

# 梁的内力

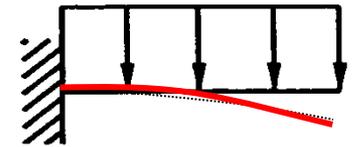
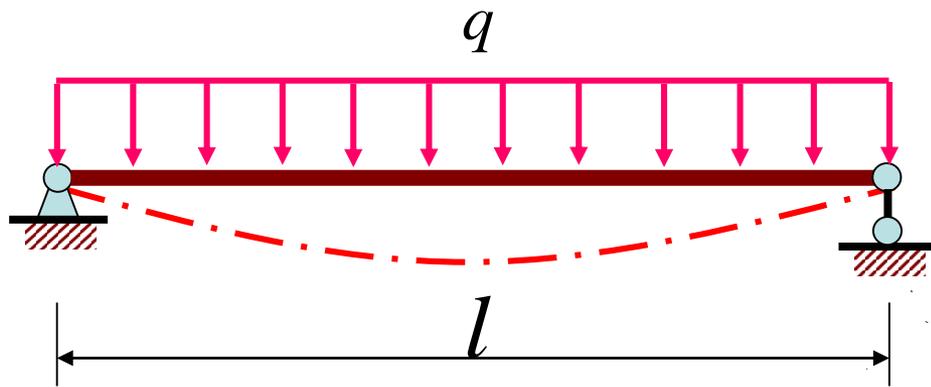
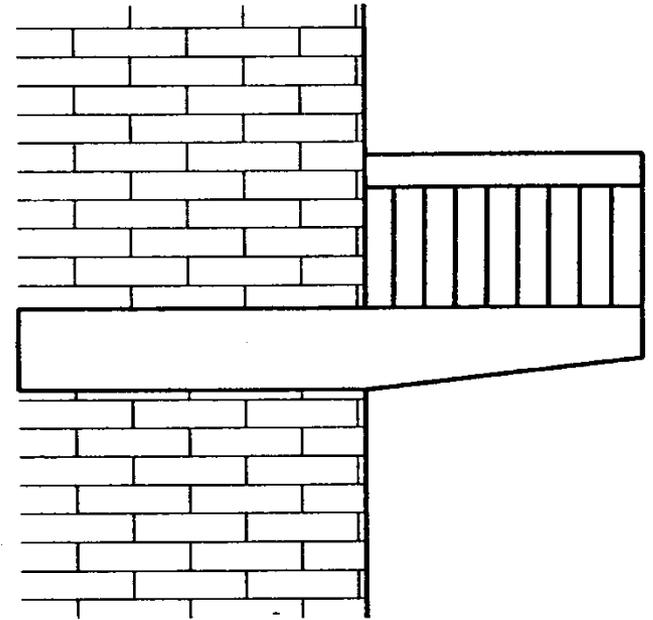
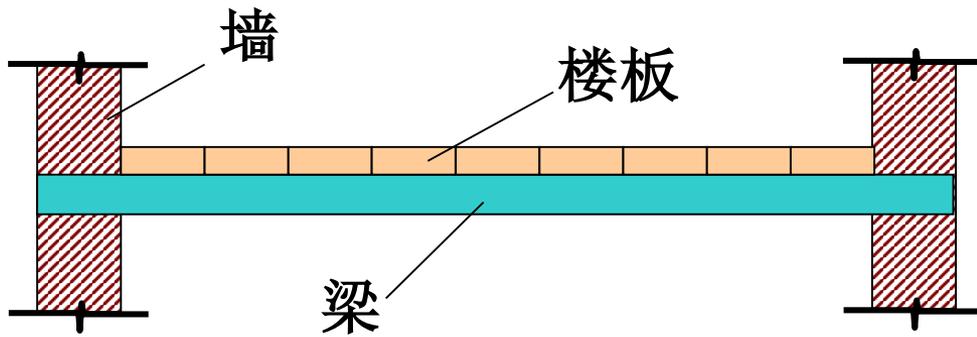
# 梁的内力

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- § 1 弯曲的概念和工程实例
- § 2 静定梁的分类
- § 3 剪力与弯矩 剪力图与弯矩图
- § 4 剪力、弯矩和分布荷载集度间的关系
- § 5 按叠加原理作弯矩图

