动量定理

§ 11-1　动量与冲量

(1)动量: $\boldsymbol{P} = m\,\boldsymbol{v}$

矢量性;瞬时性;相对性

(2)冲量: $\boldsymbol{I} = \boldsymbol{F}\,t$　　　　$\mathrm{d}\boldsymbol{I} = \boldsymbol{F}\,dt$

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt$$

$$\vec{I} = \int_{t_1}^{t_2} \left(\sum \vec{F_i}\right) dt = \sum \vec{I_i}$$

§ 11-2 动量定理

(1)质点的动量定理:

微分形式: d$\boldsymbol{P} = \boldsymbol{F}dt$

积分形式: $\overrightarrow{P_2} - \overrightarrow{P_1} = \int_{t_1}^{t_2} \vec{F} dt = \vec{I}$

(2)守恒定理:

若 $\boldsymbol{F} = 0$ 则 $\boldsymbol{P} = \boldsymbol{c}$ (恒矢量)

若 $F_x = 0$ 则 $P_x = c$ (恒量)

(3)质点系的动量定理

[1] 动量: $\boldsymbol{P}=\sum m_i \boldsymbol{v}_i$

$$质心: \vec{r_c}=\frac{\sum m_i \vec{r_i}}{\sum m_i}=\frac{\sum m_i \vec{r_i}}{M}$$

$$质心速度: \vec{v_c}=\frac{\sum m_i \vec{v_i}}{M}$$

$$\boldsymbol{P}=M\boldsymbol{v}_c$$

[2] 动量定理

:

设质点系有n个质点,第i个质点的质量为m_i速度为$\boldsymbol{v}_i$;外界物体对该质点作用的外力为$\boldsymbol{F}_i^{(e)}$,质点系内其它质点对该质点作用的内力为$\boldsymbol{F}_i^{(i)}$.则有: $\boldsymbol{R}^{(i)} = \sum \boldsymbol{F}_i^{(i)} = 0$

$$\mathrm{d}\boldsymbol{P}_i = \boldsymbol{F}_i^{(e)} dt + \boldsymbol{F}_i^{(i)} dt = \mathrm{d}\boldsymbol{I}_i^{(e)} + \mathrm{d}\boldsymbol{I}_i^{(i)}$$

微分形式: $\mathrm{d}\boldsymbol{P} = \boldsymbol{R}^{(e)} dt$

积分形式: $\boldsymbol{P}_2 - \boldsymbol{P}_1 = \boldsymbol{I}^{(e)}$

应用合矢量投影定理可将上两式投影到任一轴上.

[3]守恒定理:

若 $\boldsymbol{R}^{(e)} = 0$ 则 $\boldsymbol{P} = \boldsymbol{c}$ (恒矢量)

若 $R^{(e)}{}_x = 0$ 则 $P_x = c$ (恒量)

由上述可知:内力的主矢$\boldsymbol{R}^{(i)} = \sum \boldsymbol{F}_i^{(i)} = 0$即内力对质点系的动量无影响;而外力的主矢则影响质点系动量,即影响质心速度的变化.

11-3.质心运动定理

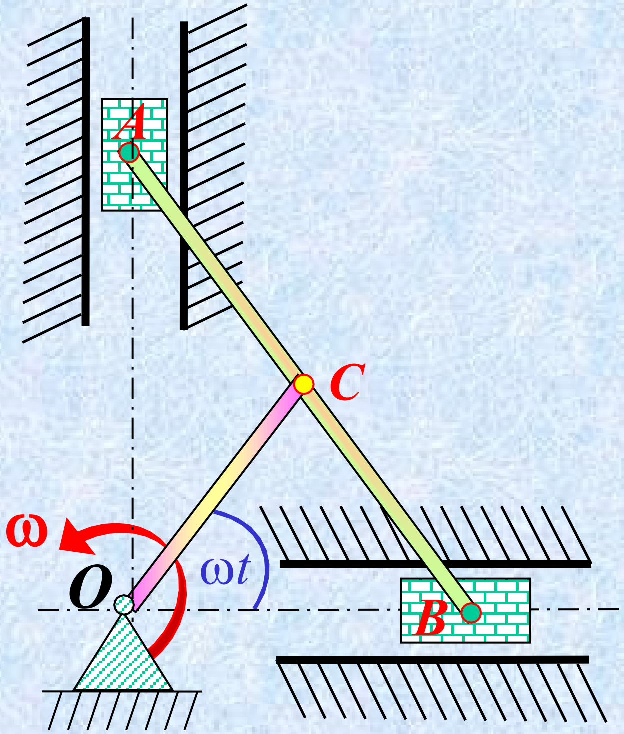
(1)运动定理: $M\boldsymbol{a_c}=\boldsymbol{R}^{(e)}$ $\quad\left\{\begin{aligned} M\ddot{x}_c &= R_x^{(e)} \\ M\ddot{y}_c &= R_y^{(e)} \\ M\ddot{z}_c &= R_z^{(e)} \end{aligned}\right.$

