2015-2019年AMC12B真题汇编含答案

2015 AMC12B

Problem 1

What is the value of $2-(-2)^{-2}$?

$2-(-2)^{-2}$的值是多少？

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ \frac{1}{16} \qquad \textbf{(C)}\ \frac{7}{4} \qquad \textbf{(D)}\ \frac{9}{4} \qquad \textbf{(E)}\ 6$

Problem 2

Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

Marie 连续做了 3 项耗时相同的任务，中间没有休息，她从下午 1:00 开始做第一个任务，在下午 2:40 完成第二个任务，她何时完成第三个任务？

$\textbf{(A)}\ \text{3:10 PM} \qquad \textbf{(B)}\ \text{3:30 PM} \qquad \textbf{(C)}\ \text{4:00 PM} \qquad \textbf{(D)}\ \text{4:10 PM} \qquad \textbf{(E)}\ \text{4:30 PM}$

Problem 3

Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

Isaac 把一个整数写了 2 遍，另一个整数写了 3 遍。这 5 个数之和为 100，其中一个数是 28，另一个数是多少？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 11 \qquad \textbf{(C)}\ 14 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 18$

Problem 4

David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

David，Hikmet，Jack，Marta，Rand，Todd 和其他 6 人参加了一场 12 人的比赛。Rand 排在 Hikamet 前面 6 位，Marta 排在 Jaok 后面 1 位，David 排在 Hikmet 的后面 2 位，Jack 排在 Todd 后面 2 位，Todd 排在 Rand 后面 1 位。Marta 排名第 6，那么谁排名第 8?

$\textbf{(A)}\ \text{David} \qquad \textbf{(B)}\ \text{Hikmet} \qquad \textbf{(C)}\ \text{Jack} \qquad \textbf{(D)}\ \text{Rand} \qquad \textbf{(E)}\ \text{Todd}$

Problem 5

The Tigers beat the Sharks 2 out of the 3 times they played. They then played N more times, and the Sharks ended up winning at least 95% of all the games played. What is the minimum possible value for N?

老虎队在和鲨鱼队的 3 场比赛中赢了 2 场，然后他们又多打了 N 场比赛，最终鲨鱼队赢了比赛总场数的至少 95%，那么 N 的最小可能值是多少?

$\textbf{(A)}\ 35 \qquad \textbf{(B)}\ 37 \qquad \textbf{(C)}\ 39 \qquad \textbf{(D)}\ 41 \qquad \textbf{(E)}\ 43$

Problem 6

Back in 1930, Tillie had to memorize her multiplication facts from 0×0 to 12×12. The multiplication table she was given had rows and columns labeled with the factors, and the products formed the body of the table. To the nearest hundredth, what fraction of the numbers in the body of the table are odd?

在 1930 年，Tillie 需要记忆从0×0到12×12的所有乘法结果。她所用的乘法表的每行每列都用乘数标记，所得的乘积形成了表的主体，那么在这张表的主体中，有多少比例的数是奇数？结果保留两位小数。

$\textbf{(A)}\ 0.21 \qquad \textbf{(B)}\ 0.25 \qquad \textbf{(C)}\ 0.46 \qquad \textbf{(D)}\ 0.50 \qquad \textbf{(E)}\ 0.75$

Problem 7

A regular 15-gon has L lines of symmetry, and the smallest positive angle for which it has rotational symmetry is R degrees. What is $L+R$?

一个正十五边形有 L 条对称轴，且具有旋转对称的最小角度是 R 度，问 $L+R$ 是多少？

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 32 \qquad \textbf{(D)}\ 39 \qquad \textbf{(E)}\ 54$

Problem 8

What is the value of $(625^{\log_5 2015})^{\frac{1}{4}}$?

$(625^{\log_5 2015})^{\frac{1}{4}}$的值是多少？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ \sqrt[4]{2015} \qquad \textbf{(C)}\ 625 \qquad \textbf{(D)}\ 2015 \qquad \textbf{(E)}\ \sqrt[4]{5^{2015}}$

Problem 9

Larry and Julius are playing a game, taking turns throwing a ball at a bottle sitting on a ledge. Larry throws first. The winner is the first person to knock the bottle off the ledge. At each turn the probability that a player knocks the bottle off the ledge is $\tfrac{1}{2}$, independently of what has happened before. What is the probability that Larry wins the game?

Larry 和 Julius 轮流玩一种用球扔倒瓶子的游戏。Larry 先扔。赢家是第一个将瓶子扔倒的人。在每一回，每个选手扔倒瓶子的概率都是$\tfrac{1}{2}$，且相互独立。问 Larry 赢得游戏的概率是多少？

$\textbf{(A)}\ \dfrac{1}{2} \qquad \textbf{(B)}\ \dfrac{3}{5} \qquad \textbf{(C)}\ \dfrac{2}{3} \qquad \textbf{(D)}\ \dfrac{3}{4} \qquad \textbf{(E)}\ \dfrac{4}{5}$

Problem 10

How many noncongruent integer-sided triangles with positive area and perimeter less than 15 are neither equilateral, isosceles, nor right triangles?