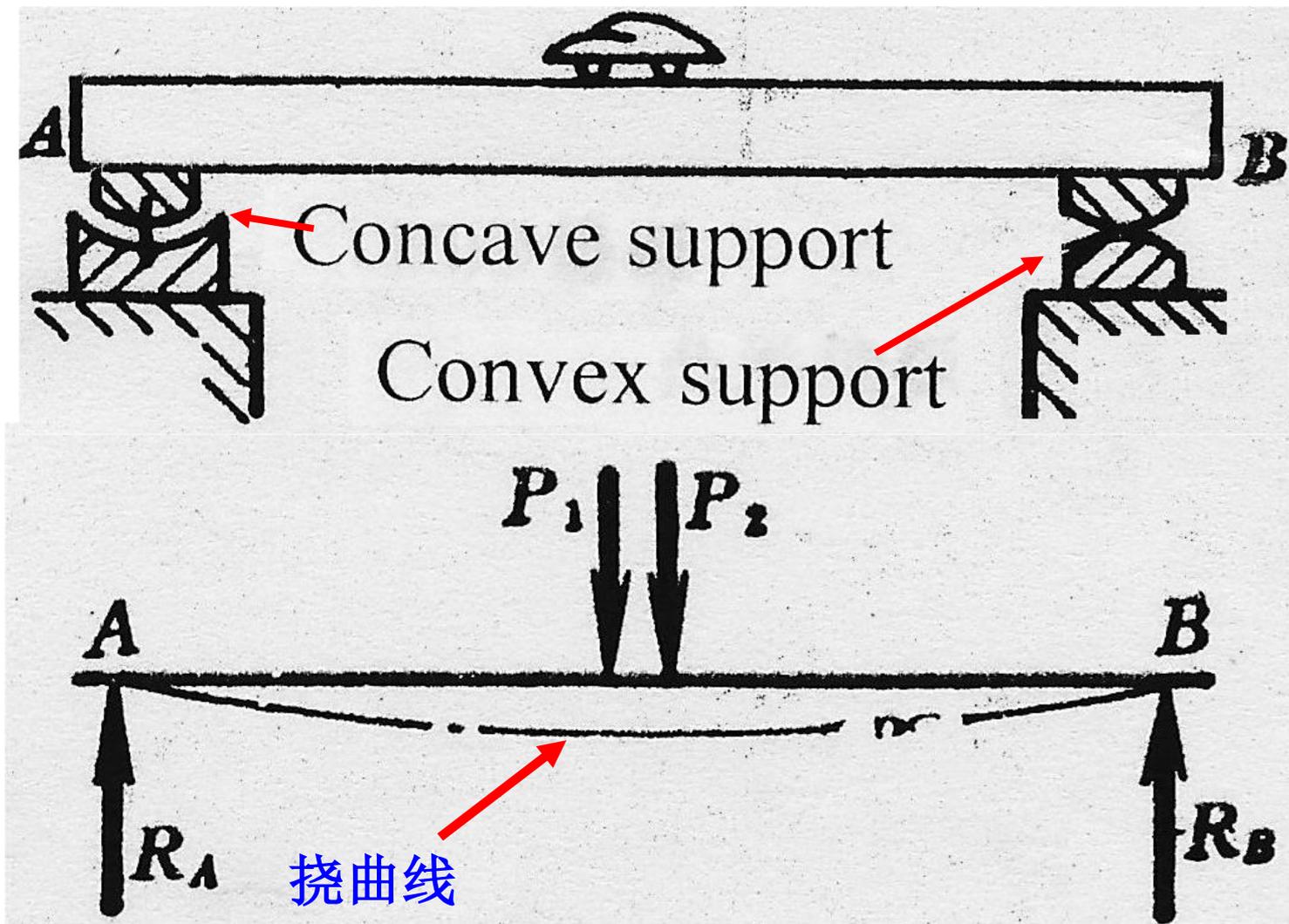
# § 1 概念和工程实例

## ※ 工程应用



# § 1 概念和工程实例

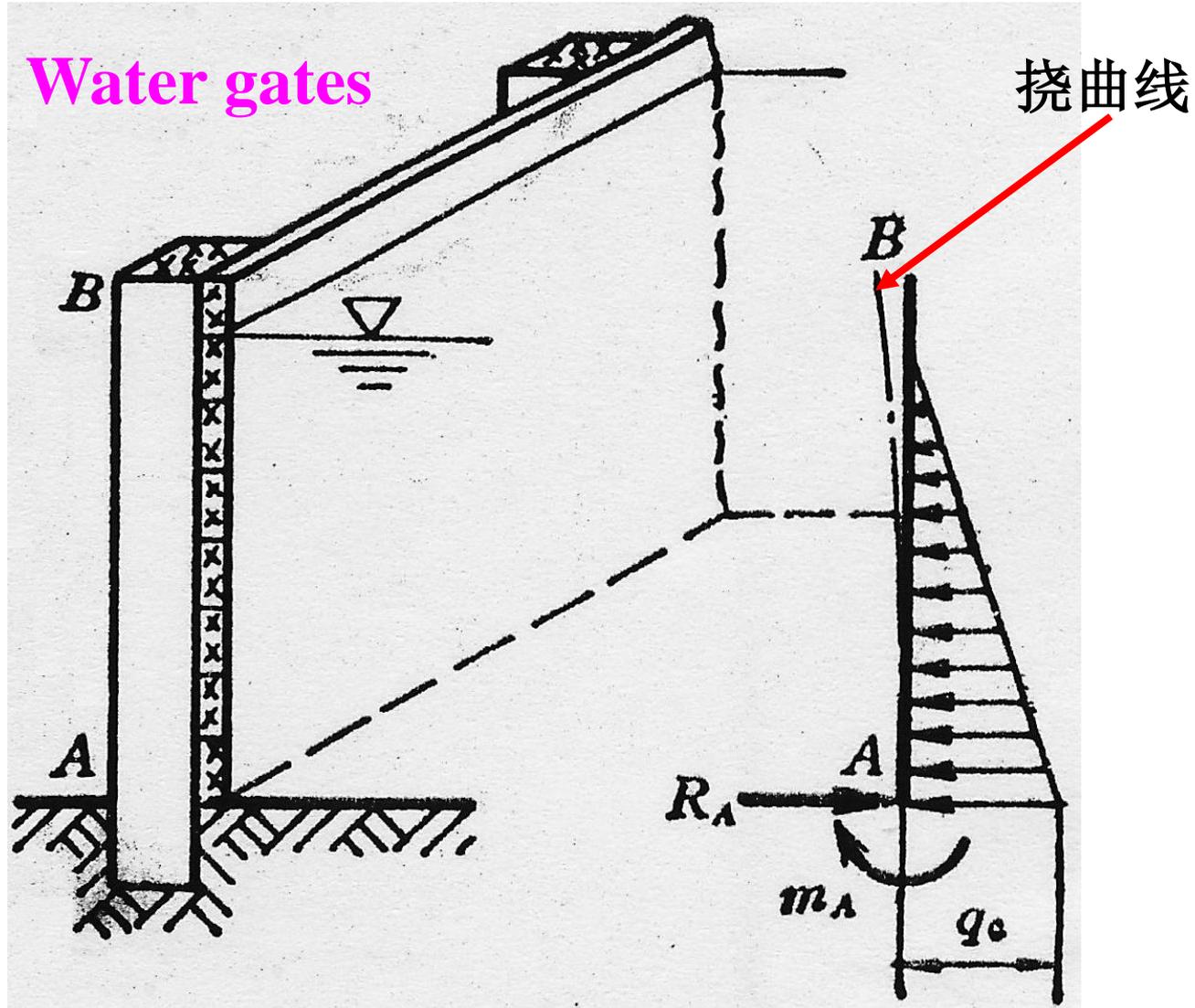
## ※ 工程应用





# § 1 概念和工程实例

## ※ 工程应用



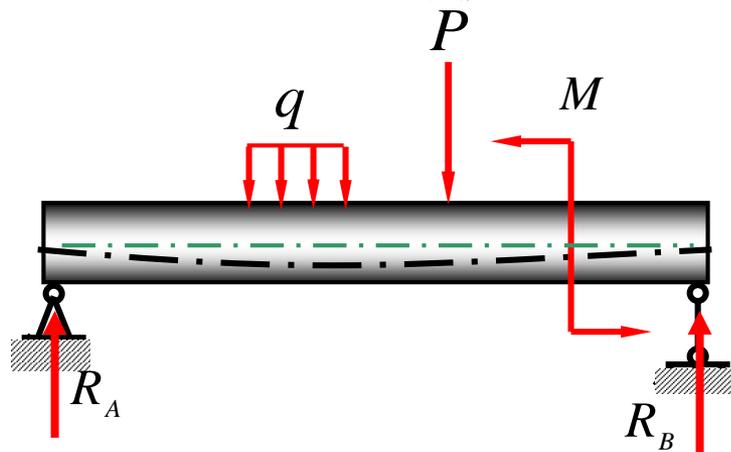
## § 1 概念和工程实例

### ※ 定义

若构件承受垂直于轴线的纵向载荷，则构件发生**弯曲变形**。

弯曲变形后，构件的初始直线轴线变成的曲线，称为**挠曲线**。

主要发生弯曲变形的构件称为**梁**。

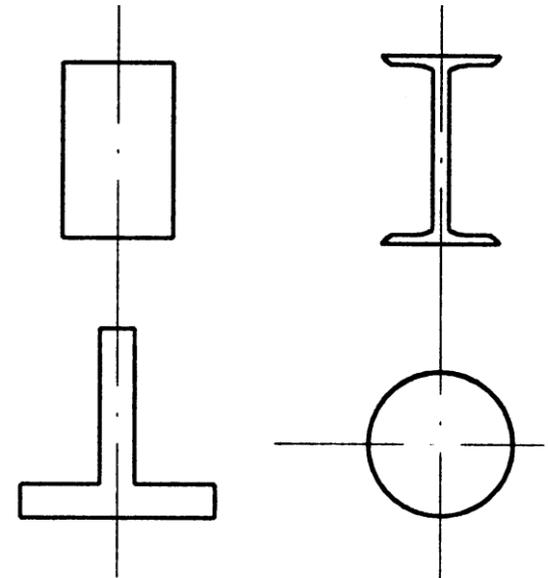
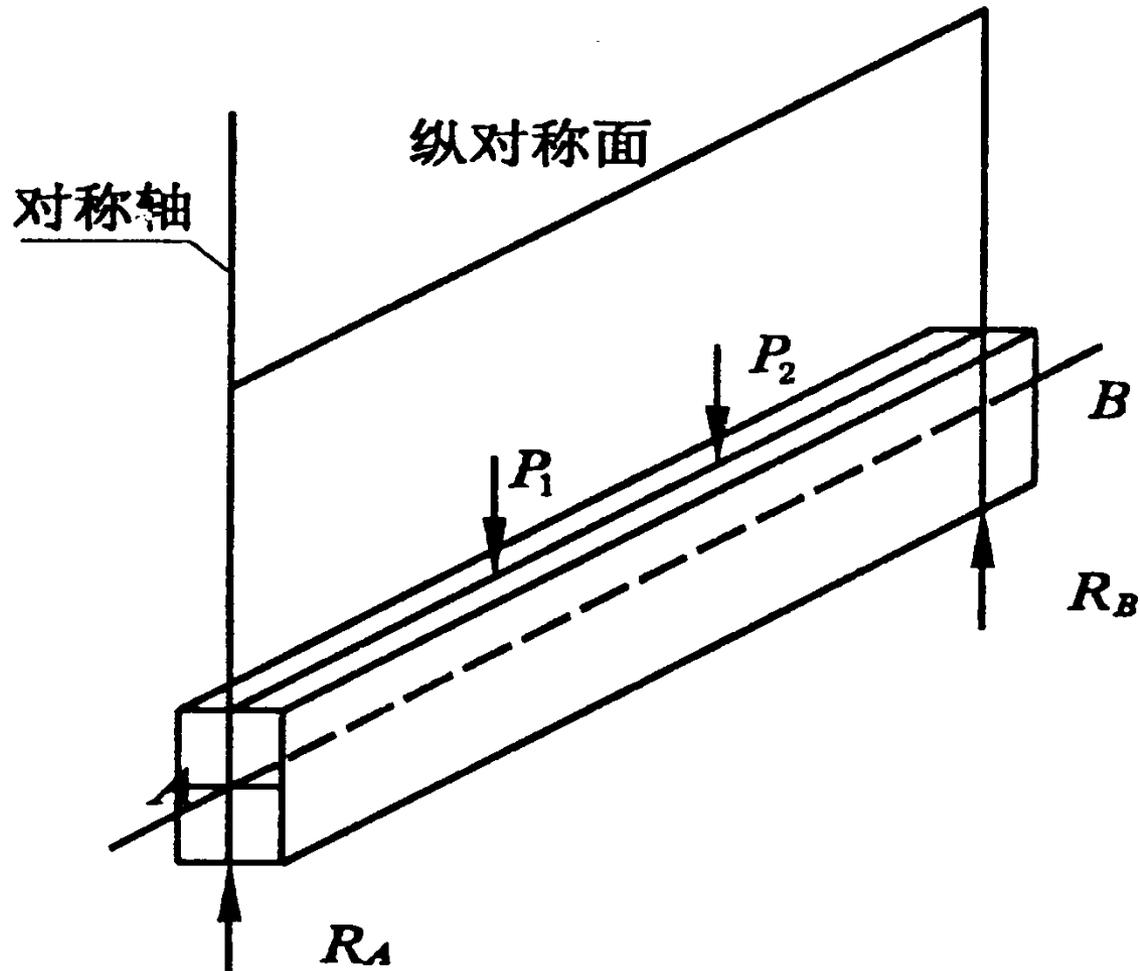


受力特点——作用于杆件上的外力都垂直于杆的轴线。

变形特点——杆轴线由直线变为一条平面的曲线。

# § 1 概念和工程实例

## ※ 定义



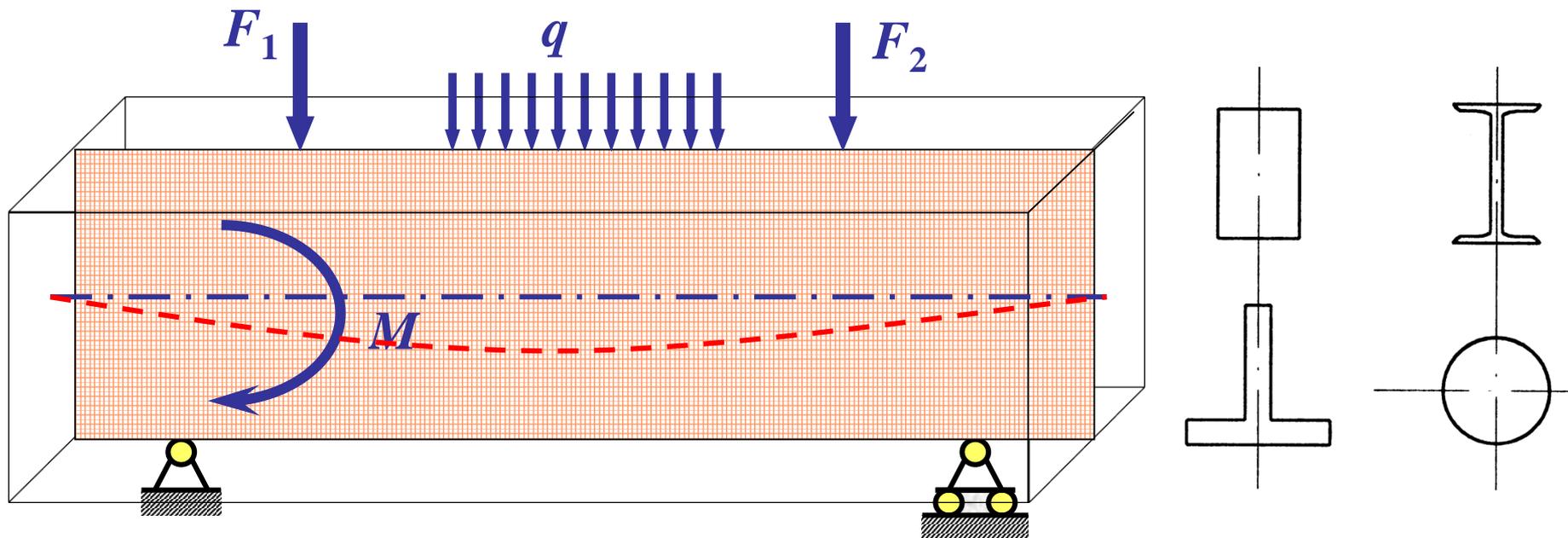
平面弯曲

## § 1 概念和工程实例

### ※ 平面弯曲的特点

**受力特点:** 外力垂直于杆轴且在梁的纵向对称面内（通过或平行形心主轴上且过弯曲中心）

**变形特点:** 杆轴线在纵向对称面内由直线变成平面曲线

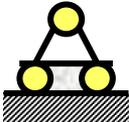


平面弯曲

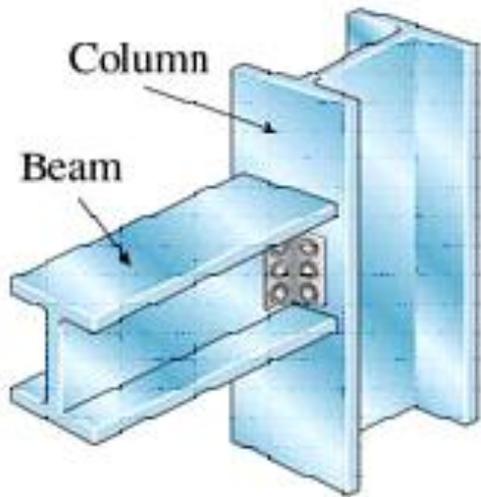
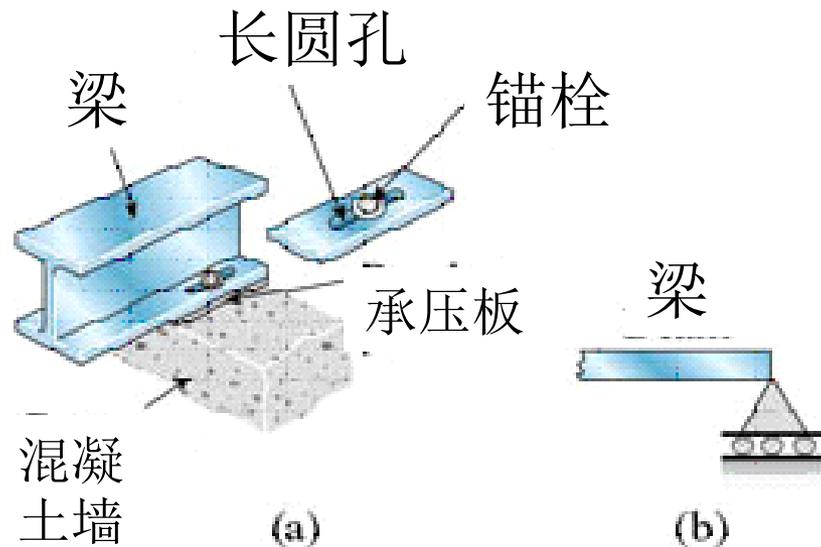
# § 2 静定梁的分类

