From partial equilibrium to general equilibrium

- In many cases, the motivation for a policy comes from its partial equilibrium effects
- However, the general equilibrium effects, due to the interrelation between markets, may offset the effectiveness of the policy in addressing the original problem and/or cause other problems - the unintended consequences

Dutch disease

natural resource booms
$\Rightarrow$ more input demand by the mining sector
$\Rightarrow$ wage $\uparrow$ due to larger induced demand for workers
$\Rightarrow$ profits in manufacturing sector $\downarrow$ due to higher labour cost

Energy Act and wheat price

promotion of renewal fuels in the 2005 Energy Policy Act
$\Rightarrow$ corn price $\uparrow$
$\Rightarrow$ production of wheat, soybean, etc $\downarrow$ due to the switch of farmers to corn
$\Rightarrow$ prices for these other agriculture produce $\uparrow$ due to declining supply

Labour market and housing supply elasticity

positive demand shock for final products
$\Rightarrow$ demand for labour $\uparrow$ wage $\uparrow$
$\Rightarrow$ labour supply $\uparrow$ because of immigrants
$\Rightarrow$ demand for housing increases $\uparrow$
$\Rightarrow$ housing price $\uparrow$
by how much? depending on the elasticity of housing supply...

School voucher

- Quality of public school and housing price
- Give voucher for private school to poor households: free money for sending kids to private schools
- What is the intended partial equilibrium effects?
- General equilibrium effect? Think about the decision of the marginal households who would be willing to live in a small apartment in a good school district if without the voucher.

Key questions and outline

Key questions

- Could all markets manage to achieve equilibrium at the same time?
- Is the equilibrium outcome efficient?

Outline

- Three activities in an economy:
  consumption, production and transaction
- Pure exchange economy
  - consumption and transaction only
  - with or without money - barter economy or competitive economy
  - Core and equilibria
- Market structure and efficiency - the Fundamental Welfare Theorems
- Introducing production
- Introducing time and uncertainty

Exchange economy: elements

- Two goods 1 and 2
- Consumers 1 and 2
  consumption $\mathbf{x}^1 = (x_1^1, x_2^1)$ and $\mathbf{x}^2 = (x_1^2, x_2^2)$
- Endowment of two commodities
  $\mathbf{e}^1 = (e_1^1, e_2^1)$ and $\mathbf{e}^2 = (e_1^2, e_2^2)$
  $\Rightarrow$ total endowment $\mathbf{e} = \mathbf{e}^1 + \mathbf{e}^2 = (e_1^1 + e_1^2, e_2^1 + e_2^2)$
- No production
- Voluntary exchange/barter

Edgeworth box

- Graphical presentation: Edgeworth box
- Two origins and four coordinates
- Feasible allocation $F(\mathbf{e}) \equiv \left\{\mathbf{x} | \sum_{i \in I} \mathbf{x}^i = \sum_{i \in I} \mathbf{e}^i\right\}$
- Preferences (indifference curves) for consumer 1 and 2: $\succsim_1$ and $\succsim_2$
- Exchange economy: $\left\{\succsim_i, \mathbf{e}^i\right\}_{i \in I}$
- **Contract curve**: tangency points of the two sets of ICs
  - What if the two goods are perfect complements for both consumers?
  - What if the two goods are perfect substitutes for both consumers?

Barter exchange equilibrium

Voluntary barter trade $\Rightarrow$ mutually beneficial trade

- Pareto efficient: there exists no other allocation $y \in F(\mathbf{e})$ such that $\mathbf{y}^i \gtrsim \mathbf{x}^i$ for all consumers i,with at least one preference relation strict
- Blocking coalitions and core
  - "Coalition S **blocks** allocation $\mathbf{x}$" means there is an allocation $\mathbf{y}$ such that $\sum_{i \in S} \mathbf{y}^i = \sum_{i \in S} \mathbf{e}^i$ and $\mathbf{y}^i \gtrsim \mathbf{x}^i$ for all $i \in S$, with at least one preference strict
  - Unblocked allocation $\Rightarrow$ Pareto efficient
  - **Core** $C(\mathbf{e})$: the set of all unblocked feasible allocations
- The exact outcome depends on the bargaining power of the two parties

Perfect competition

Instead of barter trades and bargaining over the terms of trade, all individuals take prices as given and respond to prices only

...Imagine there exists an auctioneer ...

How would the equilibrium differ?