有多少个不全等且边长为整数的三角形，满足面积为正，周长小于 15，并且既不是等边三角形，也不是等腰三角形或者直角三角形？

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Problem 11

The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

直线$12x + 5y = 60$与坐标轴形成一个三角形，这个三角形的三条高的长度之和是多少？

(A) 20 **(B)** $\frac{360}{17}$ **(C)** $\frac{107}{5}$ **(D)** $\frac{43}{2}$ **(E)** $\frac{281}{13}$

Problem 12

Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x-a)(x-b)+(x-b)(x-c)=0$?

a，b，c 是 3 个不同的 1 位数，方程$(x-a)(x-b)+(x-b)(x-c)=0$的根之和的最大值是多少？

(A) 15 **(B)** 15.5 **(C)** 16 **(D)**16.5 **(E)** 17

Problem 13

Quadrilateral $ABCD$ is inscribed in a circle with $\angle BAC = 70^\circ, \angle ADB = 40^\circ, AD = 4,$ and $BC = 6$. What is AC?

四边形 $ABCD$ 内接在一个圆内，且满足$\angle BAC = 70^\circ, \angle ADB = 40^\circ, AD = 4, BC = 6$，问 AC 是多长?

(A) $3+\sqrt{5}$ **(B)** 6 **(C)** $\frac{9}{2}\sqrt{2}$ **(D)** $8-\sqrt{2}$ **(E)** 7

Problem 14

A circle of radius 2 is centered at A. An equilateral triangle with side 4 has a vertex at A. What is the difference between the area of the region that lies inside the circle but outside the triangle and the area of the region that lies inside the triangle but outside the circle?

一个半径为 2 的圆，圆心在 A 点。一个边长为 4 的等边三角形的一个顶点是 A 点。同位于圆内但在三角形之外的区域面积和位于圆外但在三角形之内的区域面积的差是多少？

$\textbf{(A)}\ 8-\pi \qquad \textbf{(B)}\ \pi+2 \qquad \textbf{(C)}\ 2\pi-\frac{\sqrt{2}}{2} \qquad \textbf{(D)}\ 4(\pi-\sqrt{3}) \qquad \textbf{(E)}\ 2\pi-\frac{\sqrt{3}}{2}$

Problem 15

At Rachelle's school an A counts 4 points, a B 3 points, a C 2 points, and a D 1 point. Her GPA on the four classes she is taking is computed as the total sum of points divided by 4. She is certain that she will get As in both Mathematics and Science, and at least a C in each of English and History. She thinks she has a $\frac{1}{6}$ chance of getting an A in English, and a $\frac{1}{4}$ chance of getting a B. In History, she has a $\frac{1}{4}$ chance of getting an A, and a $\frac{1}{3}$ chance of getting a B, independently of what she gets in English. What is the probability that Rachelle will get a GPA of at least 3.5?

在 Rachelle 的学校，一个 A 是 4 分，一个 B 是 3 分，一个 C 是 2 分，一个 D 是 1 分。她的四门课的 GPA 分数是通过将四门课的总分除以 4 算得的。她很确定，她会在数学和科学这两门课中得 A，在英语和历史这两门课的每一门都至少是 1 个 C，她认为英语获得 A 的概率是$\frac{1}{6}$，获得 B 的概率是$\frac{1}{4}$；而对于历史这门课，她获得 A 的概率是$\frac{1}{4}$，获得 B 的概率是$\frac{1}{3}$，而与她的英语成绩无关。问 Rachelle 的 GPA 至少是 3.5 分的概率是多少？

$\textbf{(A)}\ \frac{11}{72} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ \frac{3}{16} \qquad \textbf{(D)}\ \frac{11}{24} \qquad \textbf{(E)}\ \frac{1}{2}$

Problem 16

A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?

以一个边长为 6 的正六边形的每条边为底，作出 6 个等腰三角形，且它们的腰长均为 8，再把这些等腰三角形折起来做成一个以六边形为底的棱锥。求此棱锥的体积是多少？

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 162 \qquad \textbf{(C)}\ 36\sqrt{21} \qquad \textbf{(D)}\ 18\sqrt{138} \qquad \textbf{(E)}\ 54\sqrt{21}$

Problem 17

An unfair coin lands on heads with a probability of $\frac{1}{4}$. When tossed n times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of n ?

一枚非标准硬币正面朝上的概率为$\frac{1}{4}$。当被抛了 n 次，恰好 2 次正面朝上的概率和恰好 3 次正面朝上的概率相同。问 n 是多少？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 13$

Problem 18

For every composite positive integer n, define $r(n)$ to be the sum of the factors in the prime factorization of n. For example, $r(50) = 12$ because the prime factorization of 50 is 2×5^2, and $2 + 5 + 5 = 12$. What is the range of the function r, $\{r(n) : n \text{ is a composite positive integer}\}$?

对于每个正的合数，定义$r(n)$为 n 的质因数分解中所有因子之和。例如，$r(50) = 12$，因为 50 的质因数分解为2×5^2，并且$2 + 5 + 5 = 12$。那么函数 r 的值域是多少？｛$r(n)$：n 是一个正的合数｝.