## ※ 支座类型

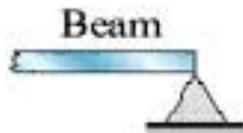
铰支座: 

滑动铰支座 

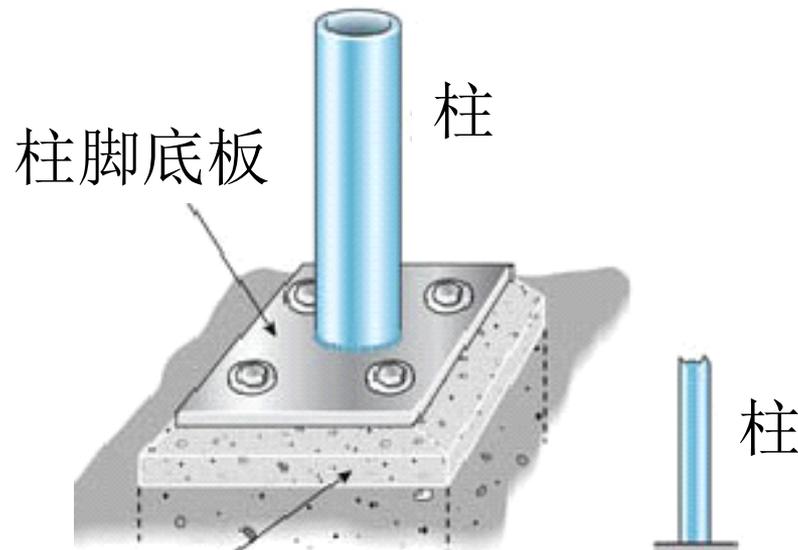
固定支座 



(c)



(d)

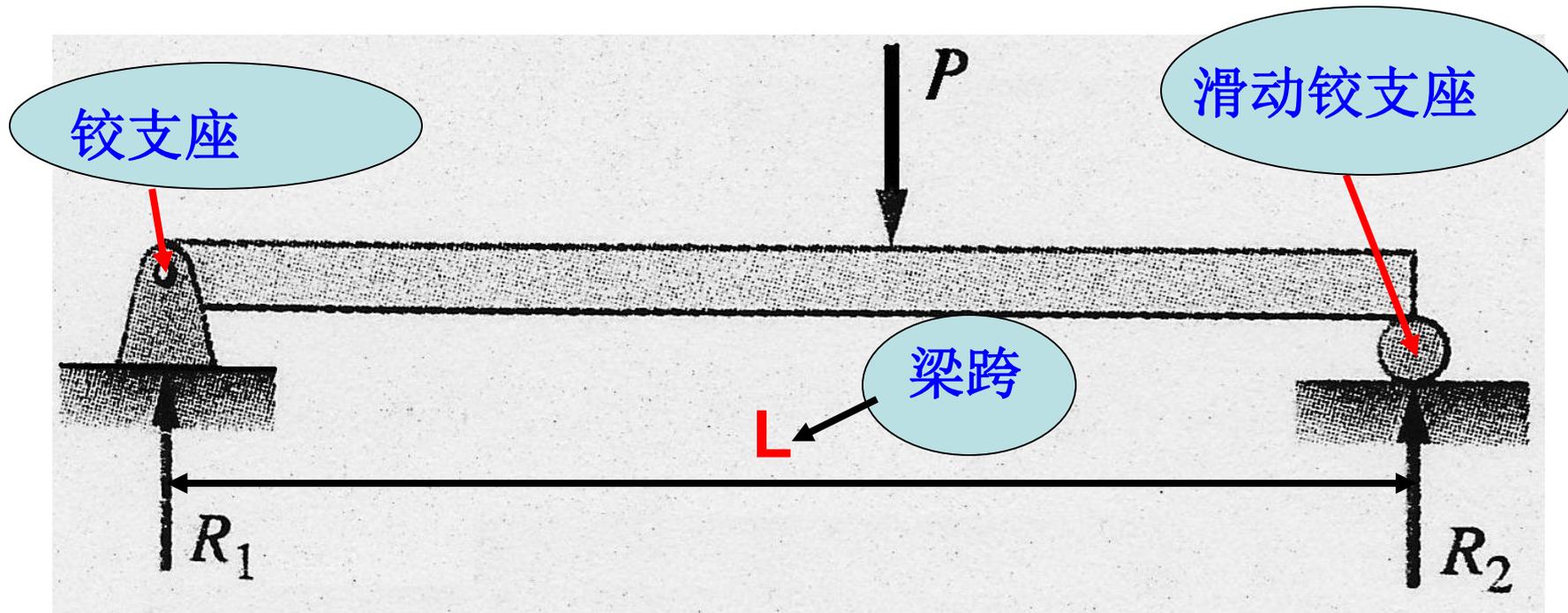


(e)

(f)