(2)守恒定理:

若 $\boldsymbol{R}^{(e)}=0$ 则 $\boldsymbol{v}_c=\boldsymbol{c}$ (恒矢量)

若 $R^{(e)}{}_x=0$ 则 $v_{cx}=c$ (恒量)

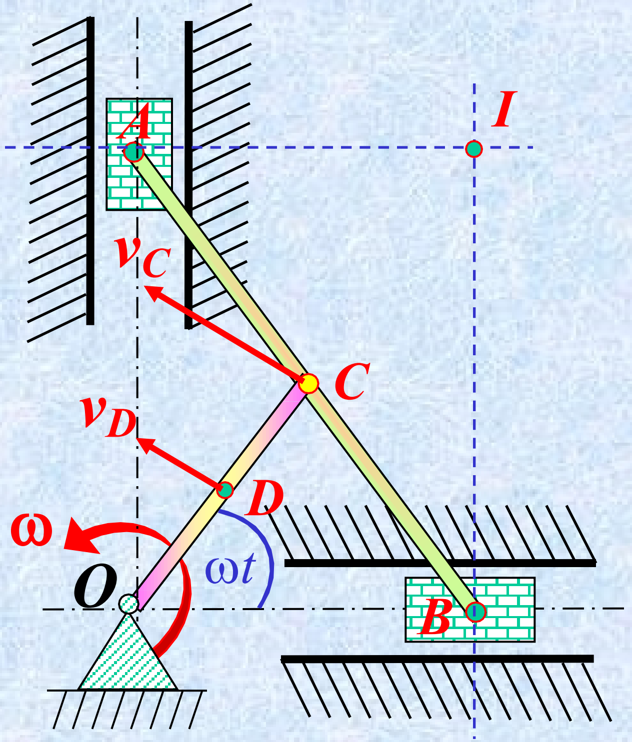
例题11-1. 图示椭圆规尺AB的质量为 $2m_1$,曲柄OC的质量为m_1 ,而滑块A和B的质量均为m_2.已知$OC=AC=CB=l$,曲柄和尺的质心分别在其中点上,曲柄绕O轴转动的角速度ω为常量.求图示瞬时系统的动量.



解:系统由四个物体组成.

滑块A和B的质心与椭圆规尺AB的质心C总是重合在一起,而AB作平面运动.瞬心为I. $IC = OC = l$

OA杆作定轴转动D为质心.



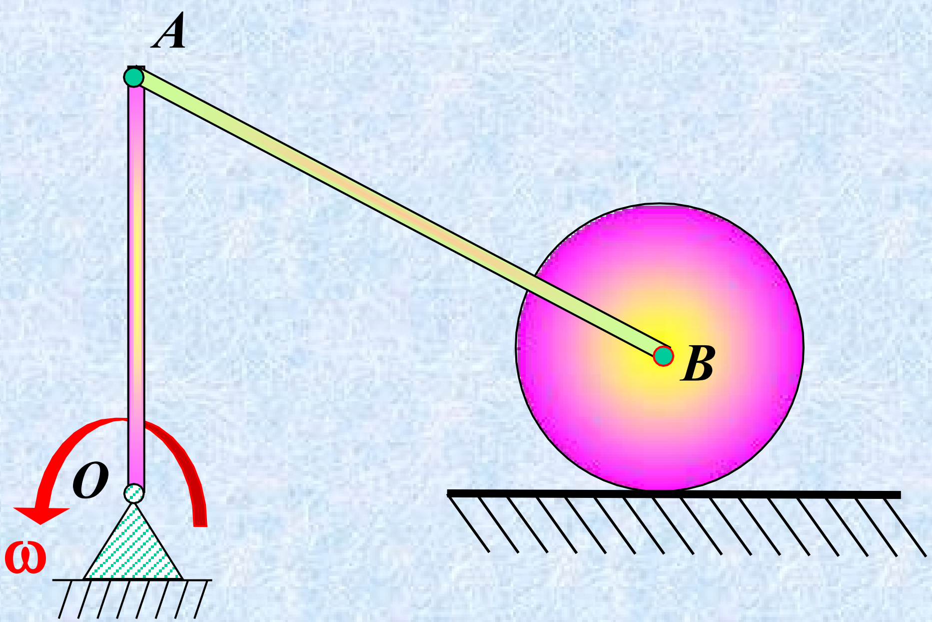
$$v_D = \frac{1}{2} l\omega \qquad v_C = l\omega$$