（A）The set of positive integers | 所有正整数组成的集合

（B）The set of compositive positive integers | 所有正合数组成的築合

（C）The set of even positive integers | 所有正的偶数组成的集合

（D）The set of integers greater than 3 | 所有大于 3 的整数集

（E）The set of integers greater than 4 | 所有大于 4 的整数集

Problem 19

In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

在$\triangle ABC$中，$\angle C = 90^\circ$，$AB = 12$，在三角形的外部作出正方形 $ABXY$和 $ACWZ$，点 X，Y，Z和 W在同一个圆上，这个三角形的周长是多少？

$\textbf{(A)}\ 12+9\sqrt{3} \qquad \textbf{(B)}\ 18+6\sqrt{3} \qquad \textbf{(C)}\ 12+12\sqrt{2} \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 32$

Problem 20

For every positive integer n, let $\text{mod}_5(n)$ be the remainder obtained when n is divided by 5. Define a function $f:\{0,1,2,3,\dots\}\times\{0,1,2,3,4\}\to\{0,1,2,3,4\}$ recursively as follows:

$$f(i,j)=\begin{cases}\text{mod}_5(j+1) & \text{if } i=0 \text{ and } 0\le j\le 4,\\ f(i-1,1) & \text{if } i\ge 1 \text{ and } j=0, \text{ and}\\ f(i-1,f(i,j-1)) & \text{if } i\ge 1 \text{ and } 1\le j\le 4.\end{cases}$$

What is $f(2015,2)$?

对于每个正整数 n， $\text{mod}_5(n)$表示 n 除以 5 所得余数，定义函数 $f:\{0,1,2,3,\dots\}\times\{0,1,2,3,4\}\to\{0,1,2,3,4\}$如下：

$$f(i,j)=\begin{cases}\text{mod}_5(j+1) & \text{if } i=0 \text{ and } 0\le j\le 4,\\ f(i-1,1) & \text{if } i\ge 1 \text{ and } j=0, \text{ and}\\ f(i-1,f(i,j-1)) & \text{if } i\ge 1 \text{ and } 1\le j\le 4.\end{cases}$$

则$f(2015,2)$是多少？

$\textbf{(A) } 0 \qquad \textbf{(B) } 1 \qquad \textbf{(C) } 2 \qquad \textbf{(D) } 3 \qquad \textbf{(E) } 4$

Problem 21

Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

Cozy 这只猫和 Dash 这条狗在有一定数目台阶的楼梯上爬楼梯，然而，Cozy 和 Dash 都是跳着爬楼梯，而不是一次一个台阶地走。Cozy 每跳一次就爬 2 个台阶（如果必要的话，他会仅仅跳过最后一个台阶）。Dash 每跳一次就爬 5 个台阶（如果最后剩下不足 5 个台阶，他就直接一步跳过去）。假设最终到达楼梯的顶部，Dash 跳的次数比 Cozy 少了 19 次，s 表示这个楼梯所有可能的台阶数的和，那么 s 的各个位上的数字之和是多少？

$\textbf{(A) } 9 \qquad \textbf{(B) } 11 \qquad \textbf{(C) } 12 \qquad \textbf{(D) } 13 \qquad \textbf{(E) } 15$

Problem 22

Six chairs are evenly spaced around a circular table. One person is seated in each chair. Each person gets up and sits down in a chair that is not the same chair and is not adjacent to the chair he or she originally occupied, so that again one person is seated in each chair. In how many ways can this be done?

6 张椅子均匀等距地绕着圆桌摆放，每张椅子坐 1 人。每个人都站起来，并且坐到新的椅子上去，新的椅子不能是这个人原来坐的那张椅子，也不能是他原来坐的椅子的左邻或者右邻的椅子。这样，每个人都各自坐到了一张新的椅子上。问一共有多少种坐法？

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 18 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 24$

Problem 23

A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1 \leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a,b,c) are possible?

一个长方形的盒子大小是$a \times b \times c$，其中 a，b，c 都是整数，且$1 \leq a \leq b \leq c$，盒子的体积和表面积在数值上是相等的，那么可能的有序对（a，b，c）有多少个？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 21 \qquad \textbf{(E)}\ 26$

Problem 24

Four circles, no two of which are congruent, have centers at A, B, C, and D, and points P and Q lie on all four circles. The radius of circle A is $\tfrac{5}{8}$ times the radius of circle B, and the radius of circle C is $\tfrac{5}{8}$ times the radius of circle D. Furthermore, $AB = CD = 39$ and $PQ = 48$. Let R be the midpoint of $\overline{PQ}$. What is $AR+BR+CR+DR$?