## § 2 静定梁的分类

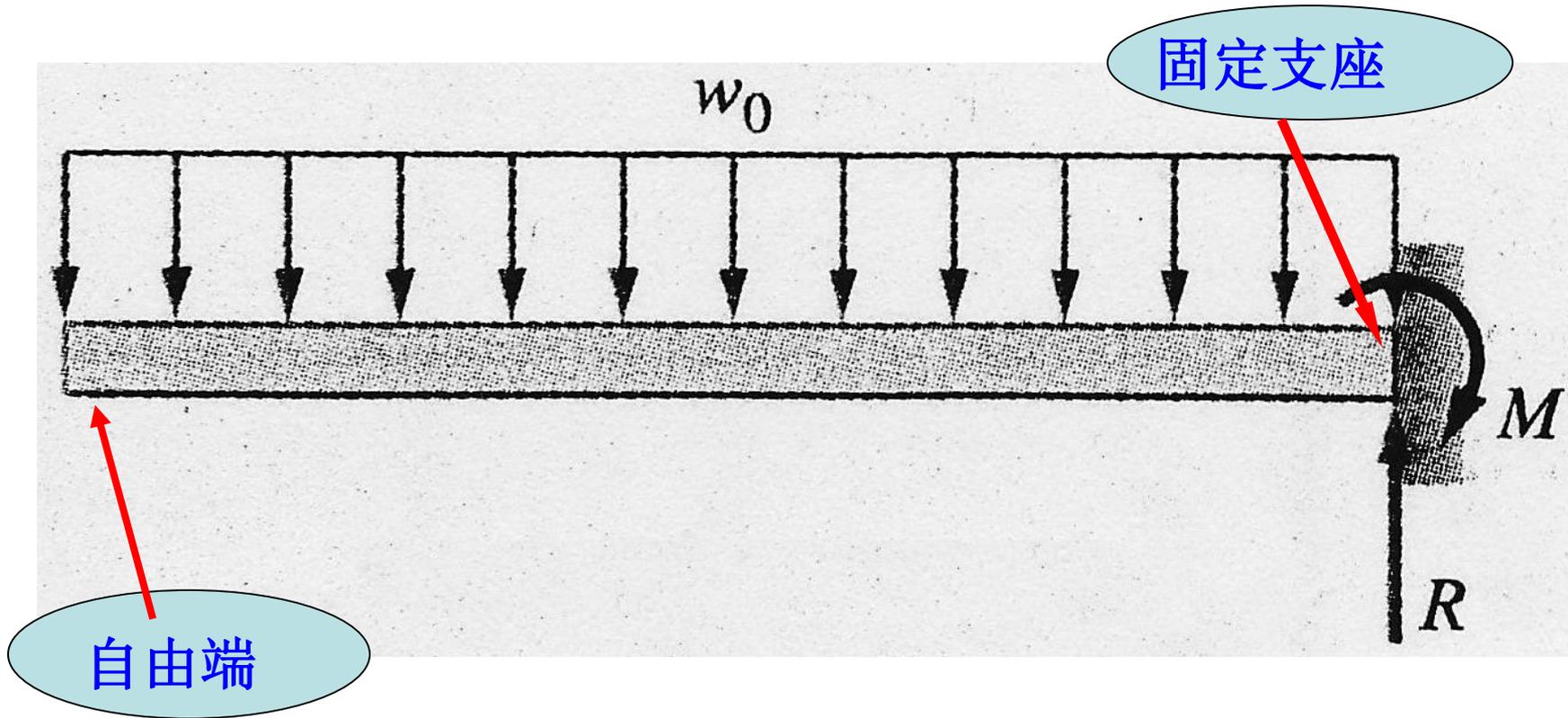
### ※ 静定梁的分类



简支梁

## § 2 静定梁的分类

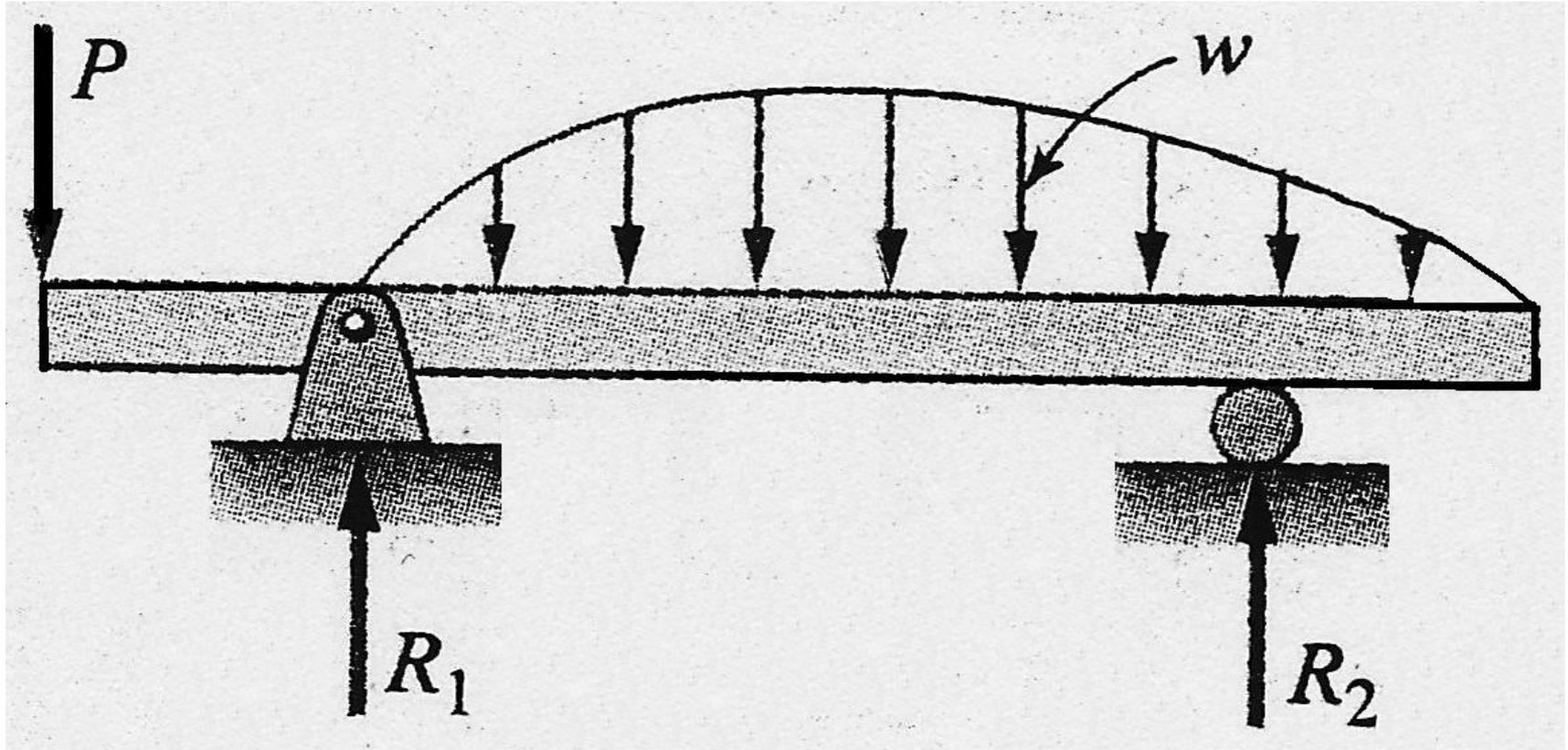
### ※ 静定梁的分类



悬臂梁

## § 2 静定梁的分类

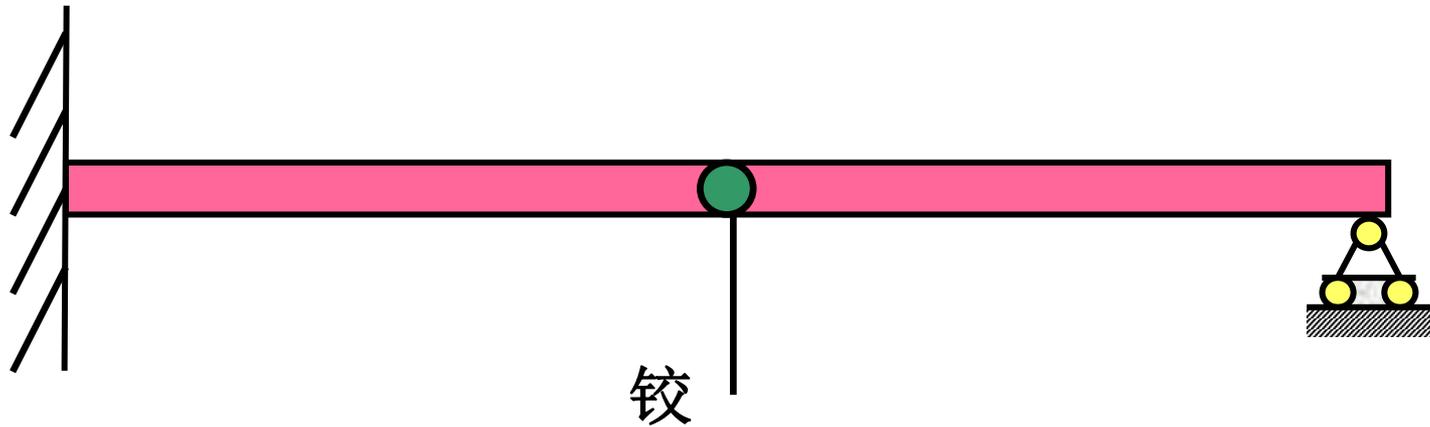
### ※ 静定梁的分类



外伸梁

## § 2 静定梁的分类

### ※ 静定梁的分类



连续梁

## § 3 剪力与弯矩 内力图

### ※ 梁的内力

$$\sum F_X = 0, \therefore F_{AX} = 0$$

$$\sum M_A = 0, F_{BY}l - Fa = 0$$

$$\sum F_Y = 0, F_{AY} - F + F_{BY} = 0$$

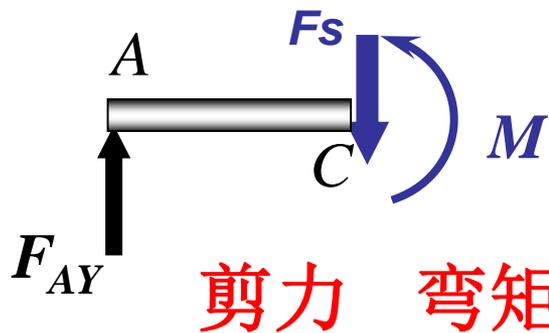
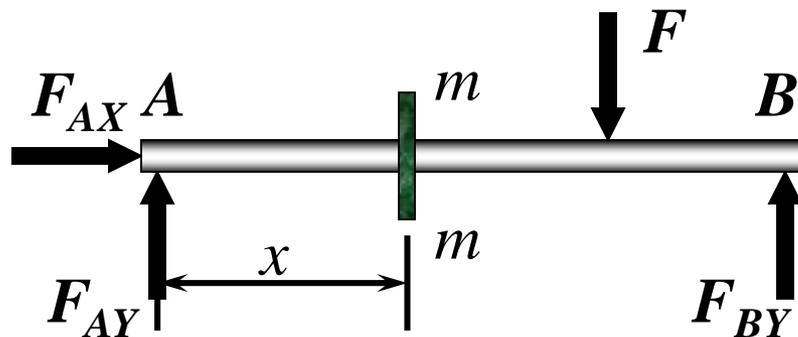
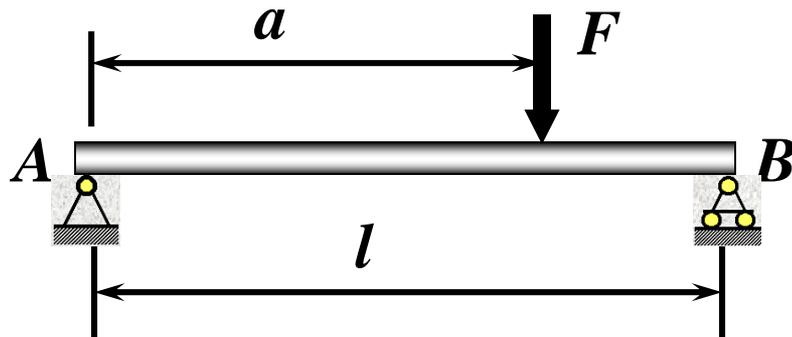
$$F_{BY} = \frac{Fa}{l}, F_{AY} = \frac{F(l-a)}{l}$$

$$\sum F_Y = 0, F_{AY} - F_s = 0.$$

$$F_s = F_{AY} = \frac{F(l-a)}{l}$$

$$\sum m_C = 0, M - F_{AY}x = 0.$$

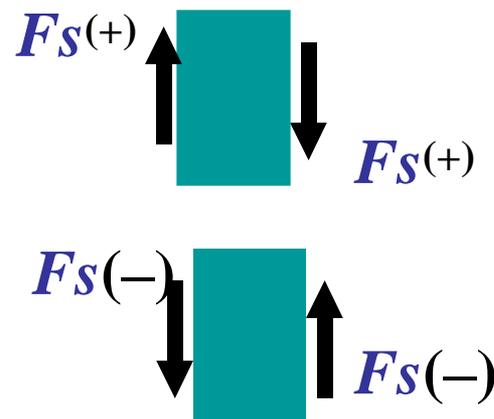
$$M = F_{AY}x = \frac{F(l-a)}{l}x$$



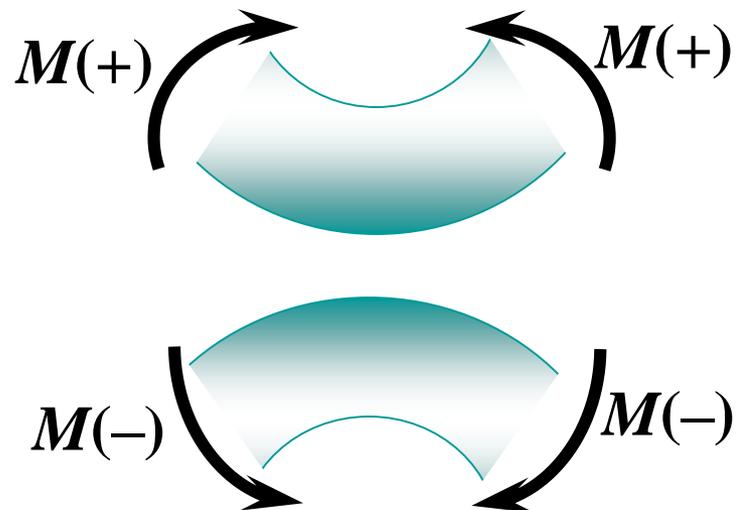
## § 3 剪力与弯矩 内力图

### ※ 剪力和弯矩的符号规定

- **剪力：** 令脱离体顺时针转动为**正**，  
反之为负



- **弯矩：** 令梁变形为向上弯曲为**正**，  
反之为负  
(笑脸为正，哭脸为负).



## § 3 剪力与弯矩 内力图

### ※ 剪力和弯矩的符号规定

取 AC 为脱离体

$$F_s = F_{AY} = \frac{F(l-a)}{l}$$

$$M = F_{AY}x = \frac{F(l-a)}{l}x$$

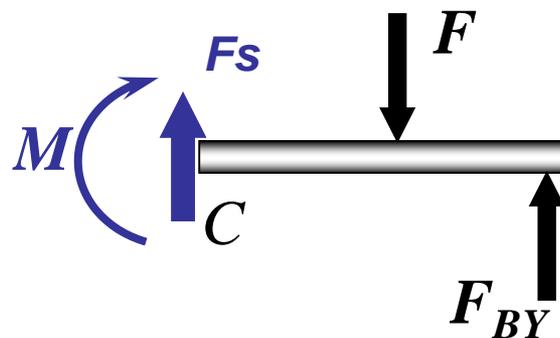
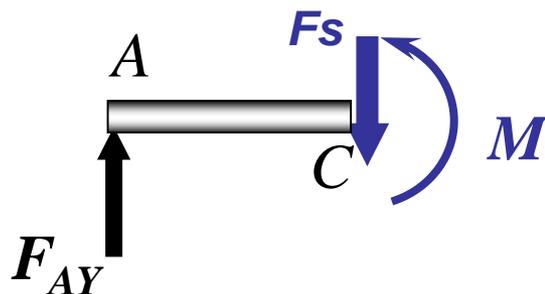
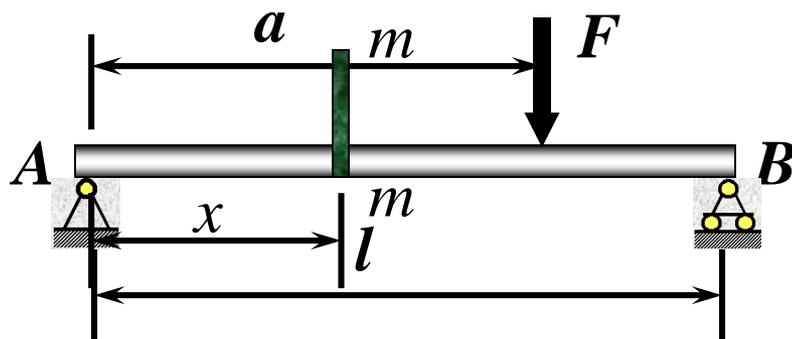
取 CB 为脱离体

$$\sum Y = 0, \quad F_s - F + F_{BY} = 0.$$

$$\sum m_C = 0,$$

$$F_{BY}(l-x) - F(a-x) - M = 0.$$

$$F_s = \frac{F(l-a)}{l}, \quad M = \frac{F(l-a)}{l}x$$



相同的大小和方向

## § 3 剪力与弯矩 内力图

### 例 1

确定截面 1-1 和 2-2 的内力

解:

