次根棒冰？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 13 \qquad \textbf{(E)}\ 15$

Problem 2

The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

两个非零实数的和是它们乘积的 4 倍，问这 2 个数的倒数之和是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 12$

Problem 3

Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?

Carroll 女士承诺任何人只要在即将到来的考试中做对所有的选择题，此次考试就能得到一个 A，由此，下面哪句话在逻辑上是对的？

(A) If Lewis did not receive an A，then he got all of the multiple choice questions wrong | 如果 Lewis 没有得到 A，那么他的所有选择题做的都是错的。

(B) If Lewis did not receive an A，then he got at least one of the multiple choice questions wrong. | 如果 Lewis 没有得到 A，那么他至少有一道选择题做的是错的。

(C) If Lewis got at least one of the multiple choice questions wrong,then he did not receive an A. | 如果 Lewis 至少有一个选择题做错了，那么他就得不到 A.

(D) If Lewis received an A，then he got all of the multiple choice questions right. | 如果 Lewis 得到一个 A，那么他所有的选择题都做对了.

(E) If Lewis reccived an A,then he got at least one of the multiple choice questions right. | 如果 Lewis 得到一个 A，那么他至少做对了一道选择题。

Problem 4

Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

Jerry 和 Silvia 想从一块正方形场地的西南角走到东北角。Jerry 先朝东走，再朝北走，走到达目标，而 Sina 朝着东北角直接走直线到达目标，Silvia 所走的路程和 Jerry 所走路程相比，少走多少，更接近下面哪个数值？

$\textbf{(A)}\ 30\% \qquad \textbf{(B)}\ 40\% \qquad \textbf{(C)}\ 50\% \qquad \textbf{(D)}\ 60\% \qquad \textbf{(E)}\ 70\%$

Problem 5

At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

一次聚会有 30 人，其中有 20 人相互之间都认识，还有 10 人任何人都不认识，如果相互之间认识的人拥抱，相互之间不认识的人握手，那么一共握手多少次？

$\textbf{(A)}\ 240 \qquad \textbf{(B)}\ 245 \qquad \textbf{(C)}\ 290 \qquad \textbf{(D)}\ 480 \qquad \textbf{(E)}\ 490$

Problem 6

Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

Joy 一共有 30 根薄竹竿，每根竹竿的长度是 1cm 到 30cm 之间的不同的整数长度. 她把长度为 3cm，7cm 和 15cm 的竹竿放在桌上，然后她想选择第 4 根竹竿，保证能和这 3 根形成一个面积为正的四边形，剩下的竹竿中，有多少根她可以选择作为第 4 根？

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 19 \qquad \textbf{(E)}\ 20$

Problem 7

Define a function on the positive integers recursively by $f(1)=2$, $f(n)=f(n-1)+1$ if n is even, and $f(n)=f(n-2)+2$ if n is odd and greater than 1. What is $f(2017)$?

定义一个在正整数上的函数，满足$f(1)=2$，当n是偶数时，则$f(n)=f(n-1)+1$，当n是大于 1 的奇数时，则$f(n)=f(n-2)+2$，问$f(2017)$是多少？

$\textbf{(A)}\ 2017 \qquad \textbf{(B)}\ 2018 \qquad \textbf{(C)}\ 4034 \qquad \textbf{(D)}\ 4035 \qquad \textbf{(E)}\ 4036$

Problem 8

The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

和线段$\overline{AB}$的距离在 3 个单位长以内的所有点，在三维空间中组成的区域的体积为 216π，AB 的长度是多少？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 24$

Problem 9

Let S be the set of points (x,y) in the coordinate plane such that two of the three quantities 3, $x+2$, and $y-4$ are equal and the third of the three quantities is no greater than the common value. Which of the following is a correct description of S?