$$\vec{P} = \overrightarrow{P_{OC}} + \overrightarrow{P_{AB}} + \overrightarrow{P_A} + \overrightarrow{P_B} \qquad P_{OC} = \frac{1}{2} m_1 l\omega$$

$$P_{AB} + P_A + P_B = (2m_1 + m_2 + m_2) l\omega = 2(m_1 + m_2) l\omega$$

$$P = (2.5m_1 + 2m_2) l\omega$$

例题11-2. 在图示系统中,均质杆OA、AB与均质轮的质量均为m,OA杆的长度为l_1,AB杆的长度为l_2,轮的半径为R,轮沿水平面作纯滚动.在图示瞬时,OA杆的角速度为ω,求整个系统的动量.

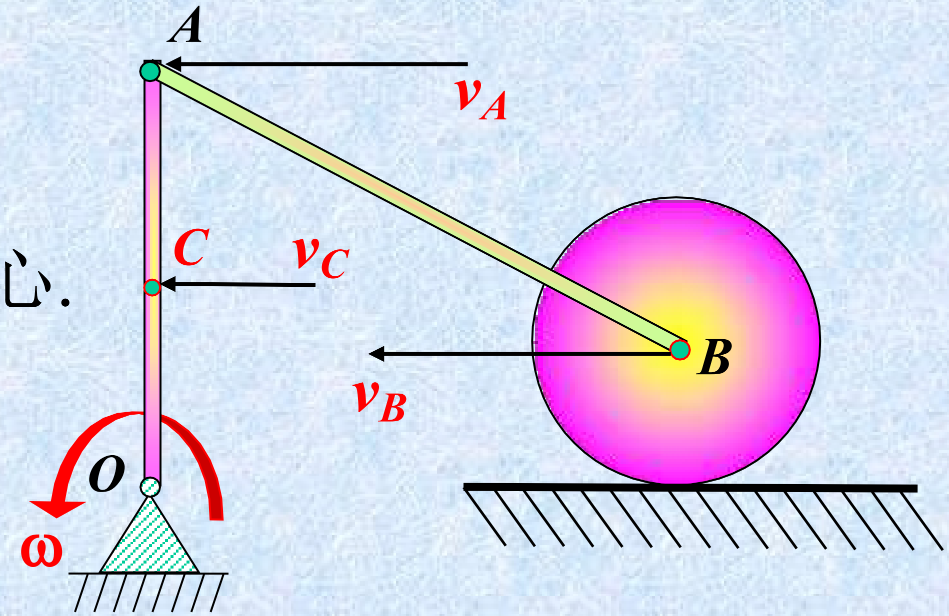


解:系统由三个物体组成.

OA杆作定轴转动C为质心.

轮作平面运动B为质心.

AB杆作瞬时平动.



$$v_C = \frac{1}{2} l_1 \omega \qquad v_A = v_B = l_1 \omega \qquad \vec{P} = \vec{P_{OA}} + \vec{P_{AB}} + \vec{P_B}$$

$$P_{OA} = \frac{1}{2} m l_1 \omega \qquad P_{AB} = m l_1 \omega \qquad P_B = m l_1 \omega$$

$$P = \frac{1}{2} m l_1 \omega + m l_1 \omega + m l_1 \omega = \frac{5}{2} m l_1 \omega$$

例题11-3. 小车重W_1= 2kN, 车上有一装沙的箱重W_2=1kN,以3.5km/h的速度在光滑直线轨道上匀速行驶.今有一重W_3= 0.5kN的物体铅垂落入沙箱中,如图.求此后小车的速度. 又设重物落人沙箱后,沙箱在小车上滑动 0.2 s 后,始与车面相对静止, 求车面与箱底间相互作用的摩擦力.