#### 1、支反力

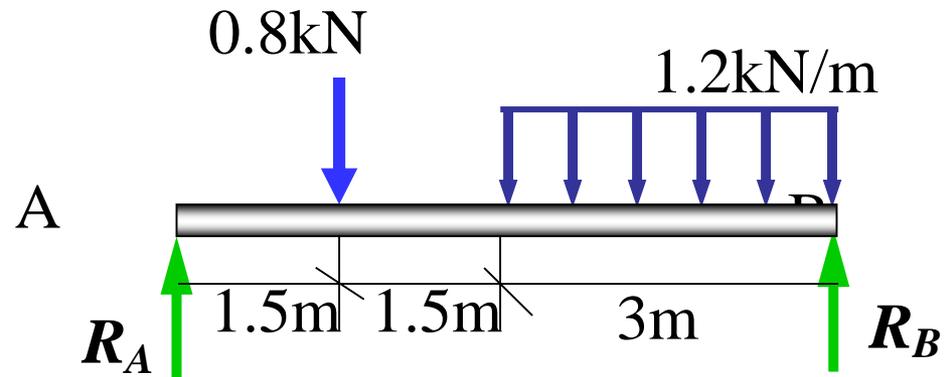
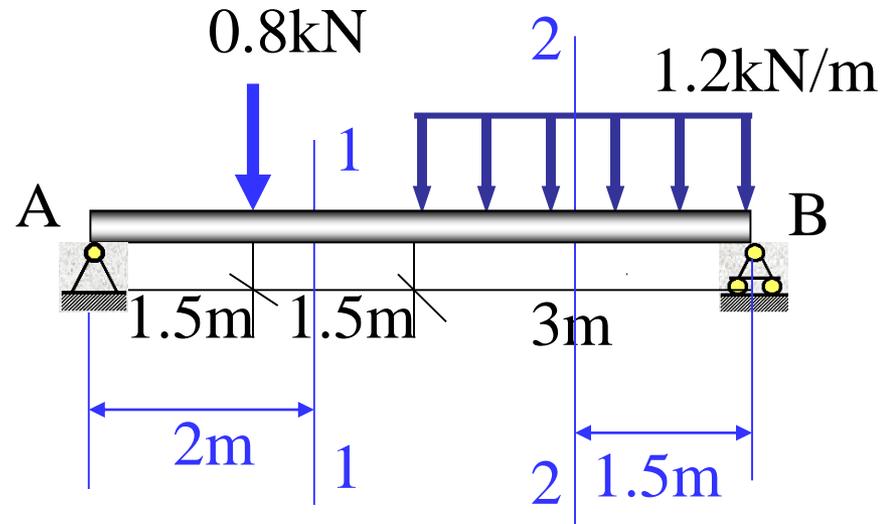
$$\sum F_Y = 0,$$

$$R_A + R_B - 0.8 - 1.2 \times 3 = 0$$

$$\sum M_B = 0,$$

$$1.2 \times 3 \times 1.5 + 0.8 \times 4.5 - R_A \times 6 = 0$$

$$R_A = 1.5 \text{ (kN)}, \quad R_B = 2.9 \text{ (kN)}$$



## § 3 剪力与弯矩 内力图

### 2、1-1截面的内力

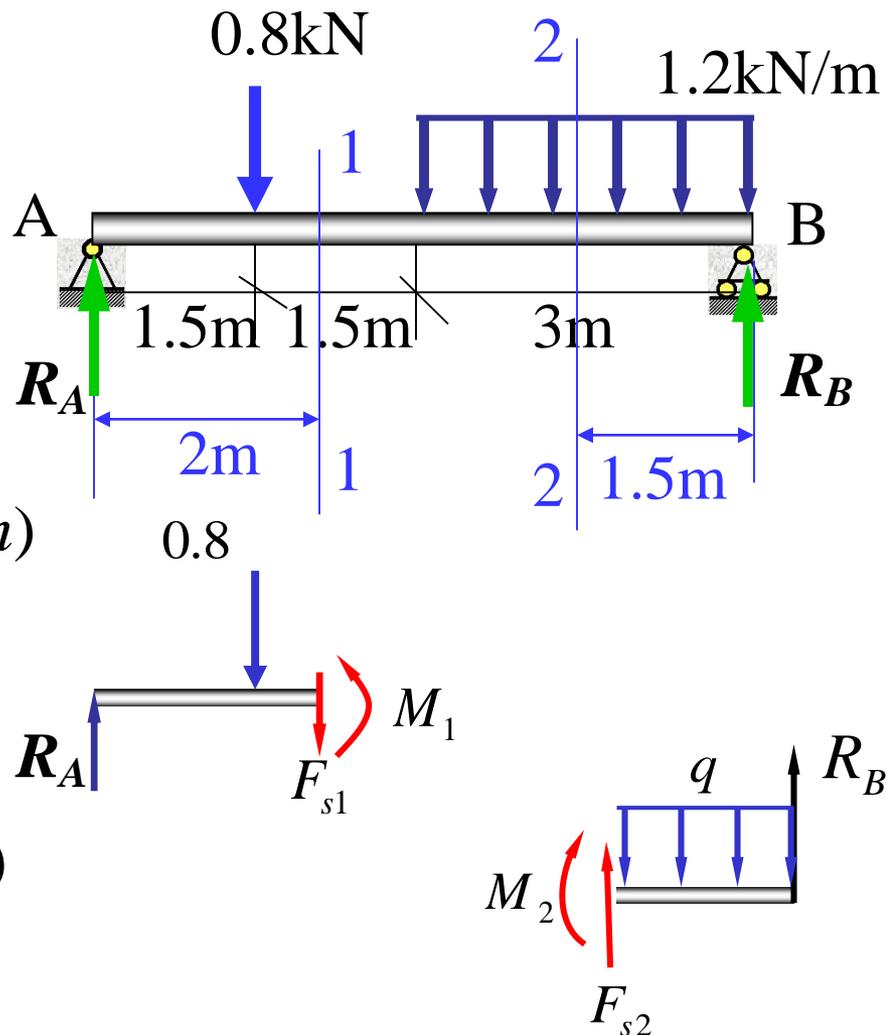
$$\begin{aligned}F_{s1} &= R_A - 0.8 \\ &= 1.5 - 0.8 = 0.7 \text{ (kN)}\end{aligned}$$

$$\begin{aligned}M_1 &= R_A \times 2 - 0.8 \times 0.5 \\ &= 1.5 \times 2 - 0.8 \times 0.5 = 2.6 \text{ (kN} \cdot \text{m)}\end{aligned}$$

### 3、2-2截面的内力

$$F_{s2} = 1.2 \times 1.5 - 2.9 = -1.1 \text{ (kN)}$$

$$\begin{aligned}M_2 &= R_B \times 1.5 - 1.2 \times 1.5 \times 0.75 \\ &= 2.9 \times 1.5 - 1.2 \times 1.5 \times 0.75 = 3.0 \text{ (kN} \cdot \text{m)}\end{aligned}$$



## § 3 剪力与弯矩 内力图

### ※ 剪力图和弯矩图

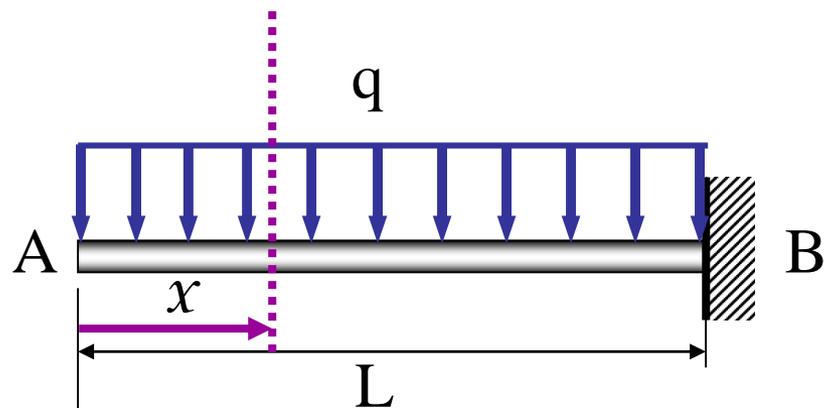
剪力方程  $F_S = F_S(x)$

弯矩方程  $M = M(x)$

反映梁的横截面上的剪力和弯矩随截面位置变化的函数式。

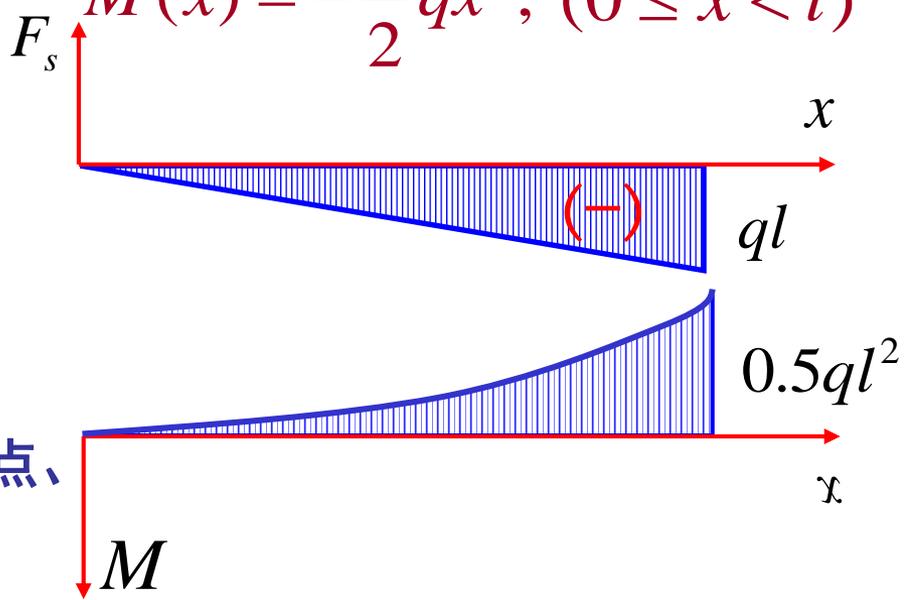
显示剪力和弯矩随截面位置的变化规律的图形则分别称为**剪力图**和**弯矩图**。