四个圆心分别为 A，B，C，D 的圆两两均不全等，点 P 和点 Q 都在这四个圆上。圆 A 的半径是圆 B 半径的$\frac{5}{8}$，圆 C 的半径是圆 D 半径的$\frac{5}{8}$，且 AB=CD=39， PQ=48，点 R 是$\overline{PQ}$的中点，那么 AR+BR+CR +DR 是多少？

$\textbf{(A)}\ 180 \qquad \textbf{(B)}\ 184 \qquad \textbf{(C)}\ 188 \qquad \textbf{(D)}\ 192 \qquad \textbf{(E)}\ 196$

Problem 25

A bee starts flying from point P_0. She flies 1 inch due east to point P_1. For $j \geq 1$, once the bee reaches point P_j, she turns 30° counterclockwise and then flies $j+1$ inches straight to point P_{j+1}. When the bee reaches P_{2015} she is exactly $a\sqrt{b}+c\sqrt{d}$ inches away from P_0, where a, b, c and d are positive integers and b and d are not divisible by the square of any prime. What is $a+b+c+d$?

一只蜜蜂从点P_0开始飞行。她先向东飞行 1 英寸到达点P_1，对于$j \geq 1$，一旦蜜蜂到达点P_j，她就逆时针转 30°，然后直线飞行$j+1$英寸到达点P_{j+1}，当蜜蜂到达点P_{2015}，她距离点P_0恰好$a\sqrt{b}+c\sqrt{d}$英寸，其中 a，b，c 和 d 都是正整数，且 b 和 d 都不能被任何质数的平方整除。问$a+b+c+d$等于多少？

(A) 2016　　**(B)** 2024　　**(C)** 2032　　**(D)** 2040　　**(E)** 2048

2015 AMC 12B Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	A	B	B	A	D	D	C	C	E	D	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	D	C	D	D	C	B	D	D	B	D	B	

2016 AMC12B

Problem 1

What is the value of $\frac{2a^{-1}+\frac{a^{-1}}{2}}{a}$ when $a=\frac{1}{2}$?

当$a=\frac{1}{2}$时，$\frac{2a^{-1}+\frac{a^{-1}}{2}}{a}$的值是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ \frac{5}{2} \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 20$

Problem 2

The harmonic mean of two numbers can be calculated as twice their product divided by their sum. The harmonic mean of 1 and 2016 is closest to which integer?

两个数的调和平均值可以由它们乘积的 2 倍除以它们的和得到。那么 1 和 2016 的调和平均值最接近下面哪个整数？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 45 \qquad \textbf{(C)}\ 504 \qquad \textbf{(D)}\ 1008 \qquad \textbf{(E)}\ 2015$

Problem 3

Let $x=-2016$. What is the value of $\Bigg\vert\Big\vert |x|-x\Big\vert-|x|\Bigg\vert-x$?

令$x=-2016$，则$\Bigg\vert\Big\vert |x|-x\Big\vert-|x|\Bigg\vert-x$的值是多少？

$\textbf{(A)}\ -2016 \qquad \textbf{(B)}\ 0 \qquad \textbf{(C)}\ 2016 \qquad \textbf{(D)}\ 4032 \qquad \textbf{(E)}\ 6048$

Problem 4

The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

两个锐角的度数比值是$5:4$，且其中一个角的补角是另一个角的补角的 2 倍，那么这 2 个角的度数之和是多少?

$\textbf{(A)}\ 75 \qquad \textbf{(B)}\ 90 \qquad \textbf{(C)}\ 135 \qquad \textbf{(D)}\ 150 \qquad \textbf{(E)}\ 270$

Problem 5

The War of 1812 started with a declaration of war on Thursday, June 18, 1812. The peace treaty to end the war was signed 919 days later, on December 24, 1814. On what day of the week was the treaty signed?

1812 英美之战以 1812 年 6 月 18 日星期四的宣战开始，结束战争的和平协议的签订是在 919 天后的 1814 年 12 月 24 日，问签订协议的那天是星期几？

（A）Friday | 周五

（B）Saturday | 周六

（C）Sunday | 周日

（D）Monday | 周一

（E）Tuesday | 周二

Problem 6

All three vertices of $\triangle ABC$ lie on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?

$\triangle ABC$的 3 个顶点位于抛物线$y = x^2$上，其中点 *A* 在原点，*BC* 和 *x* 轴平行.，三角形的面积为 64，问 BC 的长度为多少?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 16$

Problem 7

Josh writes the numbers $1, 2, 3, \ldots, 99, 100$. He marks out 1, skips the next number (2), marks out 3, and continues skipping and marking out the next number to the end of the list. Then he goes back to the start of his list, marks out the first remaining number (2), skips the next number (4), marks out 6, skips 8, marks out 10, and so on to the end. Josh continues in this manner until only one number remains. What is that number?

Josh 写下一列数字 1，2，3，…，99，100，他划掉 1，跳过 2，划掉 3，并继续跳过和划掉接下来的数字，直到这列数字的末尾。然后他返回到列表的开头，划掉剩下的第一个数字（2），跳过下一个数字（4），划掉 6，跳过 8，划掉 10，等等，一直到列表的末尾。Josh 重复这种方式，直到最后列表只剩 1 个数字。问这个数字是多少?