- Individual optimization: utility maximization with endowment
- Market clears

A formal presentation

- Consumer's problem: u^i continuous, strictly increasing and strictly quasi-concave

$$max_{\mathbf{x}^i \in \Re^n_+} \; u^i(\mathbf{x}^i) \; s.t. \; \mathbf{p} \cdot \mathbf{x}^i \leq \mathbf{p} \cdot \mathbf{e}^i$$

  $\Rightarrow$ solution $\mathbf{x}^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$

- Excess demand

$$\begin{aligned} z_k(\mathbf{p}) &\equiv \sum_{i \in I} x^i_k(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - \sum_{i \in I} e^i_k \\ \mathbf{z}(\mathbf{p}) &= (z_1(\mathbf{p}), ..., z_n(\mathbf{p})) \end{aligned}$$

  Properties: Continuity, homogeneity and Walras' Law ($\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0$)

- Walrasian equilibrium: $\mathbf{p}^*$ such that $\mathbf{z}(\mathbf{p}^*) = \mathbf{0}$
- Walras' Law $\Rightarrow$ if all $n-1$ markets are in equilibrium for some $\mathbf{p}$, then the nth market must be in equilibrium as well.

Existence of equilibrium

If each consumer's utility function is

- Continuous
- Strictly increasing
- Strictly quasi-concave

and that the aggregate endowment of each good is strictly positive, then the aggregate excess demand function would satisfy

- $\mathbf{z}(\cdot)$ is continuous
- $\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0$
- If the prices of some but not all goods are arbitrarily close to 0, then the excess demand for at least one of those goods is arbitrarily high.

and then a Walrasian Equilibrium exists.

Example: finding Walrasian equilibrium

- Preference: $u_1(x_1^1, x_2^1) = (x_1^1)^a (x_2^1)^{1-a}$ and $u_2(x_1^2, x_2^2) = (x_1^2)^b (x_1^2)^{1-b}$
- Endowment: $\mathbf{e}^1 = (1, 0)$ and $\mathbf{e}^2 = (0, 1)$
- Walrasian equilibrium $\left(\frac{p_2}{p_1}\right)^*$?

Efficiency

- Walrasian equilibrium allocation
  $\mathbf{x}(\mathbf{p}^*) = \left(\mathbf{x}^1(\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^1), ..., \mathbf{x}^I(\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^1)\right) \in F(\mathbf{e})$
- First welfare theorem: Walrasian Equilibrium $\rightarrow$ Pareto efficiency
  - No distortion of price
  - No externalities
  - No asymmetric information
  - No market power
- Second welfare theorem:
  Any Pareto efficient allocation can result from a Walrasian equilibrium (given that endowments can be redistributed in a lump sum way)

Equilibrium and the core

The set of Walrasian equilibrium allocations $W(\mathbf{e}) \subset C(\mathbf{e})$

- When there are a large number of consumers, a competitive equilibrium is achieved only if no subgroup of consumers can make themselves better off by forming a coalition.
- In other words, increasing competition from a larger group of consumers reduces bargaining powers by each individual.

Core and allocation WEA

WEA - Walrasian equilibrium allocation

Question: what is the relationship between the outcome of a barter economy - allocations in the core - and the outcome of a competitive economy - the WEAs?

- In a pure exchange economy, the set of equilibrium allocations is a subset of the set of core allocations
- As the size of the economy grows, the set of core allocations will shrink and converge to the set of equilibrium allocations

Model setup

- There are I types of consumers - consumers of the same type have the same endowment and preference
- Start with an economy in which there is only one consumer of each type, then duplicate the economy
- An r-fold replica economy ε_r: r consumers of each of the I types, which gives a total of rI consumers.
- Denote consumers of type i by iq where $q = 1, ..., r$

Edgeworth-Debreu-Scarf Limit Theorem on the Core:
If x $\in C_r$ for any $r = 1, 2, ...$, then x is a WEA for ε_1

Outline of the proof

- Equal treatment of like types in the core
  - Otherwise the "badly" treated agents can form a coalition to block the "unequal" allocation
  - Thus any allocation an r-fold replica economy is an r-fold replica of one allocation in the core of the basic economy
- Some core allocations in the basic economy can be blocked in the two-fold replica economy
  - Thus the core is shrinking as the size of the economy grows
- A WEA allocation in an r-fold replica economy $\Leftrightarrow$ a replica of a WEA in the basic economy
  - Thus the WEA prices and allocations remain "constant" as the economy grows
  - Thus the set of core allocations converge to the set of WEA allocations