注意：不能用一个函数表达的要分段，  
分段点为：集中力作用点、集中力偶作用点、  
分布力的起点、终点。



$$F_S(x) = -qx, \quad (0 \leq x < l)$$

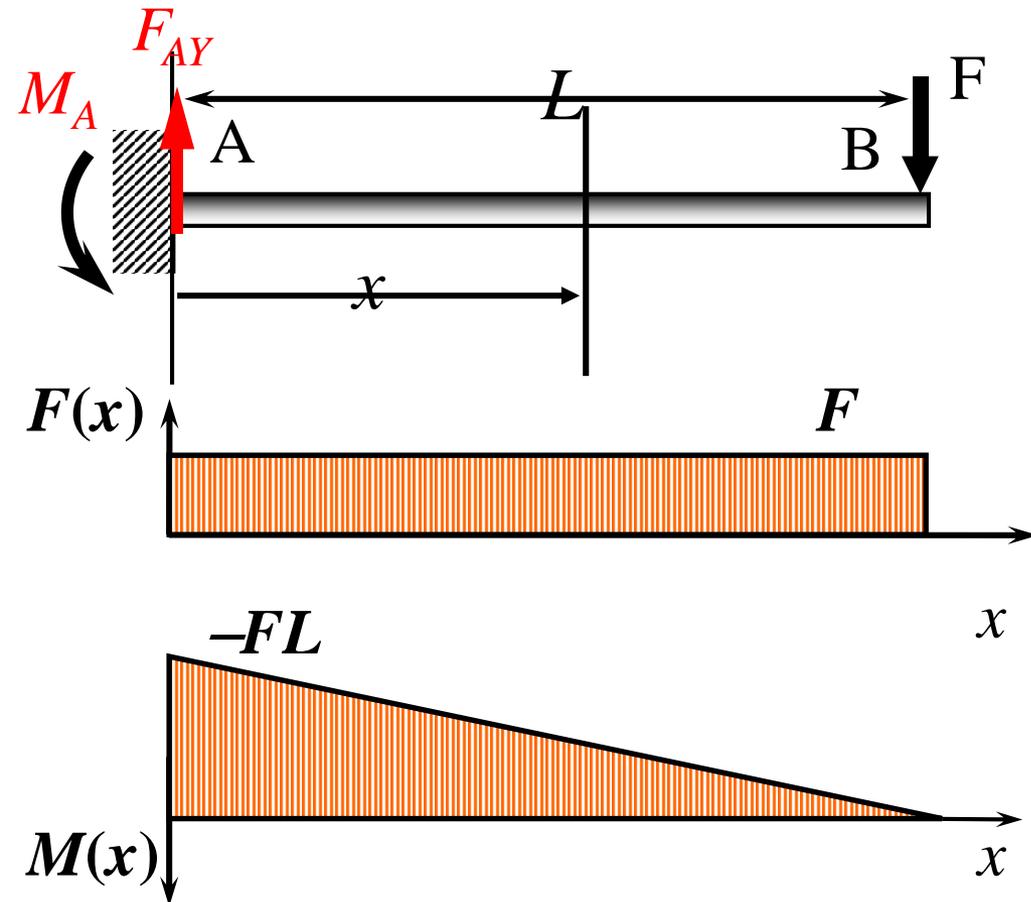
$$M(x) = -\frac{1}{2}qx^2, \quad (0 \leq x < l)$$



## § 3 剪力与弯矩 内力图

### 例 2

画出图中梁的内力图



解:

#### 1、求支反力

$$F_{AY} = F ; M_A = FL$$

#### 2、内力方程

$$F_s(x) = F_{AY} = F \quad (0 < x < l)$$

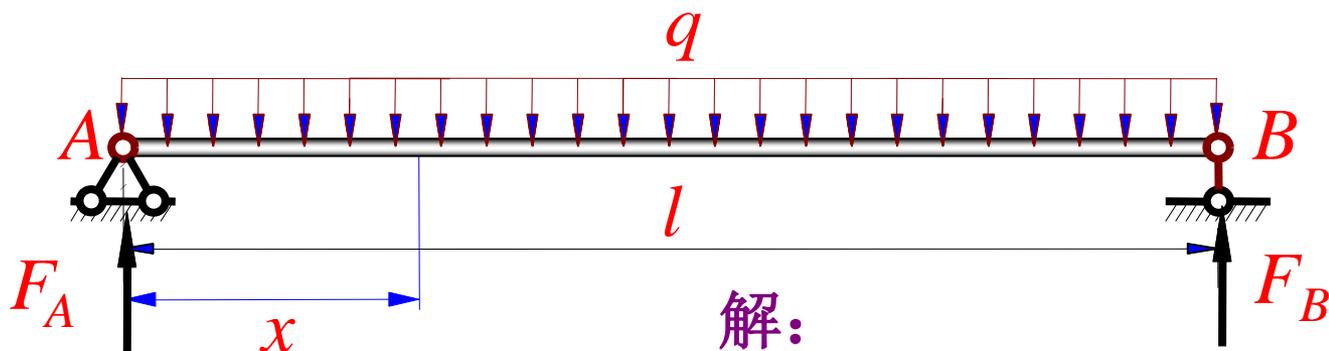
$$\begin{aligned} M(x) &= F_{AY}x - M_A \\ &= F(x - L) \quad (0 < x \leq l) \end{aligned}$$

#### 3、画内力图

## § 3 剪力与弯矩 内力图

### 例 3

画出图中梁的内力图



解:

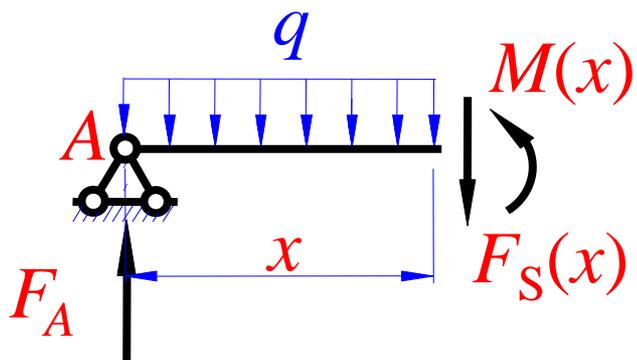
1、求支反力

$$F_A = F_B = \frac{ql}{2}$$

2、内力方程

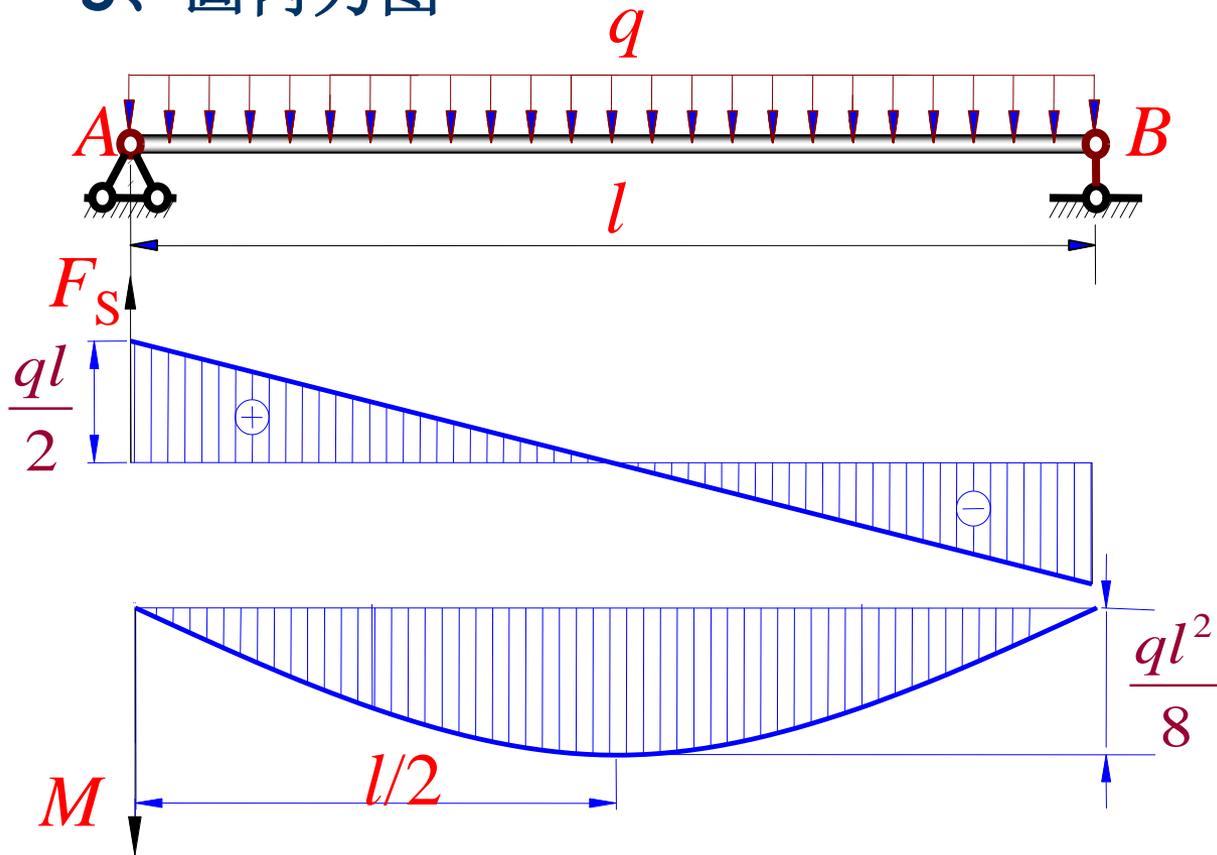
$$F_S(x) = F_A - qx = \frac{ql}{2} - qx$$