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 56 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 96$

Problem 8

A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

一个形状为正三角形，密度均匀的薄木板边长为 3 英寸，重量为 12 盎司，第二块同样木质，相同厚度，形状也为正三角形的薄木板边长为 5 英寸，下面哪个数字最接近第二块木板的重量（单位为盎司）？

$\textbf{(A)}\ 14.0 \qquad \textbf{(B)}\ 16.0 \qquad \textbf{(C)}\ 20.0 \qquad \textbf{(D)}\ 33.3 \qquad \textbf{(E)}\ 55.6$

Problem 9

Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

Carl 决定把他的长方形花园用栅栏围起来，他买了 20 个栅栏木桩，在 4 个角落各放 1 个，剩余的木桩则沿着花园的边缘均匀放置，相邻木桩之间的距离恰好是 4 码，花园的长边（包括角落）所插的木桩的数目是短边（包含角落）所插木桩数目的 2 倍，那么 Carl 的花园的面积是多少平方码?

$\textbf{(A)}\ 256 \qquad \textbf{(B)}\ 336 \qquad \textbf{(C)}\ 384 \qquad \textbf{(D)}\ 448 \qquad \textbf{(E)}\ 512$

Problem 10

A quadrilateral has vertices $P(a,b)$, $Q(b,a)$, $R(-a,-b)$, and $S(-b,-a)$, where a and b are integers with $a>b>0$. The area of $PQRS$ is 16. What is $a+b$?

一个四边形的顶点为 P（a，b），Q（b，a），R（$-a$，$-b$），S（$-b$，$-a$），其中 a 和 b 都是整数，且$a>b>0$，四边形 $PQRS$ 的面积是 16，那么$a+b$是多少？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 13$

Problem 11

How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y=\pi x$, the line $y=-0.1$ and the line $x=5.1$?

有多少个这样的正方形，满足这样的条件：它们的边和坐标轴平行，它们的顶点的坐标都是整数，且正方形位于直线$y=\pi x$，直线$y=-0.1$和直线$x=5.1$ 所包围的区域内?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 41 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 57$

Problem 12

All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

1，2，3，4，5，6，7，8，9 这 9 个数被写入 1 个3×3的方格阵列中，每个小方格一个数字，满足如果两个数是连续的，那么这两个数所在的方格共享一条边，四个角落的数字之和为 18。问中心的那个数字是多少?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 13

Alice and Bob live 10 miles apart. One day Alice looks due north from her house and sees an airplane. At the same time Bob looks due west from his house and sees the same airplane. The angle of elevation of the airplane is 30° from Alice's position and 60° from Bob's position. Which of the following is closest to the airplane's altitude, in miles?

Alice 和 Bob 的家相距 10 英里。一天 Alice 从她家向北看，看到了一架飞机。同时 Bob 从他家向西看，也看到了这架飞机。Alice 看这架飞机的仰角是 30°，Bob 的仰角是 60°。下面哪个最接近飞机的高度（单位是英里）？

$\textbf{(A)}\ 3.5 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 4.5 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 5.5$

Problem 14

The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

一个无限等比数列所有项之和为一个正实数 S，数列的第二项是 1，那么 S 的最小值是多少?

$\textbf{(A)}\ \frac{1+\sqrt{5}}{2} \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ \sqrt{5} \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 15

All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

2，3，4，5，6，7 这 6 个数被分配给一个立方体的 6 个面。每个面一个数。根据立方体的 8 个顶点，计算出包含每个顶点的 3 个面上的数字的乘积，这 8 个乘积相加所得和的最大值是多少?

$\textbf{(A)}\ 312 \qquad \textbf{(B)}\ 343 \qquad \textbf{(C)}\ 625 \qquad \textbf{(D)}\ 729 \qquad \textbf{(E)}\ 1680$

Problem 16

In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

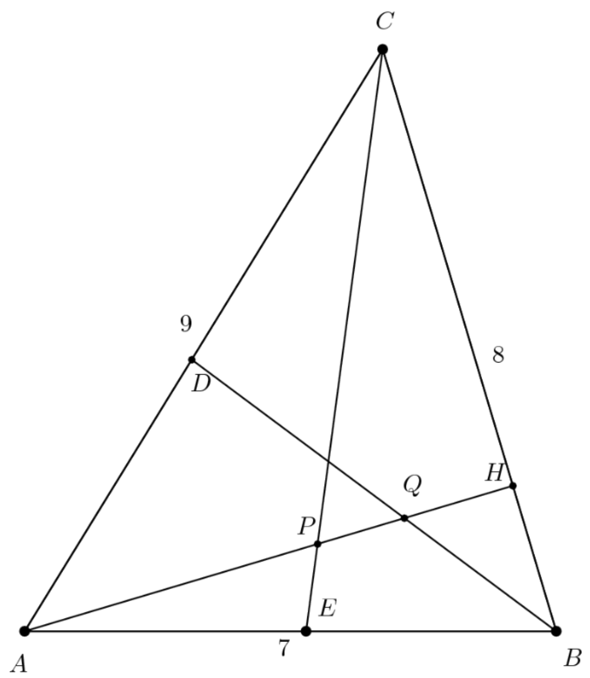
有多少种方法可以把 345 写成 2 个或更多个连续正整数组成的递增数列的所有项之和？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 17

In $\triangle ABC$ shown in the figure, $AB=7$, $BC=8$, $CA=9$, and $\overline{AH}$ is an altitude. Points D and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $\overline{BD}$ and $\overline{CE}$ are angle bisectors, intersecting $\overline{AH}$ at Q and P, respectively. What is PQ?