S 表示这三个量 3，$x+2$和$y-4$中有 2 个量相等，且第 3 个量不大于这个共同值的坐标平面内所有的点(x,y)所组成的集合，那么下面哪个是对 *S* 的正确描述？

(A) a single point丨1 个点
(B) two intersecting lines丨2 条相交的直线
(C) three lines whose pairwise intersections are three distinct points，丨3 条直线，这 3 条直线两两相交形成 3 个不同的点
(D) a triangle丨1 个三角形
(E) three rays with a common point丨有一个公共点的 3 条射线

Problem 10

Chloé chooses a real number uniformly at random from the interval $[0, 2017]$. Independently,

Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloe's number?

Chloe 从区间［0，2017］中均匀随机的选择一个实数，Laurent 独立的从区间［0，4034］中均匀随机的选择一个实数，Laurent 的数比 Chloe 的大的概率是多少？

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{2}{3} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ \frac{5}{6} \qquad \textbf{(E)}\ \frac{7}{8}$

Problem 11

Claire adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. She then discovers that she forgot to include one angle. What is the degree measure of the forgotten angle?

Claire 把一个凸多边形的内角的度数相加，得到和为 2017，后来她发现漏加了一个角，那么漏加的这个角的度数是多少度？

$\textbf{(A)}\ 37 \qquad \textbf{(B)}\ 63 \qquad \textbf{(C)}\ 117 \qquad \textbf{(D)}\ 143 \qquad \textbf{(E)}\ 163$

Problem 12

There are 10 horses, named Horse 1, Horse 2, $\ldots$, Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

一共有 10 匹马，名字分别为马 1，马 2，…，马 10，这是根据这些马沿着圆形赛道跑一圈所需要的分钟数命名的：马 k 跑 1 圈需要 k 分钟。在初始时间 0 时，所有的马都在跑道的起点，这些马开始朝着同一方向跑，并且各自速度恒定。$S > 0$（以分钟为单位）表示这 10 匹马再次同时处于起点的最短时间，在这里$S = 2520$。$T > 0$（以分钟为单位）表示至少 5 匹马再次同时处于起点的最短时间，那么 T 的各位数字之和是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 13

Driving at a constant speed, Sharon usually takes 180 minutes to drive from her house to her mother's house. One day Sharon begins the drive at her usual speed, but after driving $\frac{1}{3}$ of the way, she hits a bad snowstorm and reduces her speed by 20 miles per hour. This time the trip takes her a total of 276 minutes. How many miles is the drive from Sharon's house to her mother's house?

Sharon 平常以恒定的速度从她家开车到她妈妈家，一般需要 180 分钟。一天 Sharon 以她平常的开车速度开了$\frac{1}{3}$路程后，遭遇暴风雪，接着把速度降低了 20 英里每小时。这一次，全程开完她总共花了 276 分钟。那么从 Sharon 家到她妈妈家总共多少英里？

$\textbf{(A)}\ 132 \qquad \textbf{(B)}\ 135 \qquad \textbf{(C)}\ 138 \qquad \textbf{(D)}\ 141 \qquad \textbf{(E)}\ 144$

Problem 14

Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

Alice 拒绝坐在 Bob 或 Carla 的旁边，Derek 拒绝坐在 Eric 的旁边，在满足以上要求的情况下，他们 5 人要坐在一排 5 张椅子上，一共有多少种坐法？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 28 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 40$

Problem 15

Let $f(x) = \sin{x} + 2\cos{x} + 3\tan{x}$, using radian measure for the variable x. In what interval does the smallest positive value of x for which $f(x) = 0$ lie?

令$f(x) = \sin{x} + 2\cos{x} + 3\tan{x}$，其中未知数 x 是弧度制，使得$f(x) = 0$的最小正数 x 在下面哪个区间内？

$\textbf{(A)}\ (0,1) \qquad \textbf{(B)}\ (1,2) \qquad \textbf{(C)}\ (2,3) \qquad \textbf{(D)}\ (3,4) \qquad \textbf{(E)}\ (4,5)$