$$M(x) = F_A x - qx \times \frac{x}{2} = \frac{qlx}{2} - \frac{qx^2}{2}$$



## § 3 剪力与弯矩 内力图

### 3、画内力图



$$F_S(x) = \frac{ql}{2} - qx$$
$$M(x) = \frac{qlx}{2} - \frac{qx^2}{2}$$

$$|F_{S,\max}| = \frac{ql}{2}$$

$$M_{\max} = \frac{ql^2}{8}$$

\*结构对称、载荷对称, 则剪力图反对称, 弯矩图对称

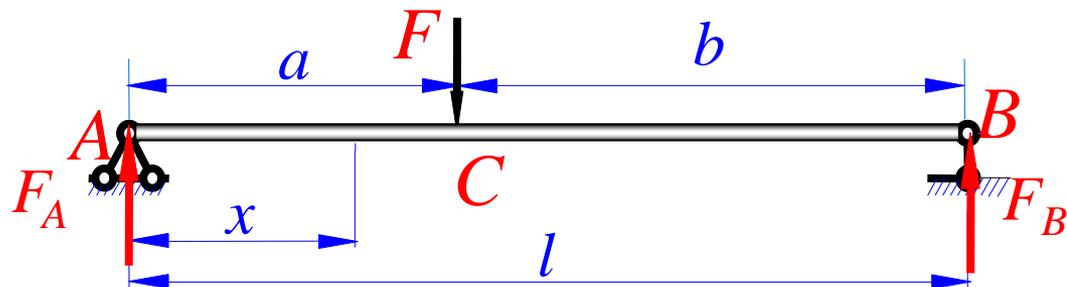
\*剪力为零的截面弯矩有极值。

## § 3 剪力与弯矩 内力图

**例 4** 画出图中所示梁的内力图

**解： 1、支反力**

$$F_A = \frac{Fb}{l} \quad F_B = \frac{Fa}{l}$$



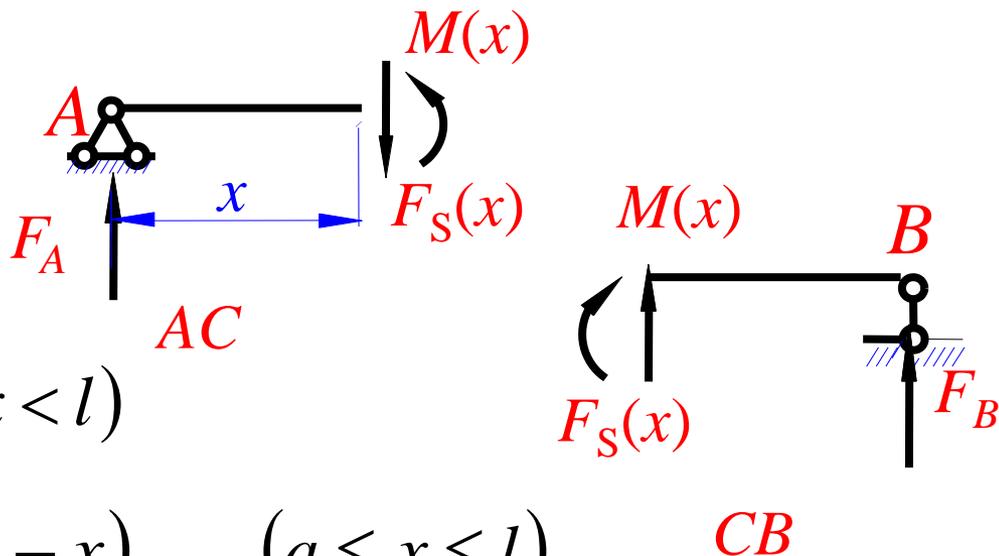
**2、内力方程**

$$F_S(x) = \frac{Fb}{l} \quad (0 < x < a)$$

$$M(x) = \frac{Fb}{l} x \quad (0 \leq x \leq a)$$

$$F_S(x) = -F_B = -\frac{Fa}{l} \quad (a < x < l)$$

$$M(x) = F_B(l - x) = \frac{Fa}{l}(l - x) \quad (a \leq x \leq l)$$



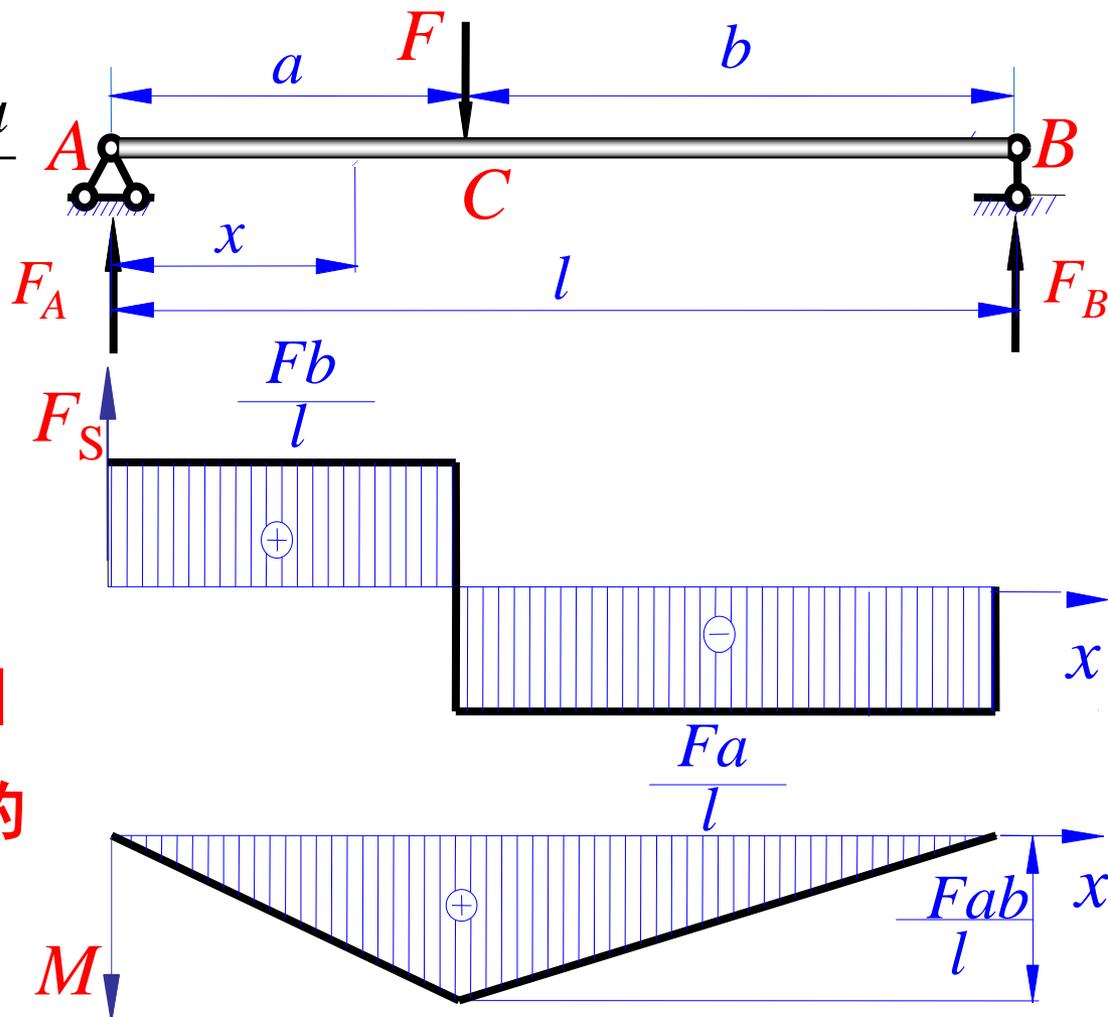
## § 3 剪力与弯矩 内力图

### 3、内力图

$$F_{S1}(x) = \frac{Fb}{l} \quad F_{S2}(x) = -\frac{Fa}{l}$$

$$M_1(x) = \frac{Fb}{l}x$$

$$M_2(x) = \frac{Fa}{l}(l-x)$$



**\*在集中力F作用处，剪力图有突变，突变值为集中力的大小；弯矩图有转折。**

**\*  $M_{\max} = \frac{Fl}{4}$ , at  $a = b = l/2$**

## § 3 剪力与弯矩 内力图

### 例 5

画出图中所示梁的内力图

解：

#### 1、支反力

$$\sum M_B = 0 \quad M_e - F_A \times l = 0$$

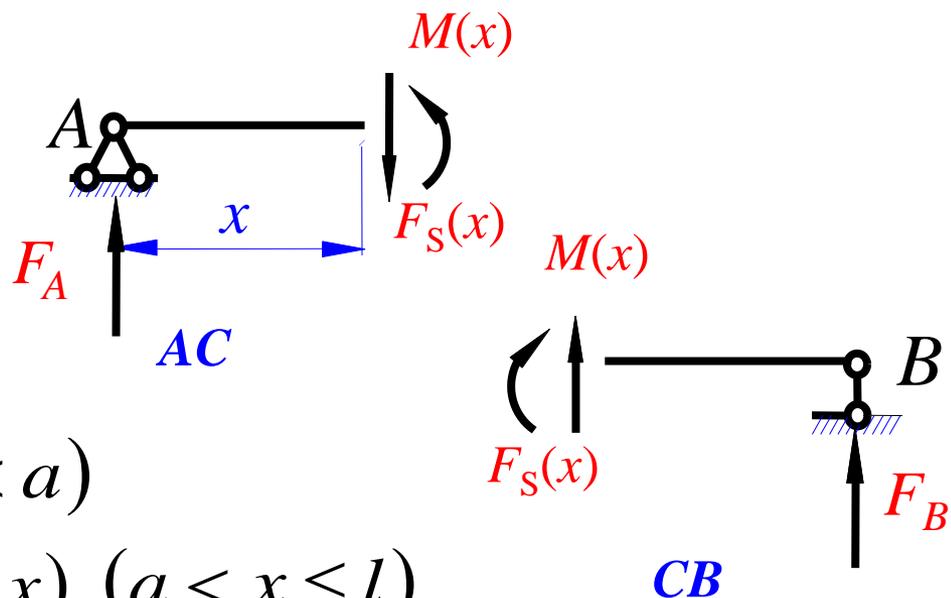
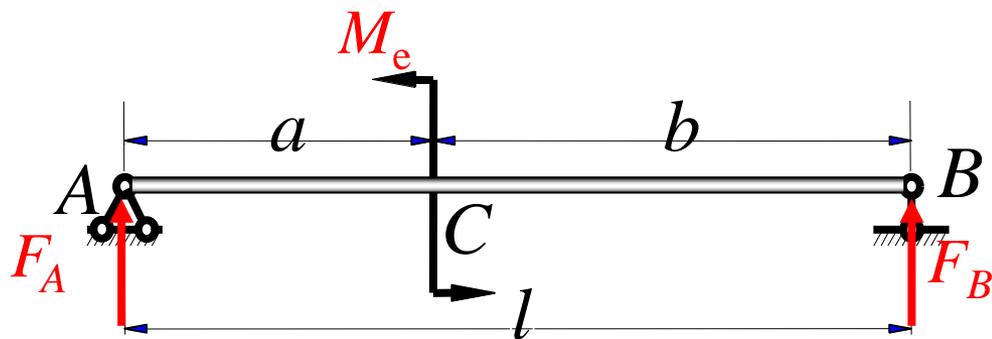
$$F_A = \frac{M_e}{l} (\uparrow) \quad F_B = \frac{M_e}{l} (\downarrow)$$