在如下图所示的$\triangle ABC$中， AB=7，BC=8，CA=9，且$\overline{AH}$是一条高。点 D 和 E 分别在边$\overline{AC}$和$\overline{AB}$上，且$\overline{BD}$和$\overline{CE}$都是角平分线，和$\overline{AH}$分别交于点 Q 和 P。那么 PQ 的长度是多少？



$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{5}{8}\sqrt{3} \qquad \textbf{(C)}\ \frac{4}{5}\sqrt{2} \qquad \textbf{(D)}\ \frac{8}{15}\sqrt{5} \qquad \textbf{(E)}\ \frac{6}{5}$

Problem 18

What is the area of the region enclosed by the graph of the equation $x^2+y^2=|x|+|y|$?

由方程$x^2+y^2=|x|+|y|$的图像所围成的区域的面积是多少？

$\textbf{(A)}\ \pi+\sqrt{2} \qquad \textbf{(B)}\ \pi+2 \qquad \textbf{(C)}\ \pi+2\sqrt{2} \qquad \textbf{(D)}\ 2\pi+\sqrt{2} \qquad \textbf{(E)}\ 2\pi+2\sqrt{2}$

Problem 19

Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which point he stops. What is the probability that all three flip their coins the same number of times?

Tom，Dick 和 Harry 在玩一个游戏，他们同时开始各自不断地抛一枚标准硬币，直到各自第一次得到正面朝上，那么他就停止抛硬币。问这三个人抛硬币的次数相同的概率是多少？

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{7} \qquad \textbf{(C)}\ \frac{1}{6} \qquad \textbf{(D)}\ \frac{1}{4} \qquad \textbf{(E)}\ \frac{1}{3}$

Problem 20

A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat A?

若干支队伍举行循环赛。在循环赛中，每支队伍和其他每支队伍恰好打一场比。每支队伍赢了 10 场比赛，输了 10 场比赛，且没有平局。问有多少个这样的｛A，B，C｝三支队伍的组合，满足 A 打败了 B，B 打败了 C，C 打败了 A?

$\textbf{(A)}\ 385 \qquad \textbf{(B)}\ 665 \qquad \textbf{(C)}\ 945 \qquad \textbf{(D)}\ 1140 \qquad \textbf{(E)}\ 1330$

Problem 21

Let $ABCD$ be a unit square. Let Q_1 be the midpoint of $\overline{CD}$. For $i = 1, 2, \dots,$ let P_i be the intersection of $\overline{AQ_i}$ and $\overline{BD}$, and let Q_{i+1} be the foot of the perpendicular from P_i to $\overline{CD}$. What is

$$\sum_{i=1}^{\infty} \text{Area of } \triangle DQ_iP_i\, ?$$

ABCD 是一个单位正方形。Q_1是$\overline{CD}$的中点。对于$i = 1, 2, \dots,$，令P_i为$\overline{AQ_i}$和$\overline{BD}$的交点。Q_{i+1}是过点P_i并垂直于$\overline{CD}$的垂线的垂足，则下式的值是多少

$$\sum_{i=1}^{\infty} \text{Area of } \triangle DQ_iP_i\, ?$$

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{1}{3} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ 1$

Problem 22

For a certain positive integer n less than 1000, the decimal equivalent of $\frac{1}{n}$ is $0.\overline{abcdef}$, a repeating decimal of period of 6, and the decimal equivalent of $\frac{1}{n+6}$ is $0.\overline{wxyz}$, a repeating decimal of period 4. In which interval does n lie?

n 是一个小于 1000 的正整数，已知$\frac{1}{n}$的小数表示是$0.\overline{abcdef}$，这是个循环周期为 6 的循环小数，而$\frac{1}{n+6}$的小数表示是$0.\overline{wxyz}$，这是个循环周期为 4 的循环小数。问 n 在哪个区间内？

$\textbf{(A)}\ [1,200] \qquad \textbf{(B)}\ [201,400] \qquad \textbf{(C)}\ [401,600] \qquad \textbf{(D)}\ [601,800] \qquad \textbf{(E)}\ [801,999]$

Problem 23

What is the volume of the region in three-dimensional space defined by the inequalities $|x|+|y|+|z|\le1$ and $|x|+|y|+|z-1|\le1$?

由不等式$|x|+|y|+|z|\le1$和$|x|+|y|+|z-1|\le1$定义的三维空间内区域的体积是多少？

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{1}{3} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ 1$

Problem 24

There are exactly $77,000$ ordered quadruplets (a,b,c,d) such that $\gcd(a,b,c,d)=77$ and $\operatorname{lcm}(a,b,c,d)=n$. What is the smallest possible value for n?

满足$\gcd(a,b,c,d)=77$和$\operatorname{lcm}(a,b,c,d)=n$的有序四元组$(a,b,c,d)$恰好共有 77，000 个，那么 n 的最小可能值是多少?

$\textbf{(A)}\ 13,860 \qquad \textbf{(B)}\ 20,790 \qquad \textbf{(C)}\ 21,560 \qquad \textbf{(D)}\ 27,720 \qquad \textbf{(E)}\ 41,580$

Problem 25

The sequence (a_n) is defined recursively by $a_0 = 1, a_1 = \sqrt[19]{2}$, and $a_n = a_{n-1}a_{n-2}^2$ for $n \geq 2$. What is the smallest positive integer k such that the product $a_1 a_2 \cdots a_k$ is an integer?

数列(a_n)定义如下：$a_0 = 1, a_1 = \sqrt[19]{2}$，且对于$n \geq 2$，有$a_n = a_{n-1}a_{n-2}^2$，使得乘积$a_1 a_2 \cdots a_k$为整数的最小整数$k$是多少？

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

2016 AMC 12B Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
D	A	D	C	B	C	D	D	B	A	D	C	E
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	D	E	D	B	B	A	B	B	A	D	A	

2017 AMC12B

Problem 1

Kymbrea's comic book collection currently has 30 comic books in it, and she is adding to her collection at the rate of 2 comic books per month. LaShawn's collection currently has 10 comic books in it, and he is adding to his collection at the rate of 6 comic books per month. After how many months will LaShawn's collection have twice as many comic books as Kymbrea's?

Kymbrea 的漫画书集目前有 30 本漫画书，并且她现在还在以每个月 2 本漫画书的速度向她的漫画书集中增添新书。Lashawn 的漫画书集目前有 10 本漫画书，并且目前他在以每个月 6 本漫画书的速度向他的漫画书集中增添新书。问经过多少个月 Lashawn 的漫画书是 Kymbrea 书的 2 倍?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Problem 2

Real numbers x, y, and z satisfy the inequalities $0<x<1$, $-1<y<0$, and $1<z<2$. Which of the following numbers is necessarily positive?

实数 x，y，z 满足不等式 $0<x<1$，$-1<y<0$，$1<z<2$，下面哪个数字是正数?

$\textbf{(A)}\ y+x^2 \qquad \textbf{(B)}\ y+xz \qquad \textbf{(C)}\ y+y^2 \qquad \textbf{(D)}\ y+2y^2 \qquad \textbf{(E)}\ y+z$

Problem 3

Supposed that x and y are nonzero real numbers such that $\frac{3x+y}{x-3y}=-2$. What is the value of $\frac{x+3y}{3x-y}$?

假设 x 和 y 是非零实数，满足 $\frac{3x+y}{x-3y}=-2$，则 $\frac{x+3y}{3x-y}$ 的值为多少？

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Problem 4

Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

Samia 骑着自行车去拜访她的朋友，平均速度为 17 公里每小时，当她走了一半的路程，一个轮胎坏了，之后她以 5 公里每小时的速度步行走完了剩余的路程，最终到她朋友家总共花了 44 分钟，Samia 步行了多少公里路程？结果保留一位小数。

$\textbf{(A)}\ 2.0 \qquad \textbf{(B)}\ 2.2 \qquad \textbf{(C)}\ 2.8 \qquad \textbf{(D)}\ 3.4 \qquad \textbf{(E)}\ 4.4$

Problem 5

The data set $[6, 19, 33, 33, 39, 41, 41, 43, 51, 57]$ has median $Q_2 = 40$, first quartile $Q_1 = 33$, and third quartile $Q_3 = 43$. An outlier in a data set is a value that is more than 1.5 times the interquartile range below the first quartile (Q_1) or more than 1.5 times the interquartile range above the third quartile (Q_3), where the interquartile range is defined as $Q_3 - Q_1$. How many outliers does this data set have?

一组数据$[6, 19, 33, 33, 39, 41, 41, 43, 51, 57]$的中位数是$Q_2 = 40$，第一个四分位数是$Q_1 = 33$，第三个四分位数是$Q_3 = 43$. 异常的数据定义为比第一个四分位数$(Q_1)$低超过 1.5 倍的四分位距，或者比第三个四分位数(Q_3)高超过 1.5 倍的四分位距，而这里的四分位距则定义为$Q_3 - Q_1$，那么这组数据有多少个异常的数?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 6

The circle having $(0,0)$ and $(8,6)$ as the endpoints of a diameter intersects the x-axis at a second point. What is the x-coordinate of this point?

一个直径两端点分别为（0，0）和（8，6）的圆和 x 轴交于第二个点。问这第二个点的 x 坐标是多少？

$\textbf{(A)}\ 4\sqrt{2} \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 5\sqrt{2} \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 6\sqrt{2}$

Problem 7

The functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π. What is the least period of the function $\cos(\sin(x))$?