#### 2、内力方程

$$F_S(x) = F_A = \frac{M_e}{l} \quad (0 < x < l)$$

$$M(x) = F_A x = \frac{M_e}{l} x \quad (0 \leq x < a)$$

$$M(x) = F_A x - M_e = -\frac{M_e}{l} (l - x) \quad (a < x \leq l)$$



## § 3 剪力与弯矩 内力图

### 3、内力图

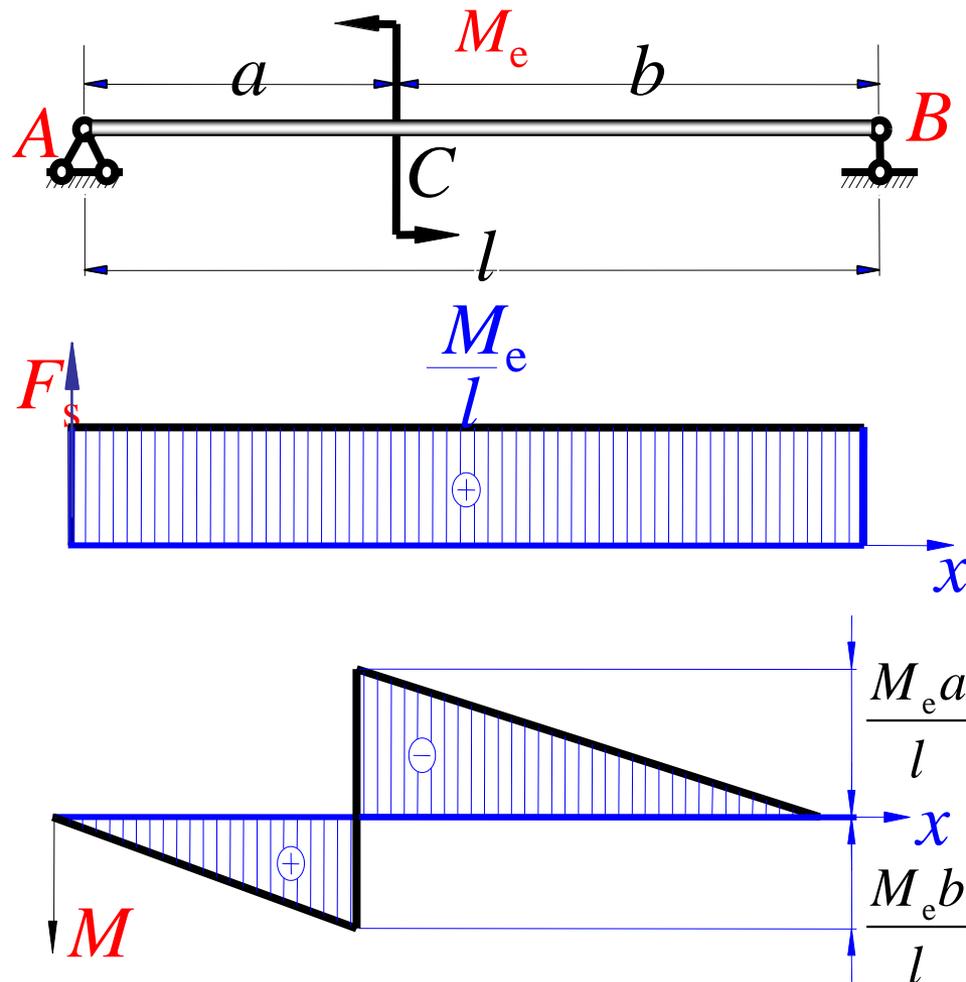
$$F_s(x) = \frac{M_e}{l}$$

$$M(x) = \frac{M_e}{l} x \quad (0 \leq x < a)$$

$$M(x) = -\frac{M_e}{l} (l - x) \quad (a < x \leq l)$$

$$b > a \text{ 时 } |M_{\max}| = \frac{M_e b}{l}$$

发生在C截面右侧。



集中力偶作用点处剪力图无影响，弯矩图有突变，突变值的大小等于集中力偶的大小。

## § 3 剪力与弯矩 内力图

### 例 6

画出图中梁的内力图

解:

#### 1、支反力

$$\sum Y = 0, \quad F_{AY} + F_{BY} - 2 - 1 \times 2 = 0$$

$$\sum M_B = 0, \quad 1 \times 2 \times 1 + 2 \times 3 - F_{AY} \times 4 = 0$$

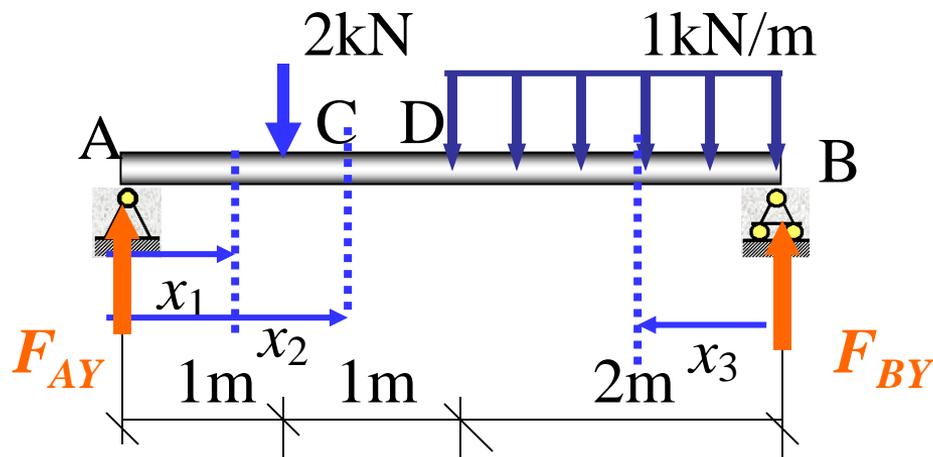
$$\therefore F_{AY} = 2(\text{kN}); \quad F_{BY} = 2(\text{kN})$$

#### 2. 内力方程

AC:

$$F_s(x_1) = F_{AY} = 2(\text{kN})$$

$$M(x_1) = F_{AY} x_1 = 2x_1(\text{kN}\cdot\text{m})$$



$$F_s(x_2) = F_{AY} - 2 = 2 - 2 = 0,$$

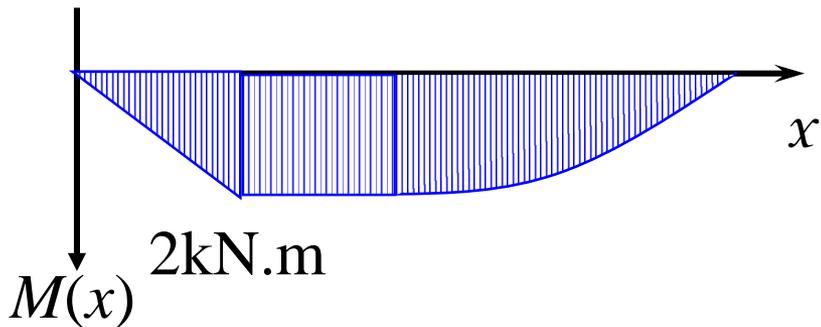
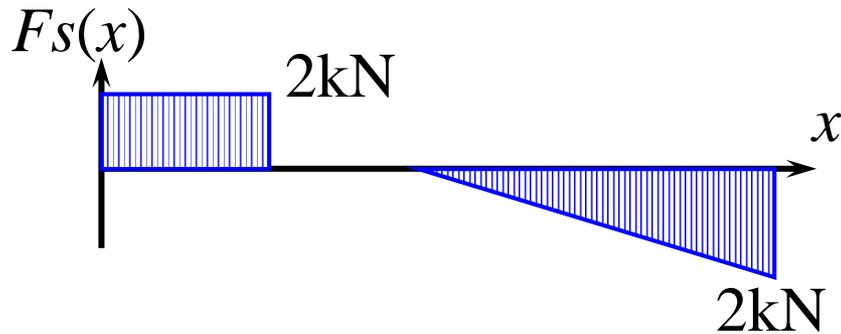
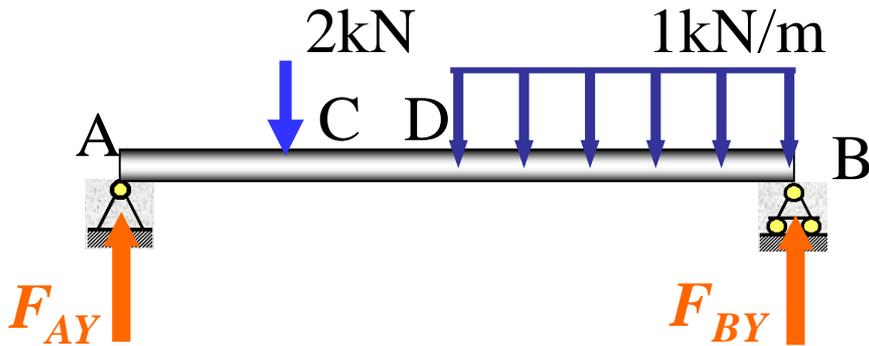
$$M(x_2) = F_{AY} x_2 - 2(x_2 - 1) = 2(\text{kN}\cdot\text{m})$$

BD:

$$F_s(x_3) = -F_{BY} + 1 \times x_3 = -2 + x_3$$

$$M(x_3) = F_{BY} x_3 - 1 \times x_3 \times \frac{x_3}{2} = 2x_3 - \frac{x_3^2}{2}$$

## § 3 剪力与弯矩 内力图



$$AC : F_s(x_1) = 2, \quad (0 < x_1 < 1)$$

$$M(x_1) = 2x_1, \quad (0 \leq x_1 \leq 1)$$

$$CD : F_s(x_2) = 0, \quad (1 < x_2 < 2)$$

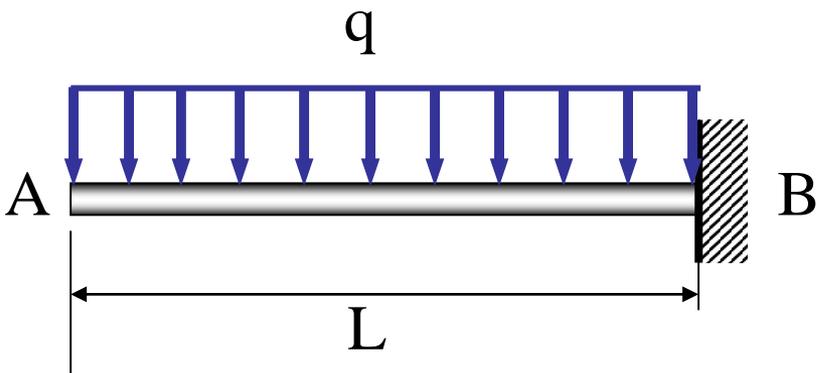
$$M(x_2) = 2, \quad (1 \leq x_2 \leq 2)$$

$$BD : F_s(x_3) = -2 + x_3, \quad (0 < x_3 \leq 2)$$

$$M(x_3) = 2x_3 - \frac{x_3^2}{2}, \quad (0 \leq x_3 \leq 2)$$

### 3、内力图

## § 4 剪力、弯矩和分布荷载集度间的关系

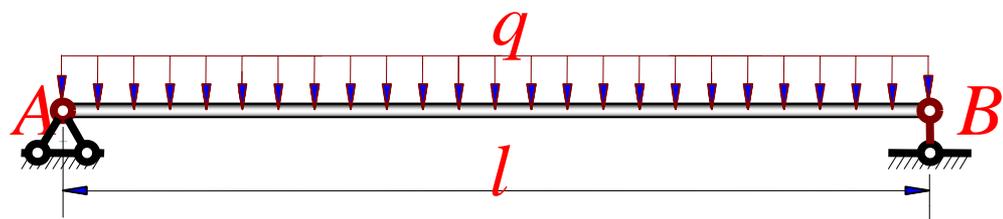


$$F_s(x) = -qx, \quad (0 \leq x < l)$$

$$M(x) = -\frac{1}{2}qx^2, \quad (0 \leq x < l)$$

$$\frac{dF_s(x)}{dx} = -q = q(x)$$

$$\frac{dM(x)}{dx} = -qx = F_s(x),$$



$$F_s(x) = F_A - qx = \frac{ql}{2} - qx$$

$$M(x) = F_A x - qx \times \frac{x}{2} = \frac{qlx}{2} - \frac{qx^2}{2}$$

$$\frac{dF_s(x)}{dx} = -q = q(x)$$

$$\frac{dM(x)}{dx} = \frac{1}{2}ql - qx = F_s(x),$$

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