函数$\sin(x)$和$\cos(x)$都是最小正周期是 2π的周期函数。那么函数$\cos(\sin(x))$的最小正周期是多少？

$\textbf{(A)}\ \frac{\pi}{2} \qquad \textbf{(B)}\ \pi \qquad \textbf{(C)}\ 2\pi \qquad \textbf{(D)}\ 4\pi \qquad$ **(E)** It's not periodic.

Problem 8

The ratio of the short side of a certain rectangle to the long side is equal to the ratio of the long side to the diagonal. What is the square of the ratio of the short side to the long side of this rectangle?

某个矩形的短边和长边之比等于长边和对角线之比，那么这个矩形的短边和长边之比的平方是多少？

$\textbf{(A)}\ \frac{\sqrt{3}-1}{2} \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{\sqrt{5}-1}{2} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{2} \qquad \textbf{(E)}\ \frac{\sqrt{6}-1}{2}$

Problem 9

A circle has center $(-10,-4)$ and radius 13. Another circle has center $(3,9)$ and radius $\sqrt{65}$. The line passing through the two points of intersection of the two circles has equation $x+y=c$. What is c?

一个圆的圆心为（-10，-4），半径是 13. 另一个圆的圆心为（3，9），半径是$\sqrt{65}$.通过这两个圆的两个交点的直线方程为$x+y=c$。那么 c 是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 3\sqrt{3} \qquad \textbf{(C)}\ 4\sqrt{2} \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \frac{13}{2}$

Problem 10

At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

在 Typico 高中，60%的学生喜欢跳舞，剩余的人不喜欢，在那些喜欢跳舞的人中，80%的人说他们喜欢跳舞，剩余的人说他们不喜欢跳舞，而在那些不喜欢跳舞的人中，90%的人说他们不喜欢跳舞，其余的人说他们喜欢跳舞，那些说自己不喜欢跳舞的学生当中，有多少比例实际上是喜欢跳舞的？

$\textbf{(A)}\ 10\% \qquad \textbf{(B)}\ 12\% \qquad \textbf{(C)}\ 20\% \qquad \textbf{(D)}\ 25\% \qquad \textbf{(E)}\ 33\frac{1}{3}\%$

Problem 11

Call a positive integer $monotonous$ if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

如果一个正整数是一位数字或者当从左往右读时，它的各个位上的数字形成一个严格递增或严格递减的数列，那么我们把此正整数称作单调的，例如，3，23578 和 987620 都是单调的，但是 88，7434 和 23557 不是。那么一共有多少个单调的正整数?

$\textbf{(A)}\ 1024 \qquad \textbf{(B)}\ 1524 \qquad \textbf{(C)}\ 1533 \qquad \textbf{(D)}\ 1536 \qquad \textbf{(E)}\ 2048$

Problem 12

What is the sum of the roots of $z^{12}=64$ that have a positive real part?

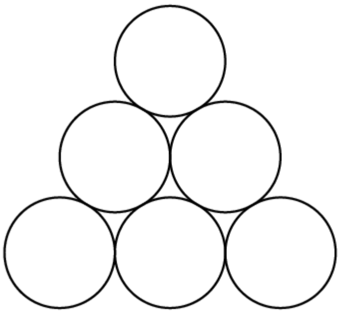
方程$z^{12}=64$的所有根中，那些实部是正数的根之和是多少？

$\textbf{(A) } 2 \qquad \textbf{(B) } 4 \qquad \textbf{(C) } \sqrt{2}+2\sqrt{3} \qquad \textbf{(D) } 2\sqrt{2}+\sqrt{6} \qquad \textbf{(E) } (1+\sqrt{3})+(1+\sqrt{3})i$

Problem 13

In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?

如下图所示，6 个圆盘中其中 3 个涂成蓝色，2 个涂成红色，1 个涂成绿色，如果两种涂色图案可以通过旋转或者对称重合，那么这 2 种涂色图案视作同一个。问一共有多少种不同的图案?



$\textbf{(A) } 6 \qquad \textbf{(B) } 8 \qquad \textbf{(C) } 9 \qquad \textbf{(D) } 12 \qquad \textbf{(E) } 15$

Problem 14

An ice-cream novelty item consists of a cup in the shape of a 4-inch-tall frustum of a right circular cone, with a 2-inch-diameter base at the bottom and a 4-inch-diameter base at the top, packed solid with ice cream, together with a solid cone of ice cream of height 4 inches, whose base, at the bottom, is the top base of the frustum. What is the total volume of the ice cream, in cubic inches?

一种花饰冰激凌由成截头椎体形状的杯子和圆锥形冰激凌组成。杯子的高度为 4 英寸，下底直径 2 英寸，上底直径 4 英寸，圆锥形冰激凌高为 4 英寸，其底部恰好就是杯子的上底。求整个冰激凌的体积是多少立方英寸？

$\textbf{(A) } 8\pi \qquad \textbf{(B) } \frac{28\pi}{3} \qquad \textbf{(C) } 12\pi \qquad \textbf{(D) } 14\pi \qquad \textbf{(E) } \frac{44\pi}{3}$

Problem 15